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# CONICAL LENS FOR 5"/54 GUN LAUNCHED MISSILE

James M. Terrell

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# THESIS

## CONICAL LENS FOR 5"/54 GUN LAUNCHED MISSILE

by

James M. Terrell

June 1981

Thesis Advisor: Allen E. Fuhs

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Under the sponsorship of the Defense Advanced Research Projects Agency (DARPA), a conical lens for a 5"/54 ramjet propelled, optically guided projectile was investigated. The resulting conical lens for the gun-launched missile (GLM) will focus parallel incident light through the lens to a design focal point. A conical lens was designed using an algorithm written for the HP-9845T desktop minicomputer. The examples

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illustrate the automated design procedure, selection of possible lenses and final lens design. Recommendations for further research are discussed.

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# Conical Lens for 5"/54 Gun Launched Missile

by

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Lieutenant, United States Navy
B.S., University of Oklahoma, 1975

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN PHYSICS

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Under the sponsorship of the Defense Advanced Research Projects Agency (DARPA), a conical lens for a 5"/54 ramjet propelled, optically guided projectile was investigated. The resulting conical lens for the gun-launched missile (GLM) will focus parallel incident light through the lens to a design focal point. A conical lens was designed using an algorithm written for the HP-9845T desktop minicomputer. The examples illustrate the automated design procedure, selection of possible lenses and final lens design. Recommendations for further research are discussed.

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## LIST OF SYMBOLS

| Symbol | Explanation | Units |
|---|---|---|
| A | GLM (x-axis) intercept | Inches |
| F | Focal point on the GLM axis | Inches |
| NORM | Acute angle of the normal to the surface measured with respect to the GLM axis | Degrees, radians |
| O | Origin; located at the vertex of the GLM conical lens | Inches |
| OP | Length of the line segment connecting point O and point P | Inches |
| P | Focal point on the GLM axis intersection of the light ray and the radius vector in the GRIN lens | Inches |
| Q | Point of intersection of the light ray and the first surface | |
| QA | Length of the line segment connecting point Q and point R | Inches |
| QP | Length of the line segment connecting point Q and point P | Inches |
| r | Radial distance from the origin | Inches |
| Rho-initial | Angle the incident ray in medium 1 makes with the GLM axis | Degrees, radians |
| (REF) | Reference direction defined as the GLM axis | |
| RT | Length of the line segment connecting point R and point T | Inches |

| Symbol | Explanation | Units |
|---|---|---|
| RO-max | Radius from the origin at which the index of refraction is maximum | Inches |
| T | Point of refraction of the ray in the lens | |
| Theta (critical) | Angle of incidence $\theta_I$ at which $\sin(\theta_R)=1$ | Degrees, radians |
| Z1 | Point at which the lens intercepts the GLM axis | Inches |
| Z3 | Intercept of the first, movable, image plane and the GLM axis | Inches |
| Z4 | Intercept of the second, fixed, image plane and the GLM axis | Inches |
| $n_1$ | Index of refraction of medium 1 | |
| $n_2$ | Index of refraction of medium 2 | |
| $n_3$ | Index of refraction of medium 3 | |
| $n=f(r)$ | Index of refraction is a function of the radius from the origin | |
| $\hat{n}_i$ | Normal direction to a surface | |
| $n_{23}$ | Ratio of $n_2/n_3$ | |
| s | Distance along the ray in the lens | Inches |
| x | Distance along the GLM axis | Inches |

| Symbol | Explanation | Units |
|---|---|---|
| $y$ | Radial distance from the GLM axis | Inches |
| $y_b$ | Upper bound of the aperture | Inches |
| $y_a$ | Lower bound of the aperture | Inches |
| $(x_i, y_i)$ | Coordinate values of the intercept of the ray and a refracting surface or an image plane | Inches |
| $(x_c, y_c)$ | Coordinate values of the intercept of the ray and the GLM axis | Inches |
| $x=f(y)$ | Distance along the GLM axis is a function of the radial distance from the GLM axis | |
| $y=f(x)$ | Radial distance from the GLM axis is a function of the distance along the GLM axis | |
| $\|x\|$ | Absolute value of x | Units of x |
| $f$ | Focal length | Inches |
| $l$ | Length of line segment QP | Inches |
| $\alpha$ | Cone half-angle | Degrees,<br>radians |
| $\alpha_2$ | $\tan(\alpha_2)$ = slope of the second surface | Degrees,<br>radians |
| $\beta$ | Angle formed by QP with respect to the GLM axis | Degrees,<br>radians |
| $\gamma$ | Complementary angle of $\theta_{R_4}$ | Degrees,<br>radians |
| $\delta$ | Angular difference between two surface normals | Degrees,<br>radians |

| Symbol | Explanation | Units |
|---|---|---|
| $\theta^*$ | Angle between two successive rays in GRIN | Degrees, radians |
| $\theta^*$ | General angle | Degrees, radians |
| $\theta_I{}^*$ | Angle of incidence with respect to the local normal | Degrees, radians |
| $\theta_R{}^*$ | Angle of refraction with respect to the local normal | Degrees, radians |
| $\pi$ | Number of radians in semi-circle | Radians |
| $\rho_I$, $\rho_0$ | Angle of incident rays in medium 1 with respect to the GLM axis | Degrees, radians |
| $\rho_1{}^*$ | Angle of the ray in the lens with respect to the GLM axis | Degrees, radians |
| $\rho_2$ | Angle of the ray in medium 3 with respect to the GLM axis | Degrees. radians |
| $\phi$ | Angle between the radius vector and the refracted ray in the GRIN lens | Degrees, radians |
| 1 | Surface number | |
| | Step of a sequence | |
| (32.5,45.6) | (x,y) coordinate position | Inches |

Subscripts not already shown:

| | | |
|---|---|---|
| 1,2,3,... | Point number | |
| | Iteration number | |
| i | Iteration number | |
| | Surface number | |

| Symbol | Explanation | Units |
|---|---|---|
| n | Iteration number | |
| max | Maximum value of the symbol | |
| next | Next quantity in the sequence | |

*: Indicates the symbol can be used with additional subscripts.

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# I. INTRODUCTION

## A. THE PROBLEM

Since World War II the presence of two conflicting requirements in missile sensor design has forced unsatisfactory compromises to be made in either the optical system, airframe or both. Examples of this are the Sidewinder air to air (AAM) and Chapparal surface to air (SAM) missile. The operational requirements are simple: from a "long" range, fly fast enough to pursue and overtake an adversary, destroy or disable the adversary and perform all of the guidance and tracking functions without assistance from the firing platform. The guidance decisions in the Sidewinder AAM are made by tracking the thermal radiation emitted from enemy aircraft. The radiation enters the missile optical system via a hemispherical dome on the nose of the missile.

Long range and high speed require sleek, low-drag airframes and efficient, high power engines. In the visible or infrared, accurate measurement of the line of sight (LOS) from the missile to the target requires optical lenses which routinely come in the shape of spheres. Spheres and similar blunt shapes have very high drag coefficients compared to conical or ogive nose shapes. Thus the Sidewinder and Chapparal missiles contain excellent optics, yet suffer in range and operational capability.

The targets expected to be encountered in surface AAW are the Soviet anti-ship missiles (ASM). These weapons have evolved from simple straight line weapons to those with large warheads, multiple seekers and flight profiles which sometimes require operational tracking systems to perform beyond their design limits.

The fleet AAW battle doctrine of today is based on a "defense in depth" concept. According to this concept the E-2C/F-14/Phoenix combat system will be the first unit to engage enemy ASM, hopefully at maximum range. The next units able to engage the ASM at extended range, assuming the target is above the radar horizon, are the units employing the SM-2 (ER) and SM-1 (ER) missiles. While within the SM-2/1 (ER) engagement envelope, the SM-1 (MR) missile enters the engagement at medium range. Finally, once the ASM has penetrated this missile cover, the defense in depth concept reduces to a "defend each unit, especially the carrier" concept. In this region the 5"/54 gun system, Basic Point Defense System, NATO Sea Sparrow and Phalanx system are brought to bear.

The battle group could conceivably consist of an aircraft carrier, three Aegis cruisers, two SM-1 (ER) cruisers, two guided missile destroyers, two guided missile frigates, two fast frigates and three destroyers; or a total of fifteen units not including logistics ships. The capital assets for the ships, aircraft and equipment are in the tens of billions of dollars and perhaps 10,000 men. These assets demand

protection. However, with three exceptions the AAW combat systems presently in the fleet, or near operational status are not designed to engage a low-flying ASM in a timely manner. The exceptions are the Aegis cruiser, NATO Sea Sparrow system and Phalanx system; even with these three systems, there exists a gap in the 1-10 nautical mile range in the defense in depth concept.

A possible attack scenario is with the battle group steaming in an AAW formation when the Soviet commander launches a series of three saturation raids of ASM's against the battle group. The second and third raid are launched to arrive at the moment when the U.S. forces are totally engaged with the previous raid. Hence, the second and third raid will substantially penetrate the battle group, with perhaps the carrier and several of the major escorts at least mission disabled. The Soviet commander can now operate at will.

Two factors are critical to the battle group commander: munition assets and engagement time. Compared to the number of threats in a saturation attack, the battle group's missile assets are limited in number with limited reload capability. The battle group commander's gun ammunition assets are large, compared to the available missiles. However, with the current gun systems the engagement time per round is extremely long, on the order of 60 seconds. If the gun engagement time could be reduced by a factor of 2, then each gun barrel could be twice as effective.

Unguided, ballastic projectiles can be replaced by gun launched missiles (GLM). The GLM flying at Mach 3.0 can intercept in 15 seconds an inbound ASM flying at Mach 1.0 at a range of 20,000 yards. This engagement time and range are superior to any close-in system presently in the fleet. Therefore the GLM enhances the battle group's capability to survive a raid which saturates the group's long range combat systems. The GLM is not designed to replace any existing systems but complement them by providing the ability to strike out and defeat the enemy in the region from 1 to 10 nautical miles. In this range interval, defense capability presently exists; however, the magazine capacity is limited. The GLM will accomplish the ASMD task by marrying existing ramjet technology and a new generation of missile optics to provide a missile which is able to defeat the Soviet ASM at a range far greater than is realizable with today's gun systems.

This thesis involved the writing of and the application of computer algorithms to accomplish the following:

1) Compute and trace monochromatic light rays through a conical lens.
2) Compute a refracting surface to focus monochromatic light rays to a point on the GLM axis.
3) Apply an optimization computer algorithm to the lens design problem.
4) Apply gradient index optics (GRIN) to the lens design.

## B. ASSUMPTIONS AND SIGN CONVENTION

The computer algorithms are written based on Snell's law without approximations. All rays are assumed to be transmitted without loss of energy in the medium. When total internal reflection occurs, the ray is assumed to stop at the point of reflection. The algorithms are written to provide a basic understanding of the optics involved. Flexible computer programs have been written with the capability of adding ray tracing ability by adding subroutines.

The medium ahead of the GLM is assumed to be air. Flying at Mach 3.0 the GLM will produce shock waves which will act like a refracting surface [1]. This thesis assumes the air to be homogenous, isotropic, linear, time independent (HILT) and free of shock waves in order to investigate the fundamental problem of designing a conical lens. The lens medium and the medium after the lens are assumed to be composed of an HILT material unless otherwise noted.

The sign convention used is a right-handed system with distances positive to the right and up as seen by the reader. All angles are positive for a counterclockwise rotation from the horizontal axis to the point in question. When used in derivation of geometrical relationships, the absolute values of angles are used in all trigonometric expressions.

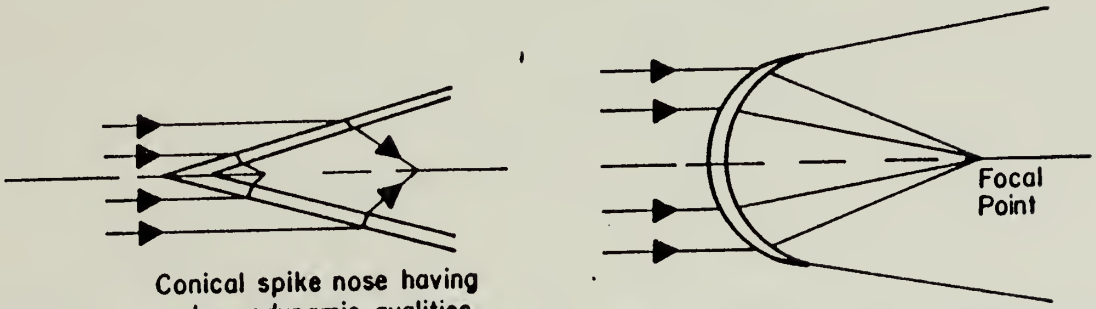
All linear dimensions are inches unless otherwise noted.

## II. FIRST SURFACE

A spiked nose used as a ramjet inlet optimizes the aerodynamic qualities yet degrades the optical qualities. Figure 1 [2] illustrates the inability in the past of a spiked nose to focus light rays while a sperical lens accomplishes an excellent focus. Thus, optimizing the aerodynamic qualities degrades the optical qualities and vice versa.

Replacing the solid propellant rocket motor now used on most missiles with a ramjet engine further complicates the compromise between optical quality and aerodynamic efficiency. Replacing the rocket with an air-breathing ramjet is motivated by fuel economy and the ability to throttle the ramjet. The ability to throttle allows the missile to fly a thrust equal drag trajectory. This type of trajectory results in a vacuum-like trajectory with less sensitivity to wind and a marked increase in maximum range [3].

Ramjets operate with inlets configured for cruise performance at the design Mach number. The configuration of the inlet shown in figure 2 is based on a design Mach number of 3.0 [3]. The spike of the inlet is a combination of a cone and curved geometry to provide high pressure recovery during supersonic operation [3].



Conical spike nose having good aerodynamic qualities and poor optical qualities.

Blunt nose having poor aerodynamic qualities and good optiaal qualities.

Figure 1. Spiked vs. Blunt Ramjet Inlets. (Reproduced from Ref. 2, p 19)

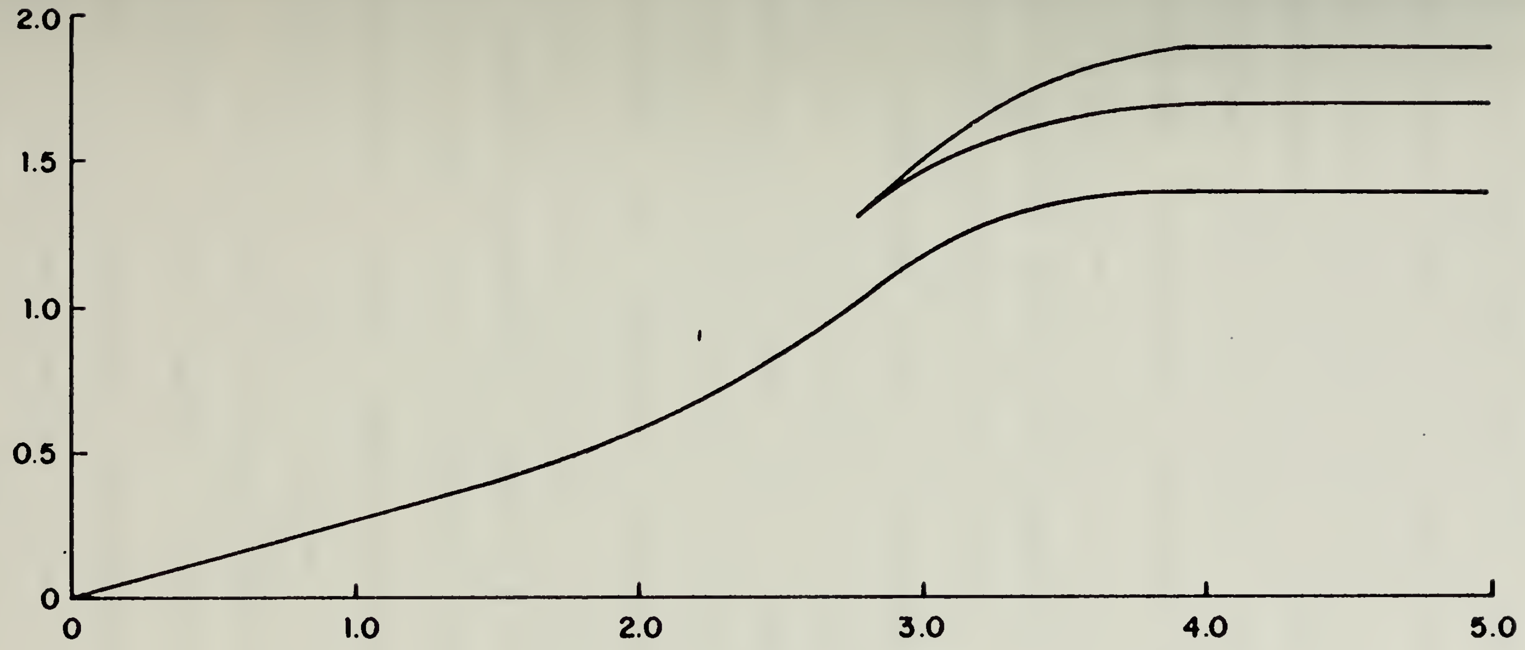


Figure 2. Dimensions of 15° Tip Axisymmetric Isentropic Inlet Designed for Mach 3. (Reproduced from Ref. 3, p 37).

The inlet spike geometry is a complex surface which further complicates the task of the lens system which is to form an image of the target on the GLM axis. The inlet is a fluid mechanical device, called a diffuser, used to decelerate the air flow to a subsonic Mach number. The diffuser converts the kinetic energy of the flow into an increase in pressure of the air, known as pressure recovery, and decelerates the flow to a subsonic Mach number. Diffusers are required because the combustion of fuel is more efficient at high pressure and low velocity.

A desirable inlet is one which decelerates the flow at constant entropy, produces no shocks, creates zero drag exclusive of ram drag, and is insensitive to angle of attack. Inlets which operate off the design Mach number may have additive drag. Additive drag is the sum of the forces acting along the streamline A-B shown in figure 3 [4]. The pressure multiplied by the area of the streamtube A-B is a force which is in a direction opposite to that of flight and therefore is drag. Examples of other types of supersonic diffusers are shown in figures 4 and 5 [4].

The marriage of missile optics and a controllable engine promises a potent weapon. Preliminary feasibility studies have shown that ramjet engines, control systems, guidance laws and warhead designs can be integrated into a 5"/54 projectile [2,3,5,6]. The optical system has been identified as a major subsystem which requires additional investigation.

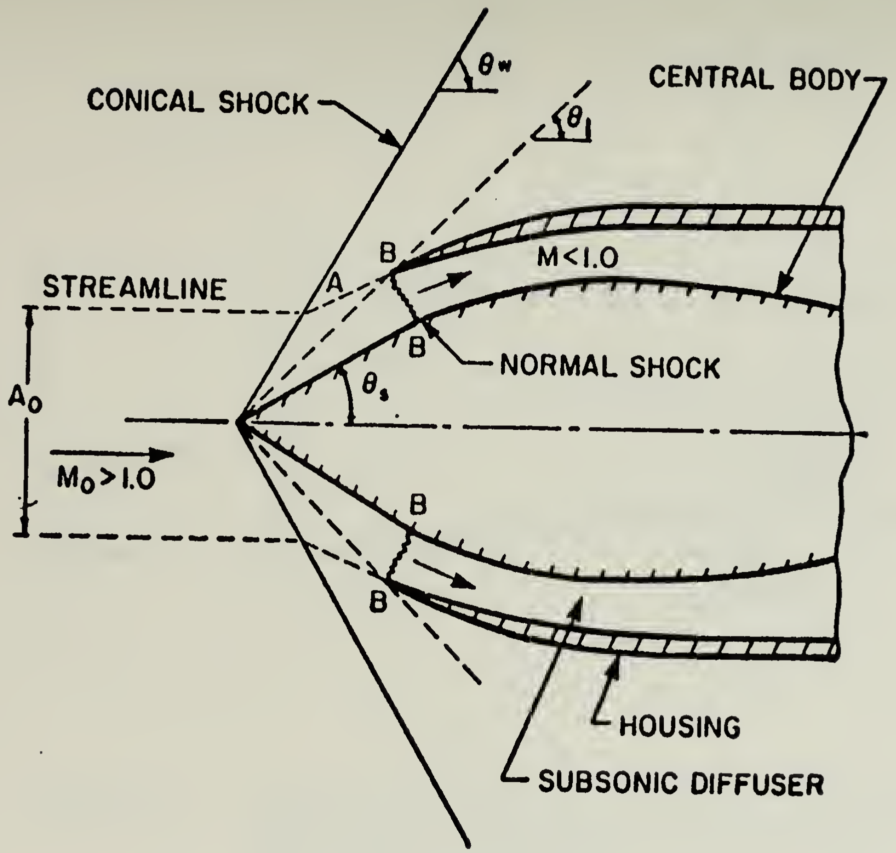


Figure 3. Schematic Diagram of a Conical Shock (Oswatitsch Type) Supersonic Diffuser. (Reproduced from Ref. 4, p. 245)

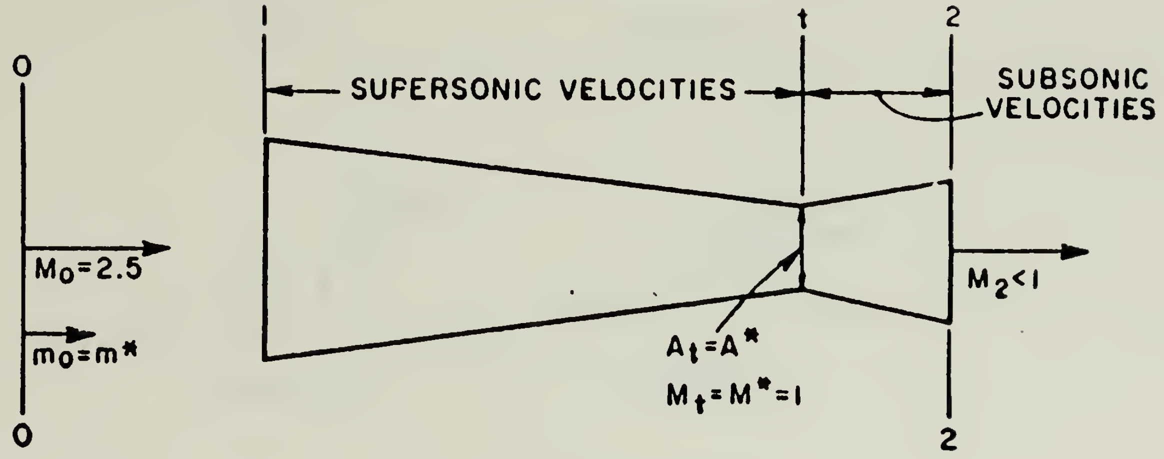
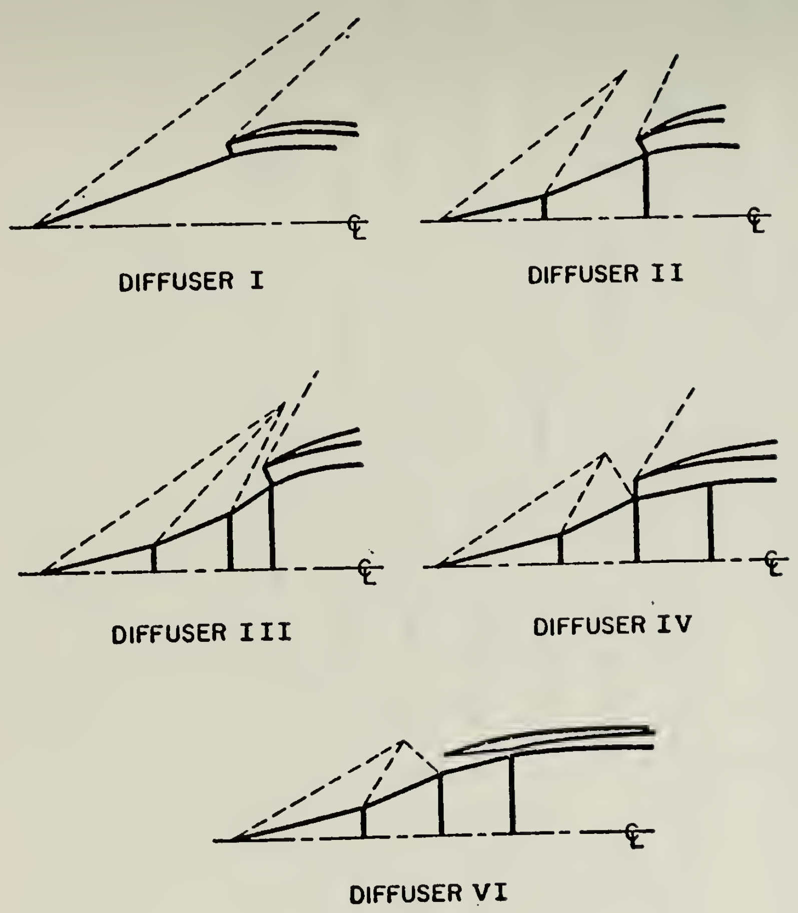


Figure 4. Scheme of Using a REVERSED Converging-Diverging Isentropic Nozzle as a Supersonic Diffuser. (Reproduced from Ref. 4, p. 239)



DIFFUSER I

DIFFUSER II

DIFFUSER III

DIFFUSER IV

DIFFUSER VI

Figure 5. Different Conical Shock Supersonic Diffuser Configurations Tested by Oswatitsch. (Reproduced from Ref. 4, p. 247)

Specifically, the lens, which is required to focus the image of the target, is the stumbling block to an efficient ramjet design. A blunt nose cannot be used because of poor pressure recovery and large additive drag. Therefore, some type of an inlet using a spike must be used. The spike must also be a lens capable of forming an image on the GLM axis. The lens must have a large enough aperture and field of view to provide the GLM with sufficient data to maintain and accurately measure the line of sight.

This thesis is based on an inlet design by Brown [3] using data from Faro [7] shown in figure 2. The inlet is a $15^{\circ}$ conical tip, axisymmetric, isentropic spike for a design Mach number of 3.0. The spike studied in the thesis is a cone having approximately the same length-to-diameter ratio as the inlet in figure 2 [3]. Hence, the first optical surface used for ray tracing is a cone with a $21^{\circ}$ half-angle.

## III. SECOND SURFACE GENERATION: DIRECT SOLUTION

The direct solution to designing a conical lens consists of solving Snell's Law at a point T shown in figures 6 and 7 along the refracted ray within the lens. Point T is located below the line QP in figure 6. When T is in this position relative to QP, the geometry is referred to as the low region. When T is in the position shown in figure 7, the geometry is referred to as the high region. When T is on the line QP, i.e. $\rho_1 = \beta$, the low region geometry is assumed.

The following discussion is the derivation of expressions for $\rho_2$, the angle with respect to the GLM axis of the ray refracted at T which passes through the design focal point P, and $\alpha_2$, the angle which the tangent line at T makes with respect to the GLM axis. The derivation will only discuss the low region, because the procedure for the high region is identical and only the results for the high region will be given. The magnitude of a line segment is shown by stating the endpoints of the line segment in uppercase letters or by a single lower case letter.

Let $QP=\ell$, $QT=s$ and $OP=f$; then according to the law of sines $\sin\alpha/\ell = (\sin(\pi-[\alpha+|\beta|]))/f$. This relation simplifies to

$$\ell = \frac{f \sin\alpha}{\sin(\alpha+|\beta|)} \qquad (1)$$

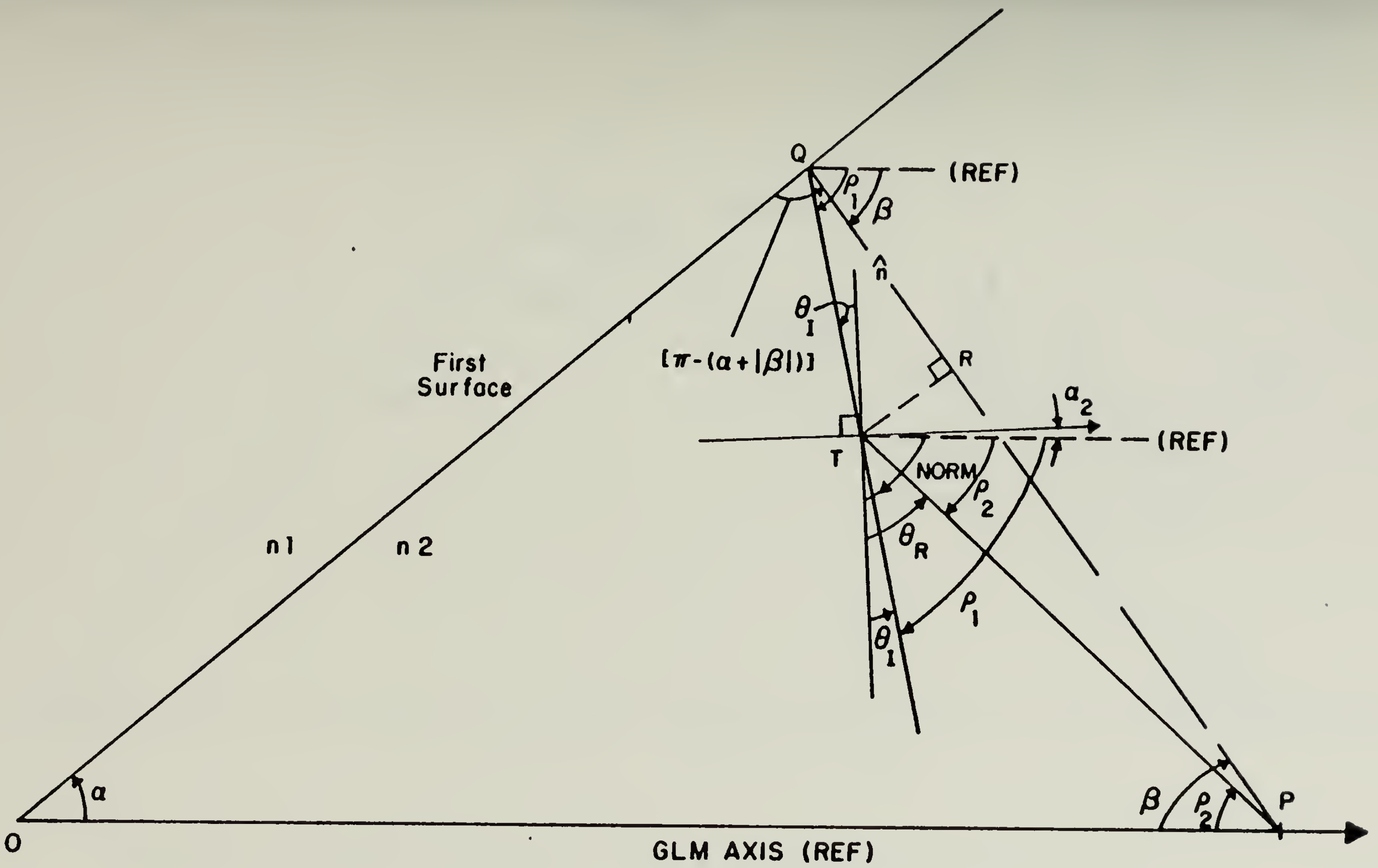


Figure 6. Direct Solution: Low Region Geometry for $n_2 > n_3$.

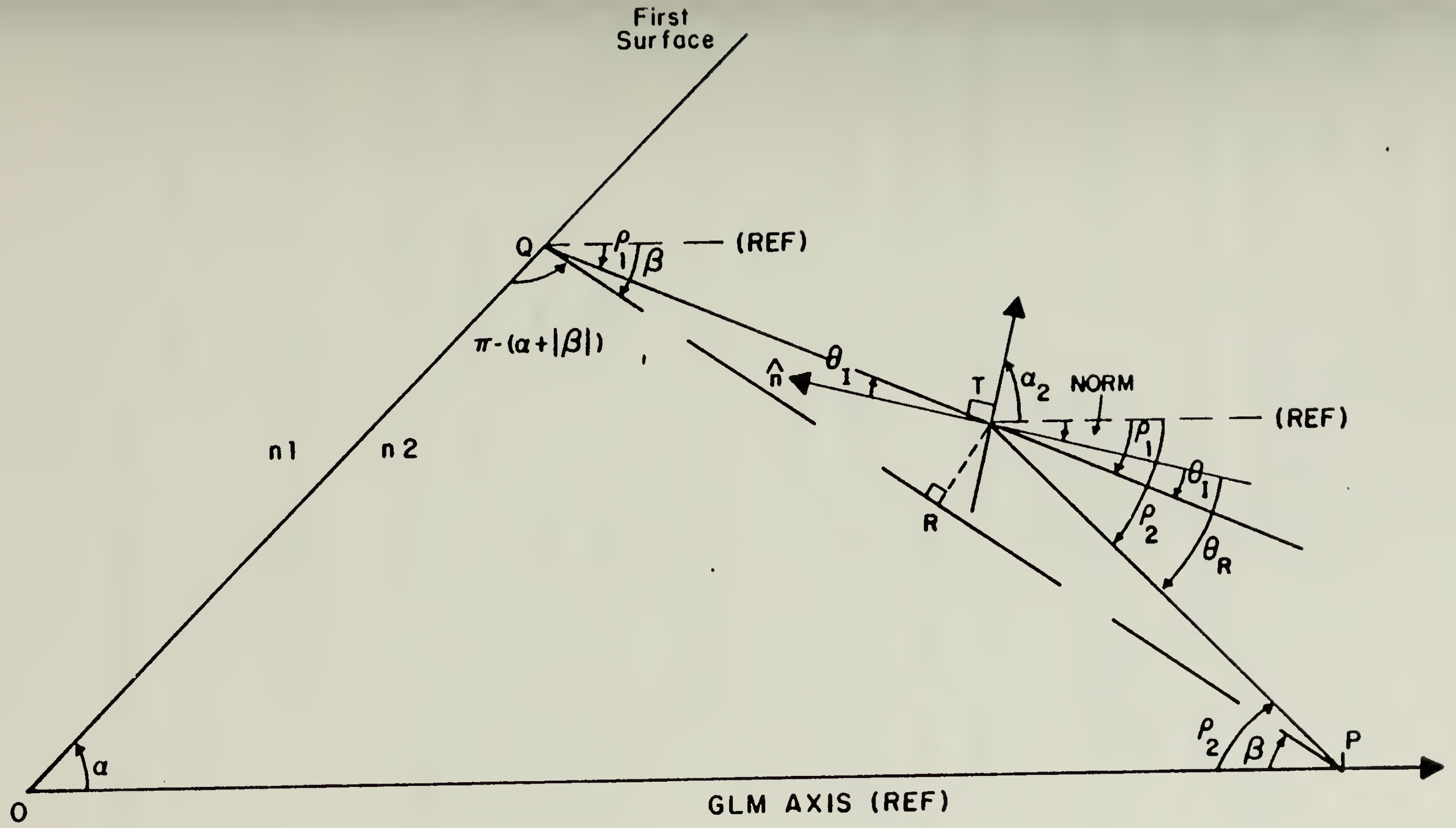


Figure 7. Direct Solution: High Region Geometry for $n_2>n_3$.

Next, note that

$$RT = s \sin(|\rho_1| - |\beta|) \tag{2}$$

and

$$RT = (PQ-QR)\tan(|\beta|-|\rho_2|) \tag{3}$$

Now $PQ-QR=\ell-s\cos(|\rho_1|-|\beta_1|)$. Substituting this into equation (3) and solving for $\tan(|\beta|-|\rho_2|)$ yields

$$\tan(|\beta|-|\rho_2|) = \left\{ \frac{s \sin(|\rho_1|-|\beta|)}{\frac{f \sin\alpha}{\sin(\alpha+|\beta|)} - s\cos(|\rho_1|-|\beta|)} \right\} \tag{4}$$

Consequently the formula for $|\rho_2|$ becomes $|\beta|$ - arctan{}. Using equation (4), $\rho_1$, the angle with respect to the GLM axis of the ray refracted at Q, apply Snell's Law at T. Thus, referring to figure 6

$$\sin\theta_R = \frac{n_2}{n_3} \sin\theta_I = n_{23}\sin\theta_I \tag{5}$$

where $\theta_R = |NORM|-|\rho_2|$ and $\theta_I = |NORM|-|\rho_1|$. Substituting these expressions into equation (5) yields a relation involving the sine of two angles. Expanding the sine expressions and solving for $\tan|NORM|$ yields

$$\tan|NORM| = \left\{ \frac{\sin|\rho_2| - n_{23}\sin|\rho_1|}{\cos|\rho_2| - n_{23}\cos|\rho_1|} \right\} \tag{6}$$

and $|NORM|$ = arctan{ }. Equation (6) must be greater than or equal to zero for the expression to be valid. Then

$$\alpha_2 = \frac{\pi}{2} - |NORM| \tag{7}$$

The result of the derivation for the high region yields

$$\tan(|\rho_2|-|\beta|) = \left\{ \frac{s \sin(|\beta|-|\rho_1|)}{\dfrac{f \sin\alpha}{\sin(\alpha+|\beta|)} - s \cos(|\beta|-|\rho_1|)} \right\} \qquad (8)$$

$$\tan|NORM| = \frac{n_{23} \sin|\rho_1| - \sin|\rho_2|}{n_{23} \cos|\rho_1| - \cos|\rho_2|} \qquad (9)$$

and solving for $\alpha_2$ gives

$$\alpha_2 = \frac{\pi}{2} - |NORM| \qquad (10)$$

Therefore, $\alpha_2$ is a complicated relationship involving incident ray position, denoted by $\beta$, cone half angle $\alpha$, design focal point P, the refractive indices $n_1$, $n_2$, and $n_3$ and position $s$ along the ray within the lens. In order for a systematic study to be conducted, the parameters $\alpha$, $f$, $n_1$, $n_2$ and $n_3$ are assigned specific values. The parameter $\beta$ is determined by the position of point Q on the first surface. The angle of $\rho_1$, is determined by the incident ray angle with respect to the first surface, $n_1$, and $n_2$. Therefore, with the values for the parameters, either fixed or calculated, a design chart relating $\alpha_2$ with the position along the ray in the lens $s$ shown in figure 8 can be generated. The numbered curves correspond to numbered rays entering the lens. The rays are numbered consecutively beginning with the one nearest the GLM axis.

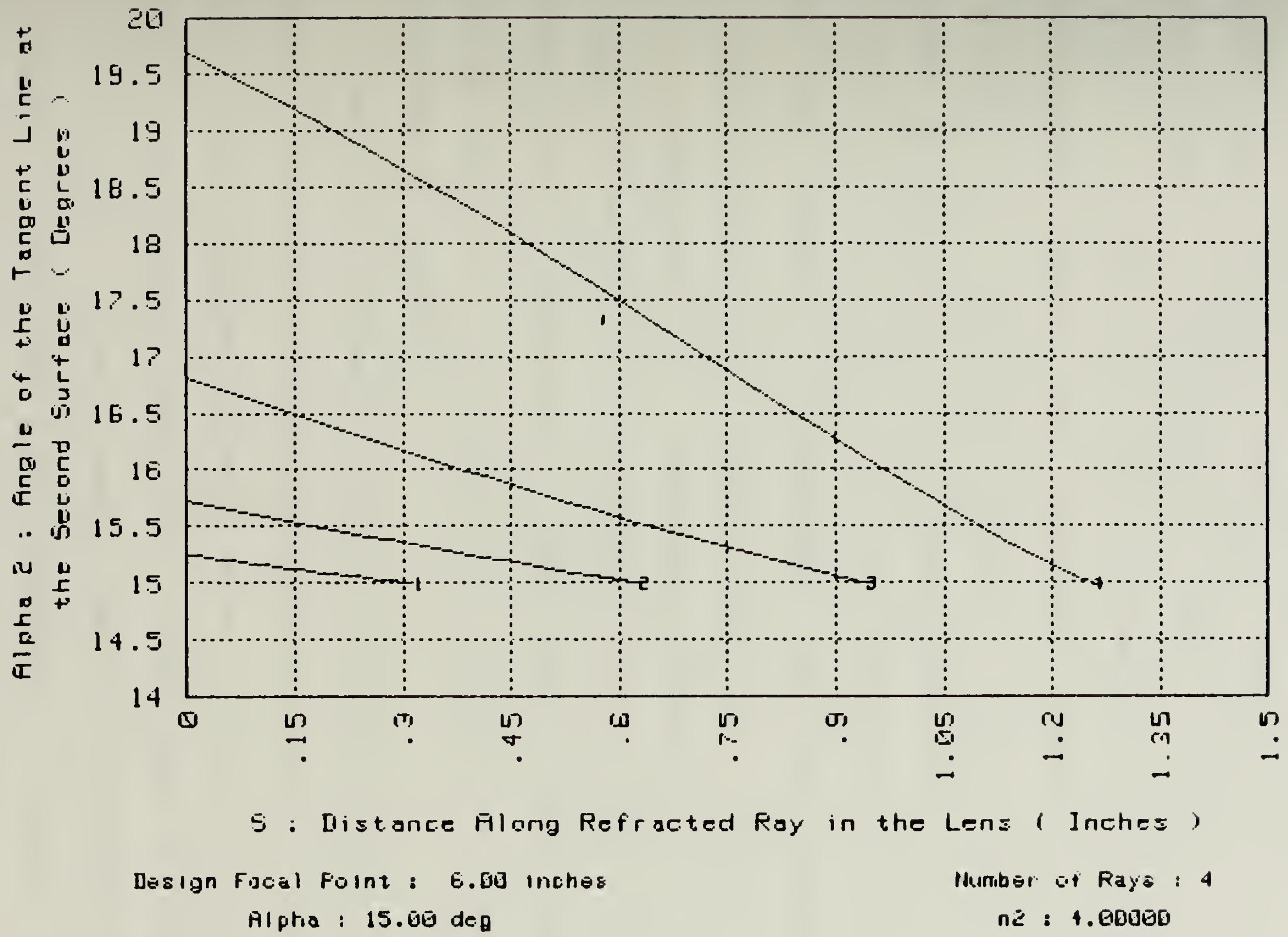


Figure 8. Sample Design Chart for a Conical Lens Design.

The actual design process involves selecting a starting point, T1, on a given ray and noting the magnitude of $s_1$ as shown in figure 9. With that value of $s_1$, enter the design chart in figure 8 and read the value of $\alpha_{2_1}$ from curve number 1. Returning to figure 9 extend a line from T1 using $\alpha_{2_1}$ until that line intercepts the next ray at T2. Measure the value of $s_2$ and enter figure 8 to obtain the value of $\alpha_{2_2}$ from curve number 2. Return to figure 9 and extend a line from T2 to intercept the next ray, etc. This process continues until all of the rays are intercepted or the first surface is intercepted. Once the intercept point T and the angle of the tangent line $\alpha_2$ is known, the ray can be refracted at T. Equations (4) and (7) for the low region and equations (8) and (10) for the high region accomplish the refraction. The result of each refraction by design result in a refracted ray through the focal point P as shown in figure 10.

The design procedure does not produce a single surface which can be used to refract light through the focal point. The surface designed depends on which ray was chosen as the initial ray and the value of $s$ along that ray where T1 is positioned. Thus, a small family of surfaces illustrated in figure 11 can easily be calculated to provide a choice of lens second surfaces to the optical engineer.

The lens designer must choose from the family of second surfaces generated by the design procedure. In order for a

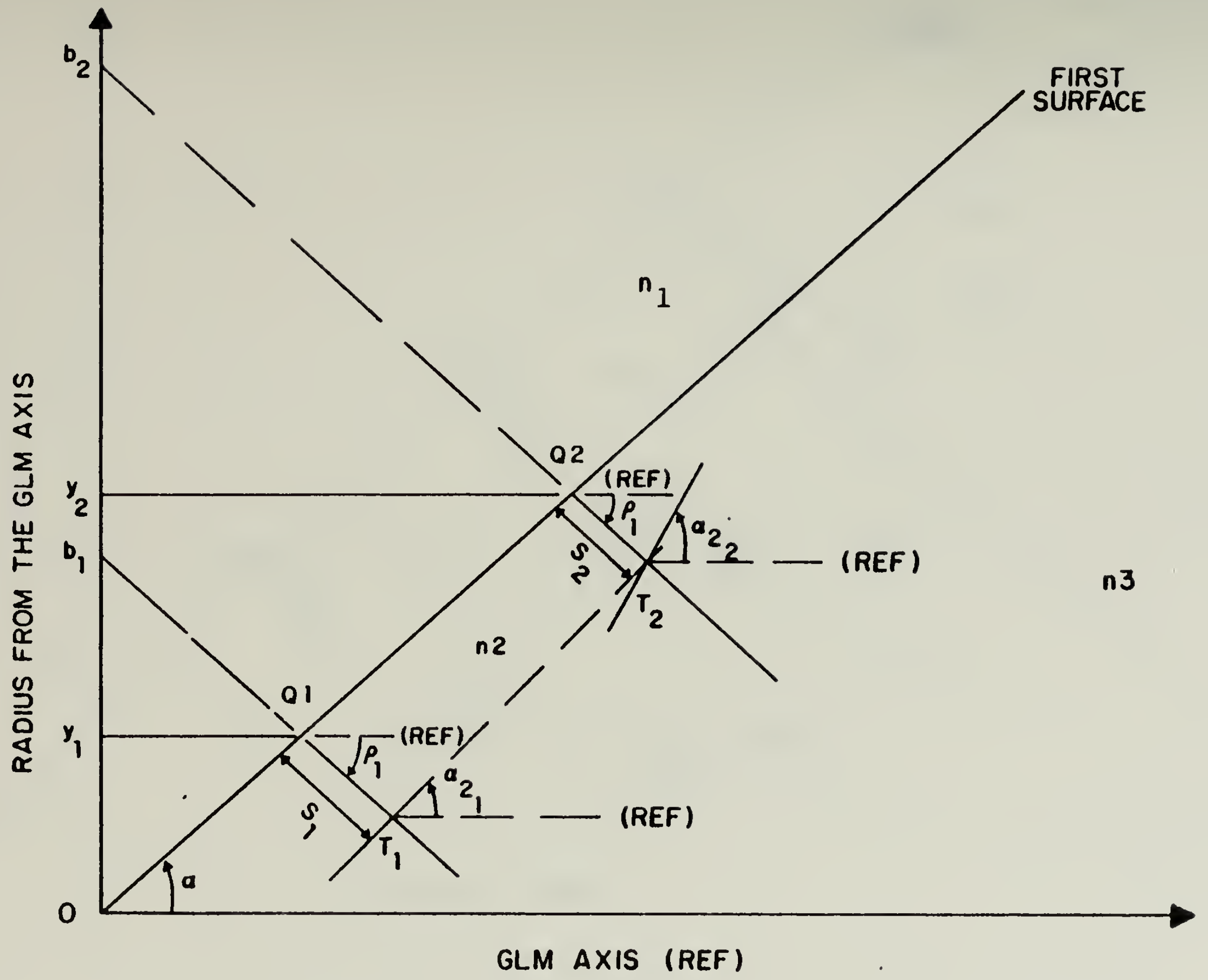


Figure 9. Geometric Relationship for the Calculation of $\alpha_2$ versus s.

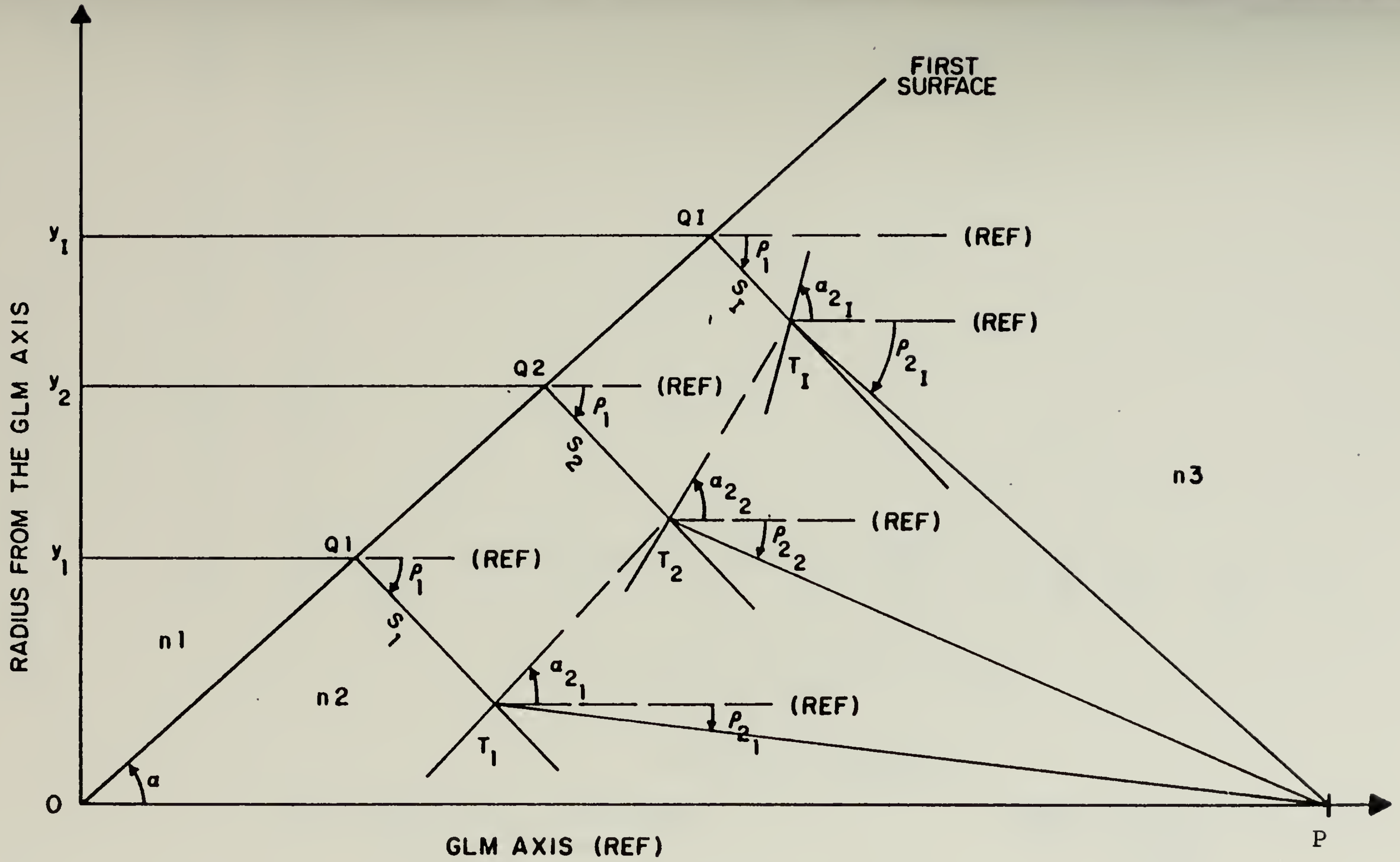


Figure 10. Light Rays Refracted Through the Design Focal Point.

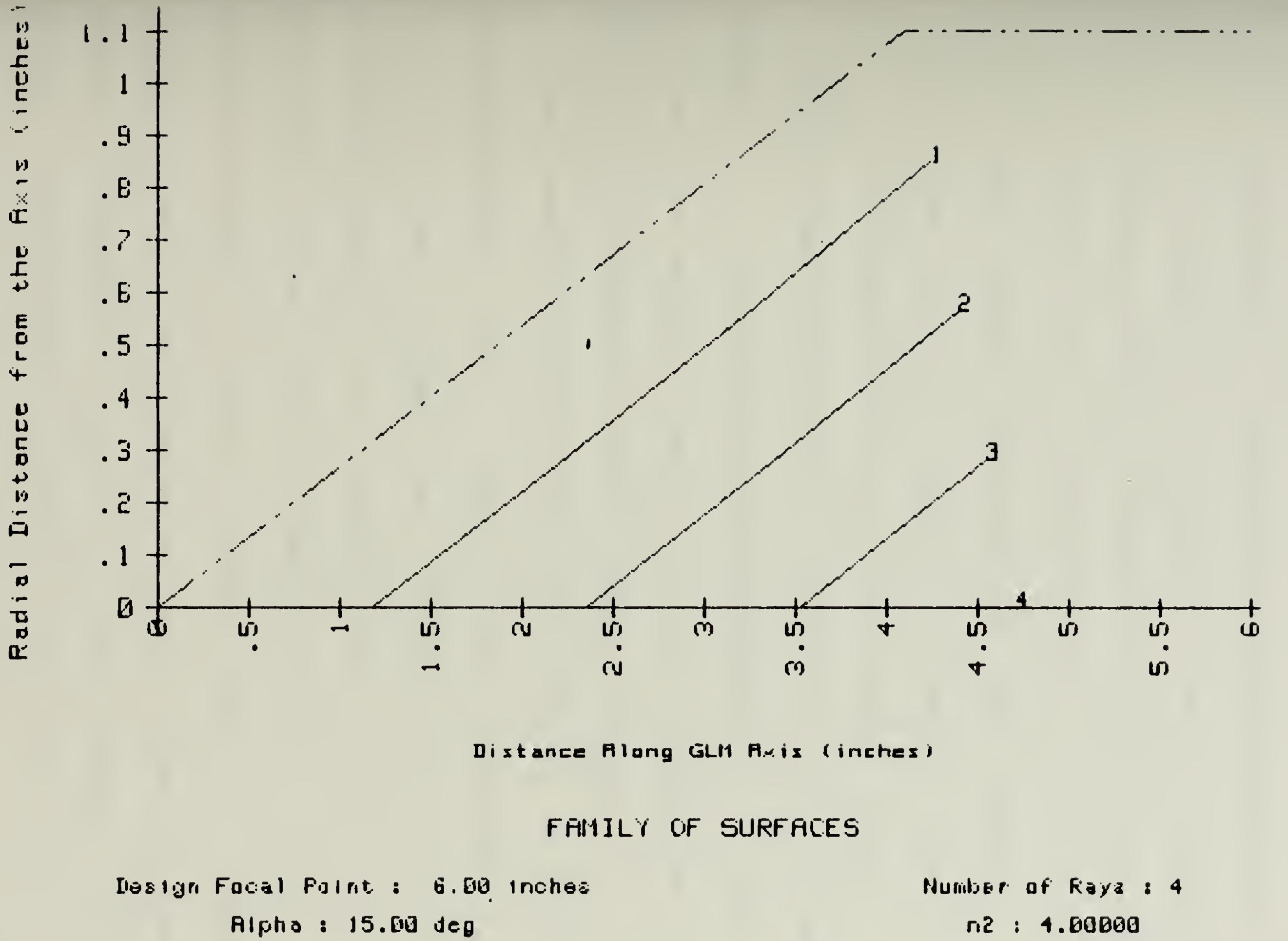


Figure 11. Example of a Family of Second Surfaces Calculated from the Design Chart in Figure 8. The Surfaces Numbers 1, 2, 3 and 4 are Correlated Between the Curve Numbers in this Figure and Figure 8.

consistent set of decisions to be made a set of design criteria for this thesis are:

1) that the lens must be as thin as possible.
2) that the lens have as large an aperture as possible. The aperture is defined to be the difference in radius between the maximum and minimum radii from the GLM axis at which light rays enter the lens and are refracted through the focal point.
3) that the focal length to diameter ratio, f/d, is a minimum and in no case greater than 4.

Using the stated design criteria, surface number 1 in figure 11 is the logical choice because it is the thinnest and has the maximum aperture of the 5 surfaces provided. All of the surfaces have the same f/d ratio of 2.73.

Each member of the family of surfaces is generated by using straight lines. Since a real surface will not be a series of flat surfaces but a smooth continuous surface with no discontinuities, an inherent error exists in the design. The error can be reduced substantially by using a large number of rays to provide a large number of line segments to approximate a continuous surface. The designer must now choose the number of rays to be used, repeat the design process and generate the design chart for the final lens design using, for example, 26 rays in figure 12. Using figure 12, the designer calculates the final lens design in figure 13 beginning with the starting point which is the

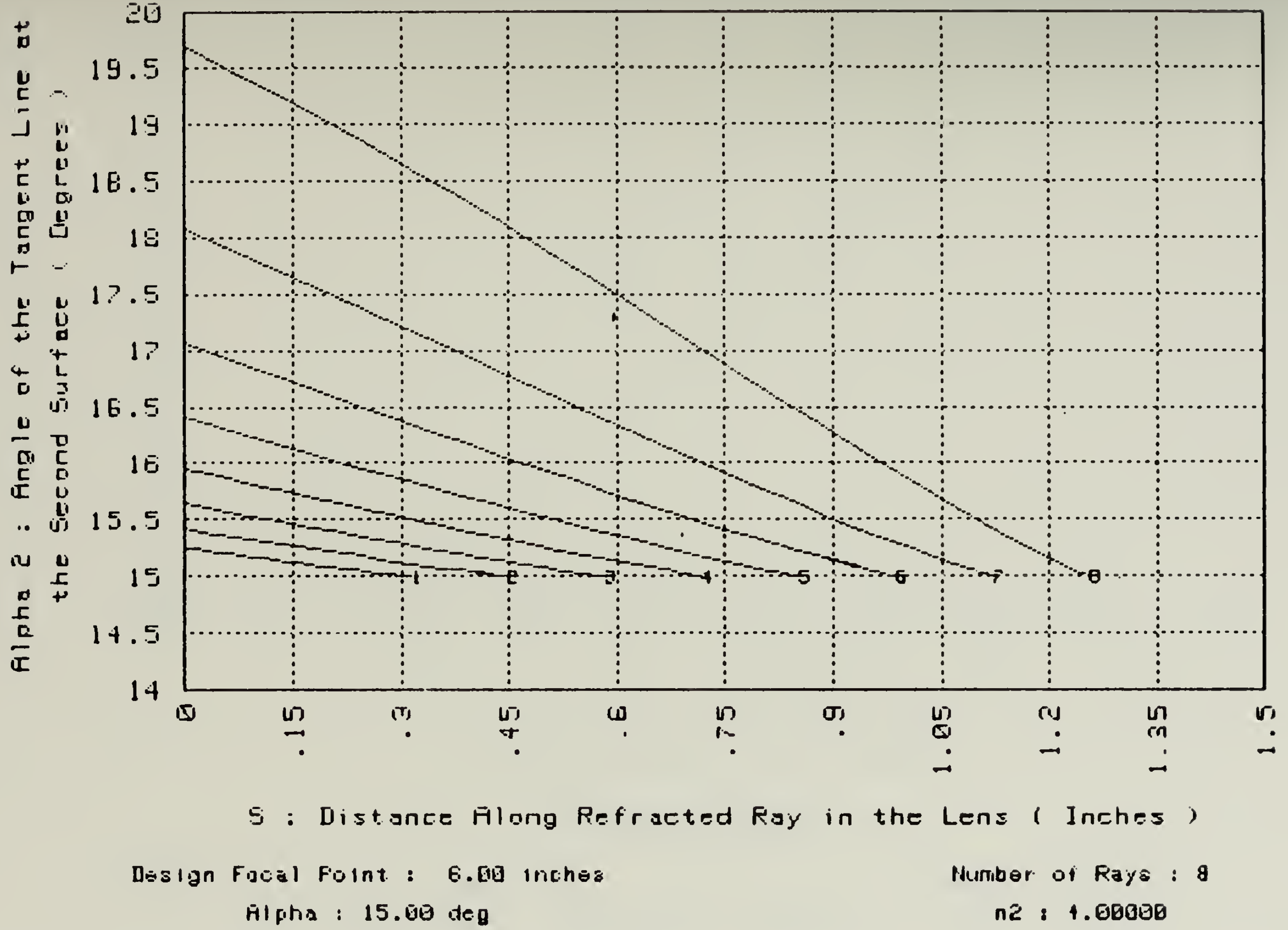


Figure 12. Example Design Chart for the Final Design of the Second Surface Using a Particular Surface Chosen in Figure 11.

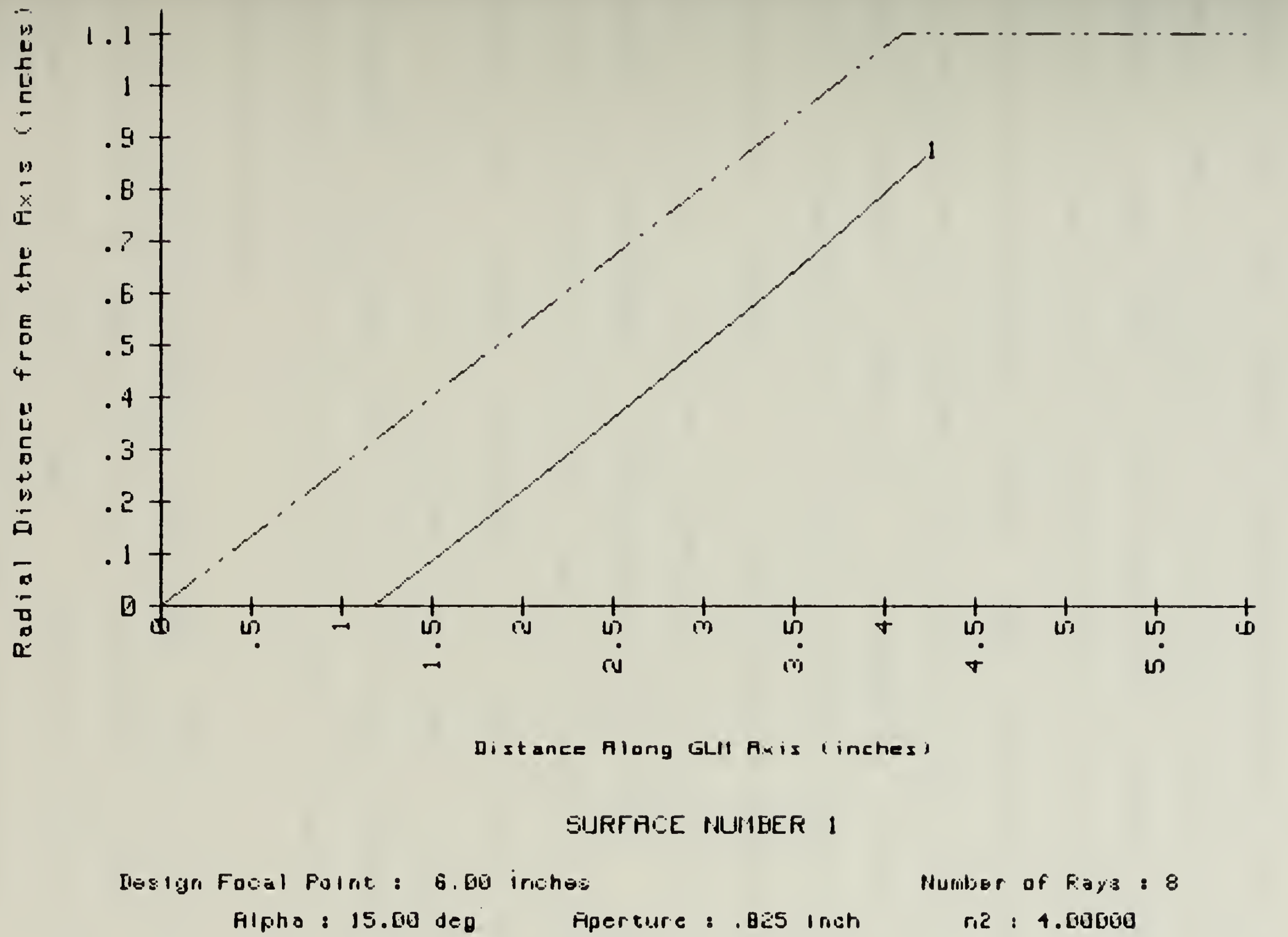


Figure 13. Example of the Final Design of the Second Surface for the Surface Chosen in Figure 11.

end of a curve number 1 in figure 12. Thus T1 is near the GLM axis and $\alpha_2 = 21^\circ$.

A ray diagram shown in figure 14 illustrates the paths of the light rays through the lens generated using the design chart in figure 12. A histogram showing the distribution of light rays on the image plane reveals that the light rays do pass through one design focal point plus or minus an half-increment width. The width of an increment is defined as the maximum radius of the GLM divided by the desired number of intervals. For example, if the maximum radius of the GLM is 1.1 inch and 100 intervals from the GLM axis to a radius of 1.1 inch then the increment width is 0.011 inch. In order to position an interval on the origin, an increment is centered on the origin. The effect of this operation is to add a half-increment at the maximum value of the radius. Therefore, for the entire GLM a total of $2 \times 100 + 1 = 201$ increments are used to generate a histogram. The fact that all of the rays intersect the focal point shows the calculations are correct for this particular lens.

An automated computer algorithm, CHART, has been written to construct a design chart and calculate a family of surfaces from that design chart. CHART waits until the operator chooses a particular surface from the presentation and the number of rays desired to calculate a final surface. Next, CHART will compute the final surface and present it to the

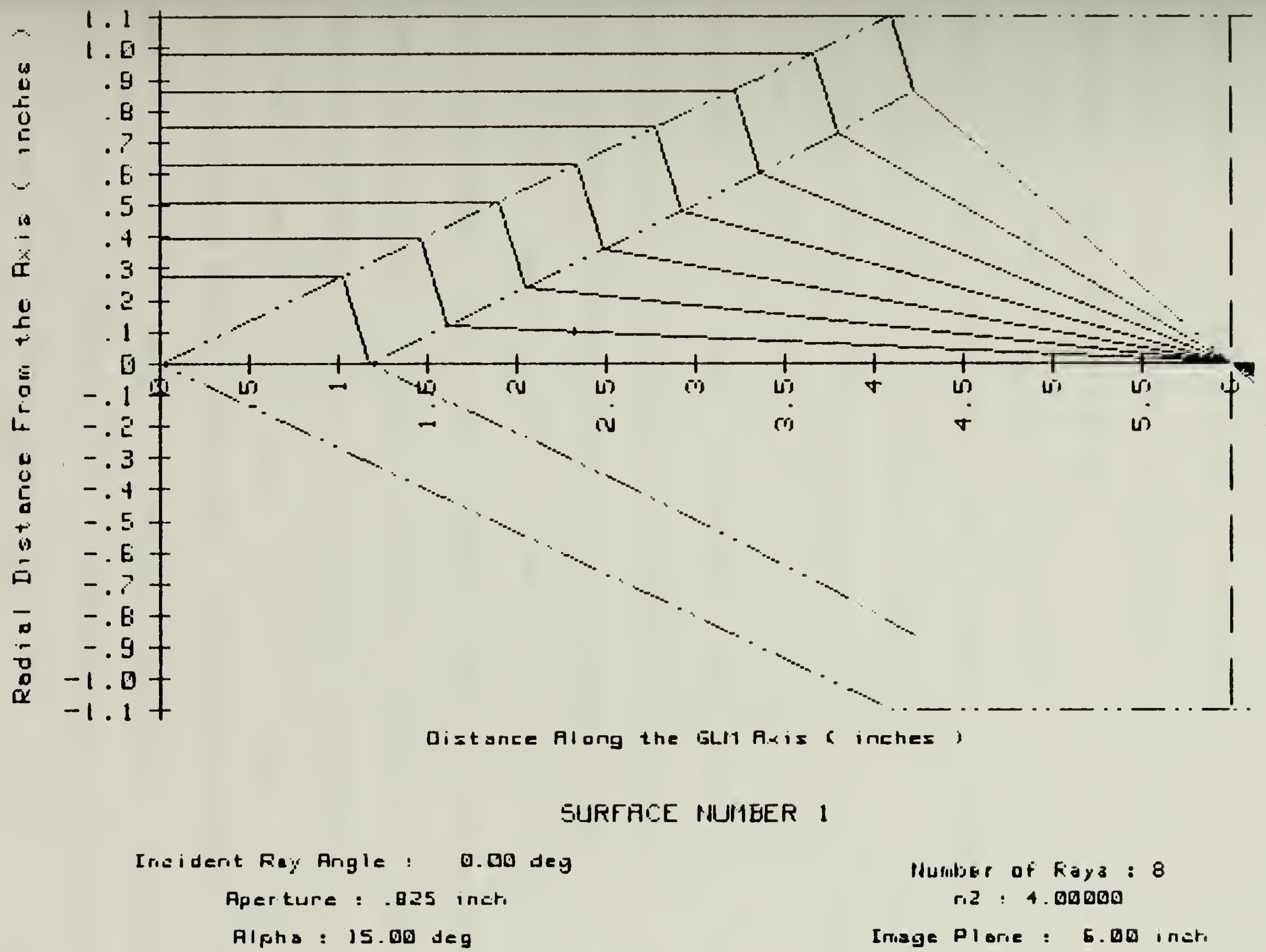
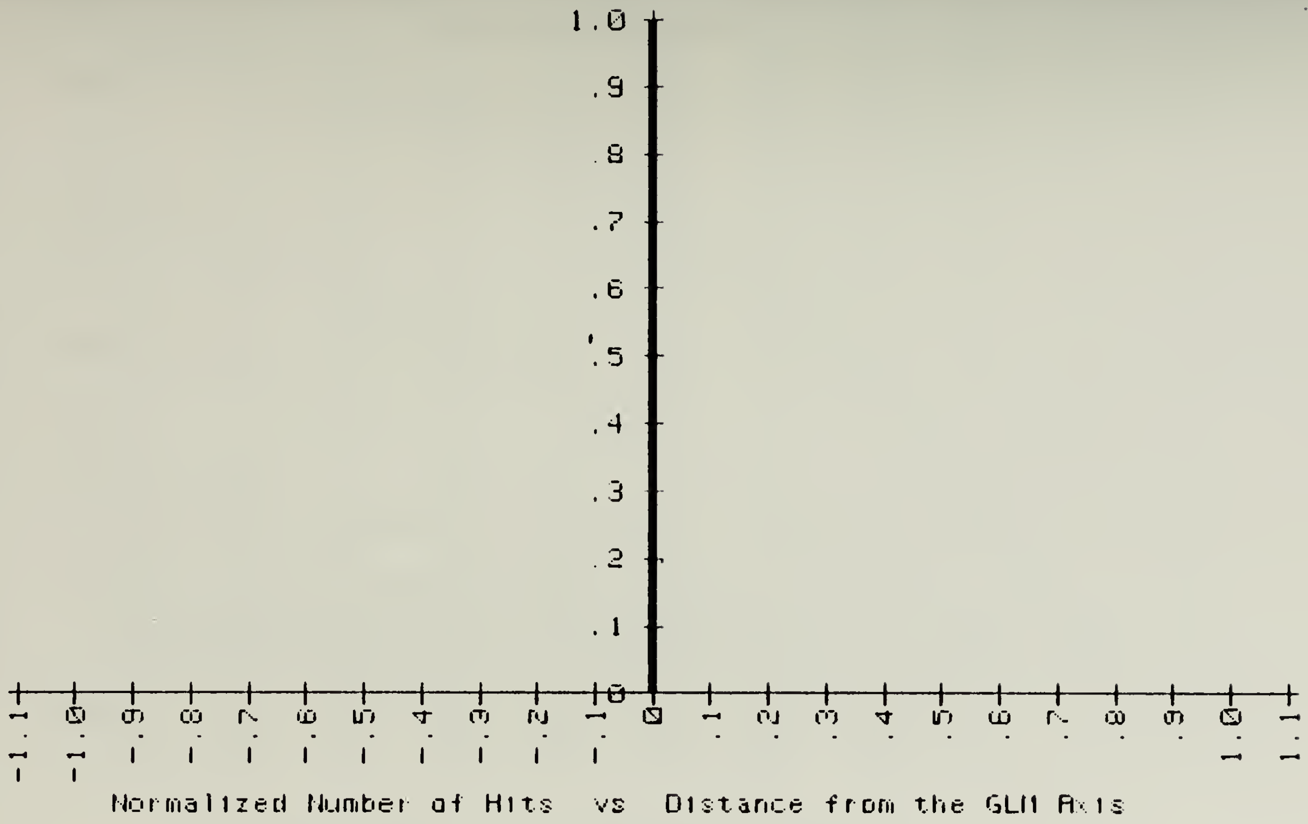


Figure 14. Example Ray Diagram Showing the Results of the Lens Design Procedure.

operator. The operator can choose to stop or draw a ray diagram to show the effectiveness of the design. If the ray diagram is chosen a histogram of ray distribution on the image plane is presented to the operator. If the operator wants to design a lens using another surface, the entire procedure must be repeated.

The following figures are a sample run of CHART to design a lens. Tables containing the initial parameter values and calculated data are located in Appendix A. The design chart in figure 16 was generated using the initial parameter information in Table A-II. The family of second surfaces in figures 18 and 19 were calculated using the end of each numbered curve in figure 16 as the starting point T1 for each numbered second surface in figures 17 and 18. The information used to draw figures 17 and 18 is in Table A-III. The rays shown in figure 18 were used to generate the design chart and family of second surfaces figures 17 and 18. The maximum length of a ray in the lens is defined as QA, the distance from the intercept of the incident ray and the first surface to the GLM axis. The calculated information used to draw the family of second surfaces in figures 17 and 18 is found in Table A-IV. A comparison of QA and the value of s used as the starting point T1 for each family, $s_{max}$, is found in Table A-V.



SURFACE NUMBER 1

Number of Hits on the Image Plane : 8

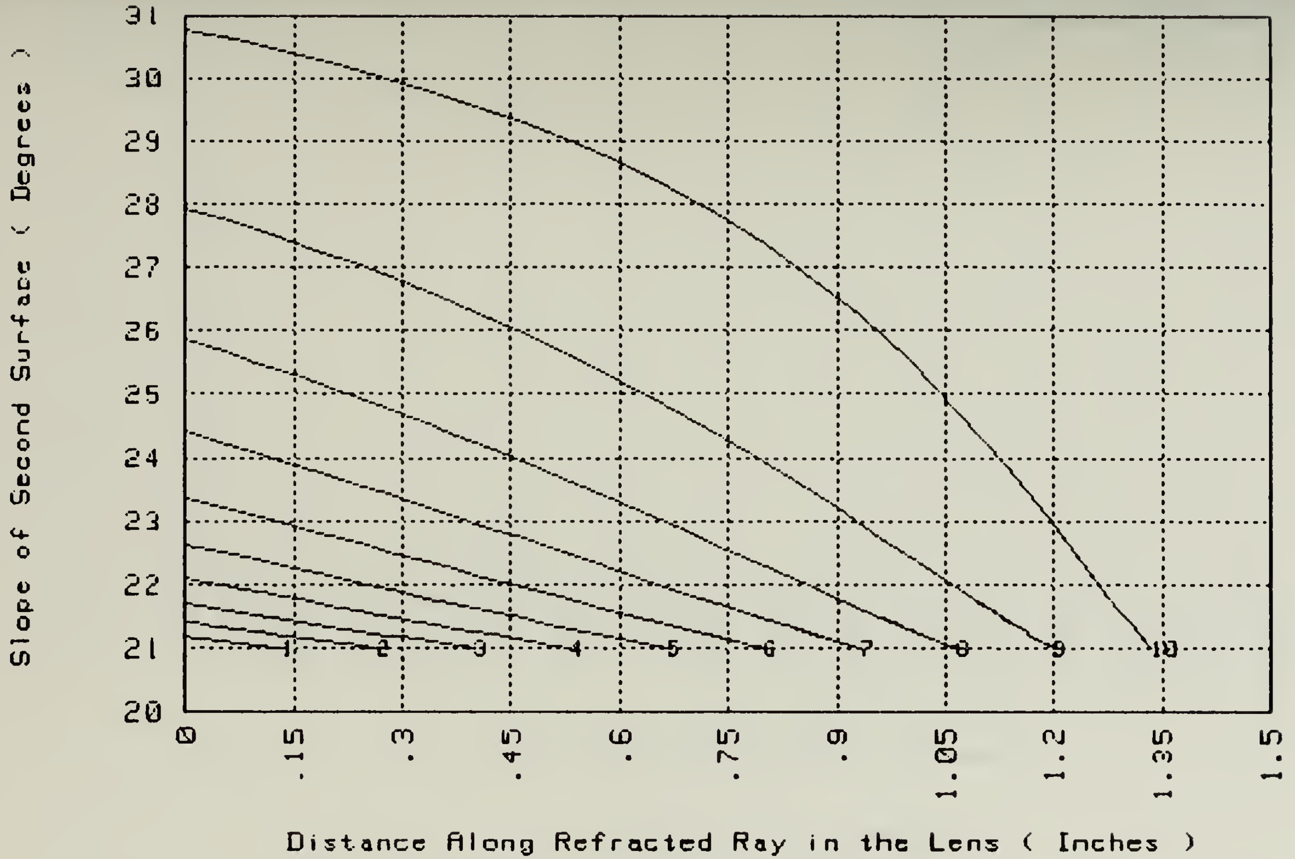
Number of Increments [0,1.1] : 100

Number of Rays : 8

% of Rays to Image Plane : 100.00

Image Plane : 6.00 inches

Figure 15. Example of the Histogram of the Ray Distribution on the Image Plane in Figure 10. Compare with Figures C-3, C-7, C-13 and C-14 of Appendix C.



Design Focal Point : 4.00 inches

Number of Rays : 10

Alpha : 21.00 deg

n2 : 1.00000

Figure 16. Initial Design Chart Showing the Angle of the Tangent Line at the Second Surface, $\alpha_2$, as a Function of Distance Along the Ray in the Lens, s.

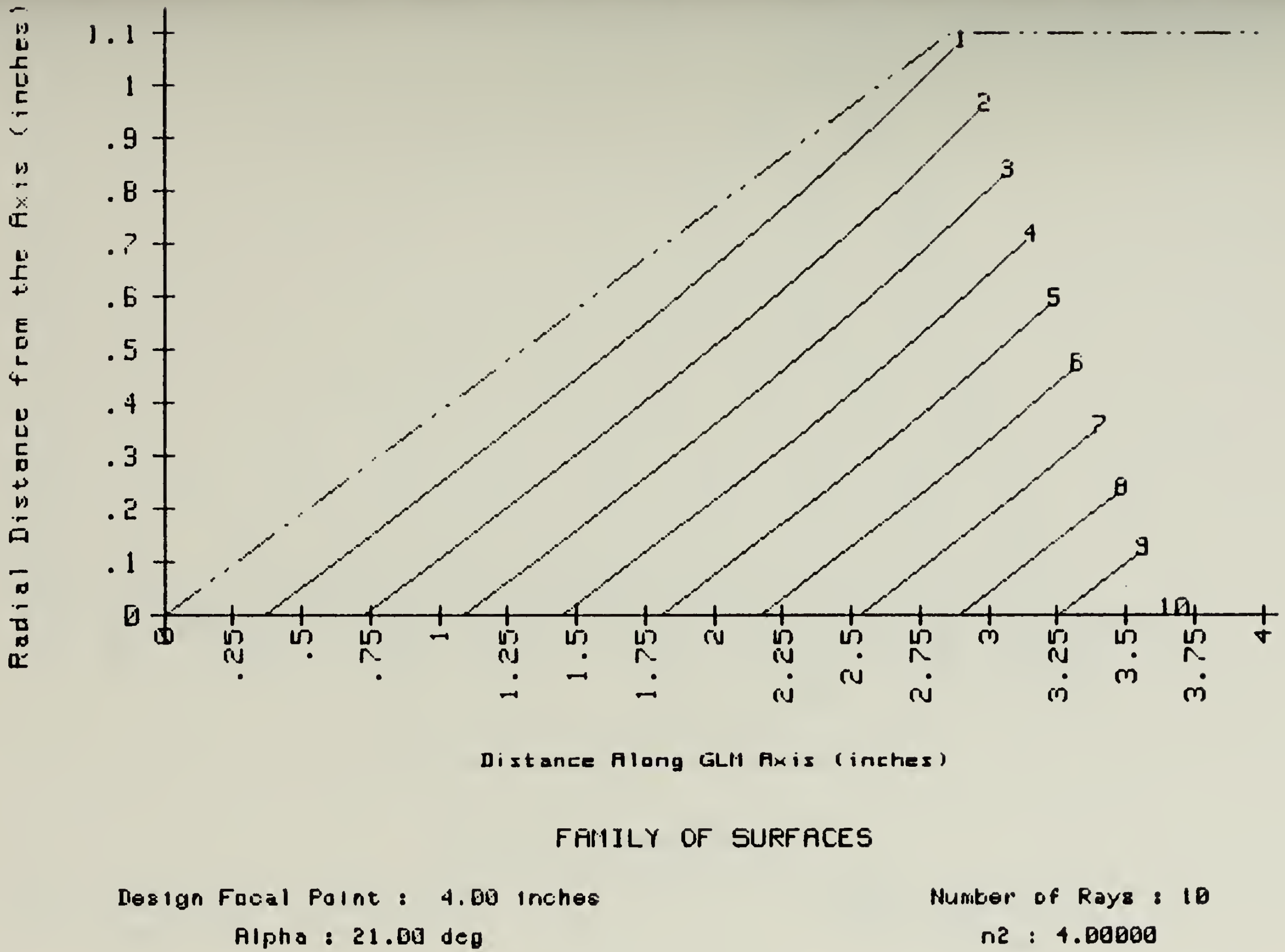


Figure 17. Family of Second Surfaces Generated from the Design Chart in Figure 16.

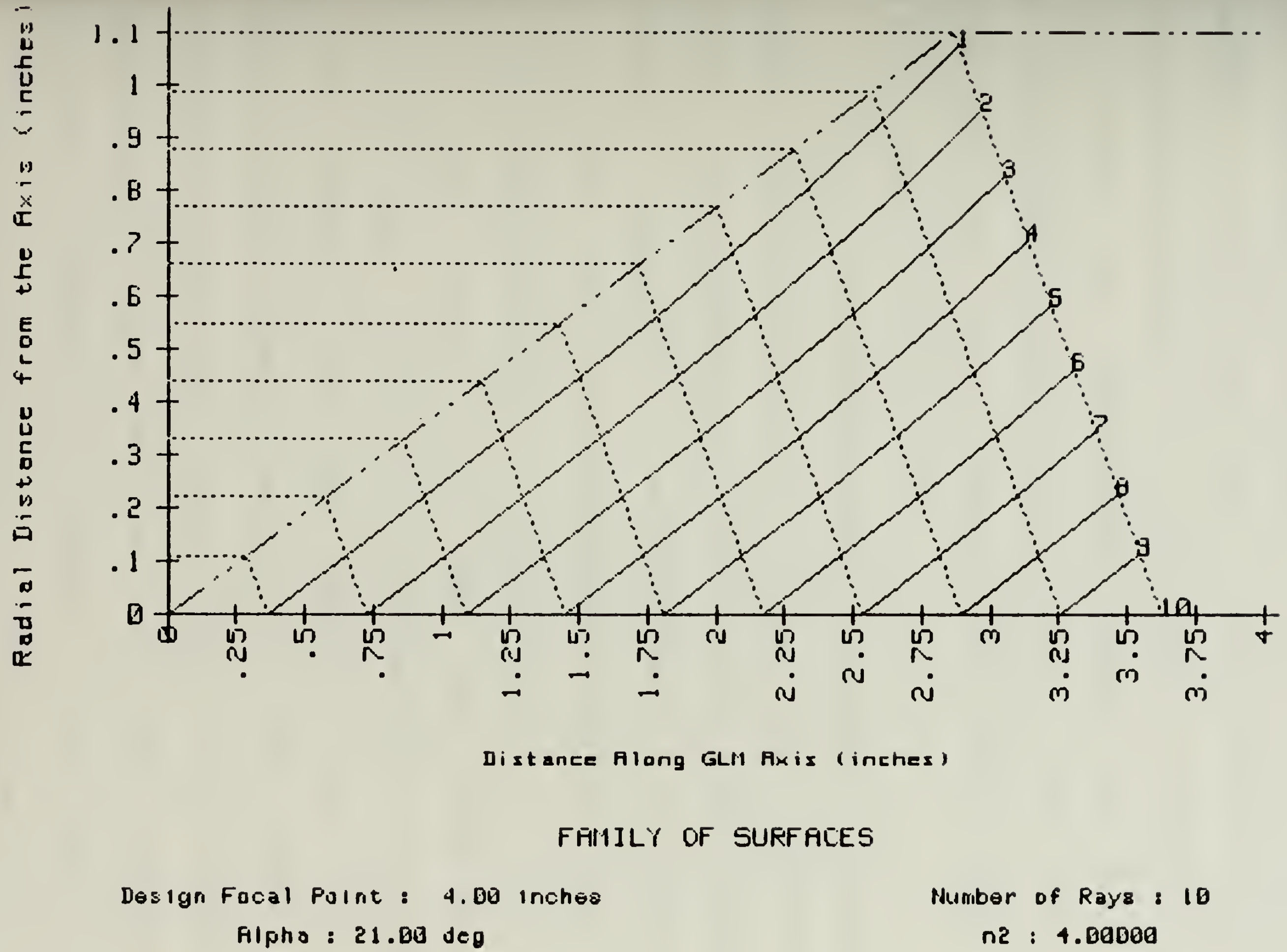


Figure 18. Family of Second Surfaces Generated from the Design Chart in Figure 16. The Rays are Included to Provide a Geometric Perspective.

Surface number 1 was chosen because the surface satisfied all of the design criteria. The final design chart shown in figure 19 was generated using 26 rays. The number of rays actually entered into the computer was 25, CHART then calculates the aperture of the lens using the chosen surface. If the lower limit of the aperture is not the GLM axis, CHART adds one ray to the number entered to use as the starting ray. This method insure a ray will be drawn at the maximum and minimum boundaries of the aperture. The calculated data used to generate the design chart in figure 19 is listed in Table A-VI.

The final lens design shown in figure 20 consists of a surface defined by 26 points rather than 10 with the initial design. The coordinate values for the final surface are found in Table A-VII. The refinement of the surface obtained by using 26 rays is illustrated by comparing the end points of the surfaces listed in Tables A-VII and A-IV. The changes in x, y are (-0.009, 0.013) inch. The change in s is (-0.016) inch. Hence, the lens designer should be aware of the tendency of the final surface to move towards the first surface, possibly resulting in the design surface intercepting the first surface prior to the GLM wall at the maximum radius from the axis. This effect results in a decrease in aperture, a violation of the design criteria.

The final design surface is then drawn within an outline of the GLM shown in figure 21 illustrating the symmetry of

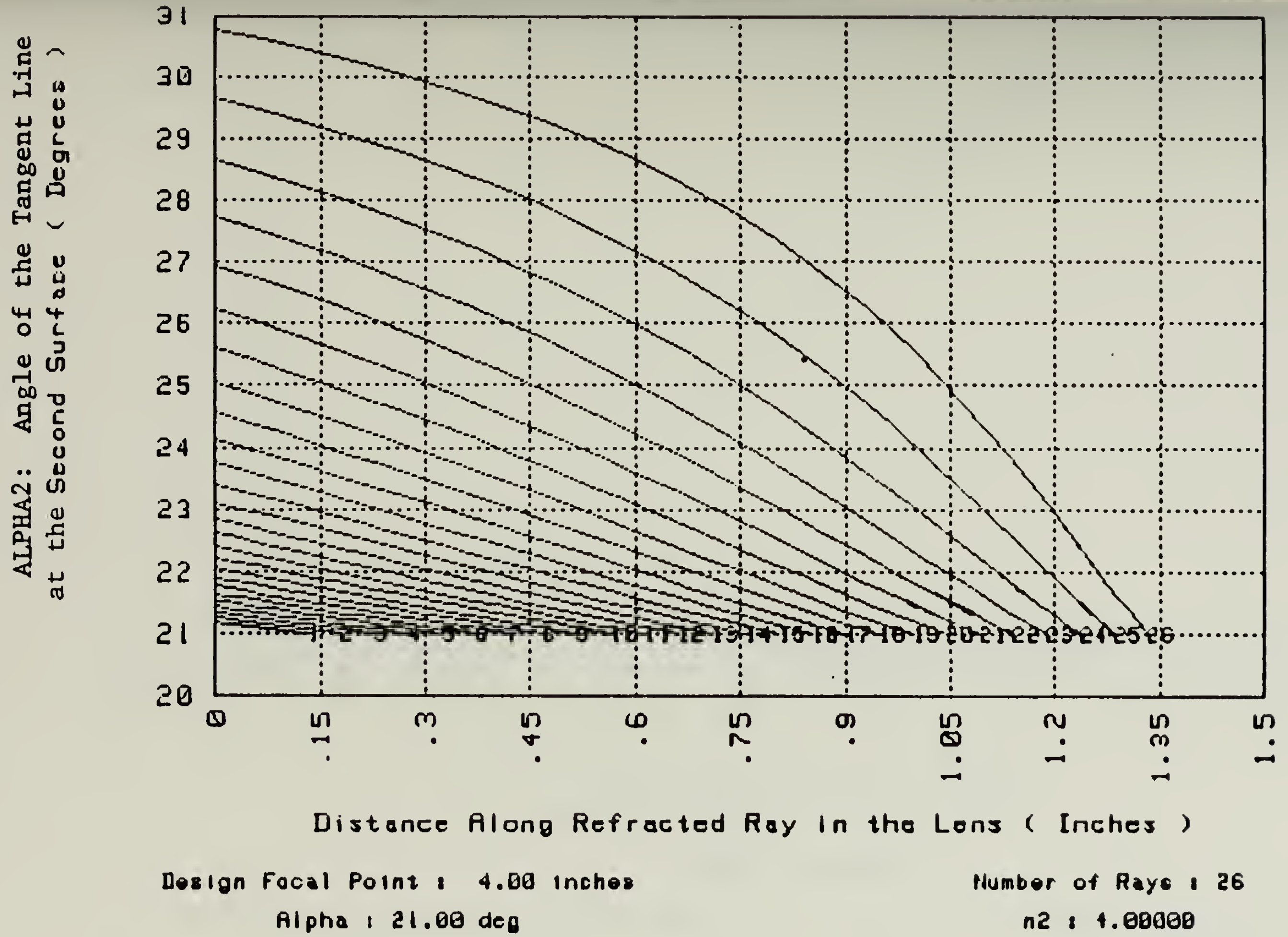


Figure 19. Final Design Chart Used to Calculate the Surface Number 1 in Figures 17 and 18.

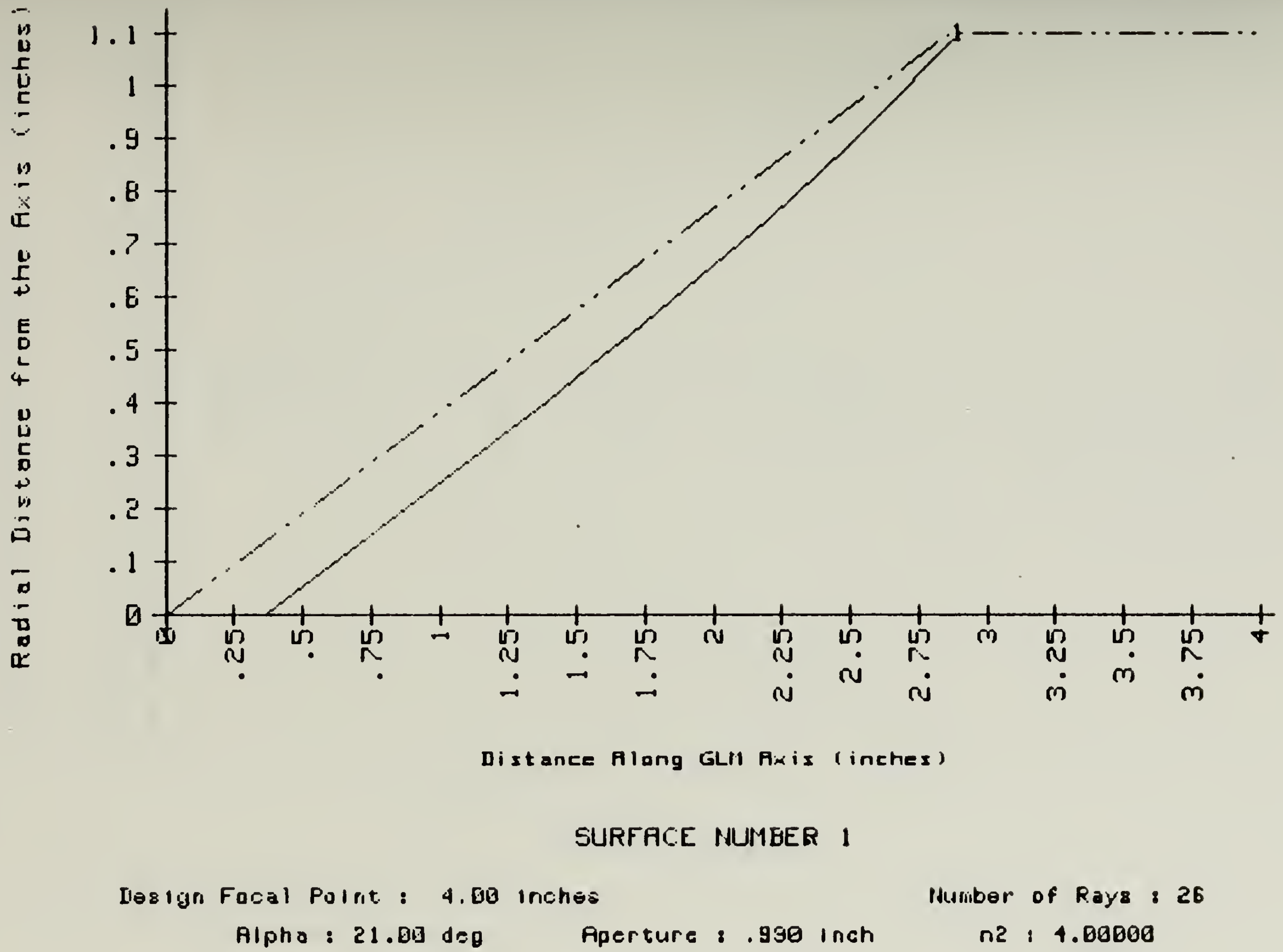


Figure 20. Final Design of the Second Surface of the Conical Lens Calculated from the Design Chart in Figure 19.

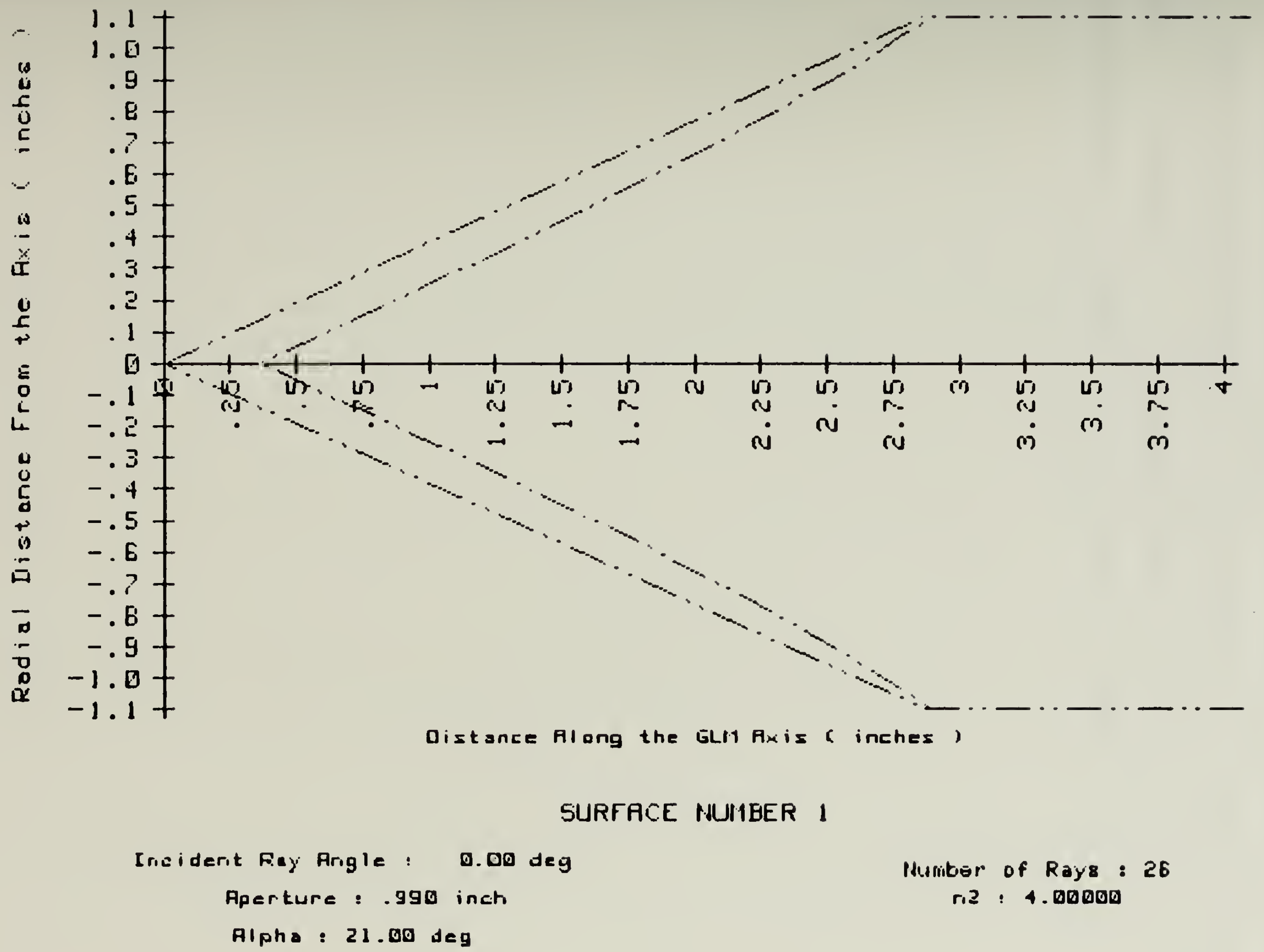


Figure 21. Final Lens Design Illustrating the Symmetry and Position of the Lens in the GLM.

the design and the location of the lens within the missile. A ray diagram using the final lens design shown in figure 22 with the distribution of the rays on the image plane shown in figure 23. The coordinate values used to draw the ray diagram in figure 22 are listed in Table A-VIII.

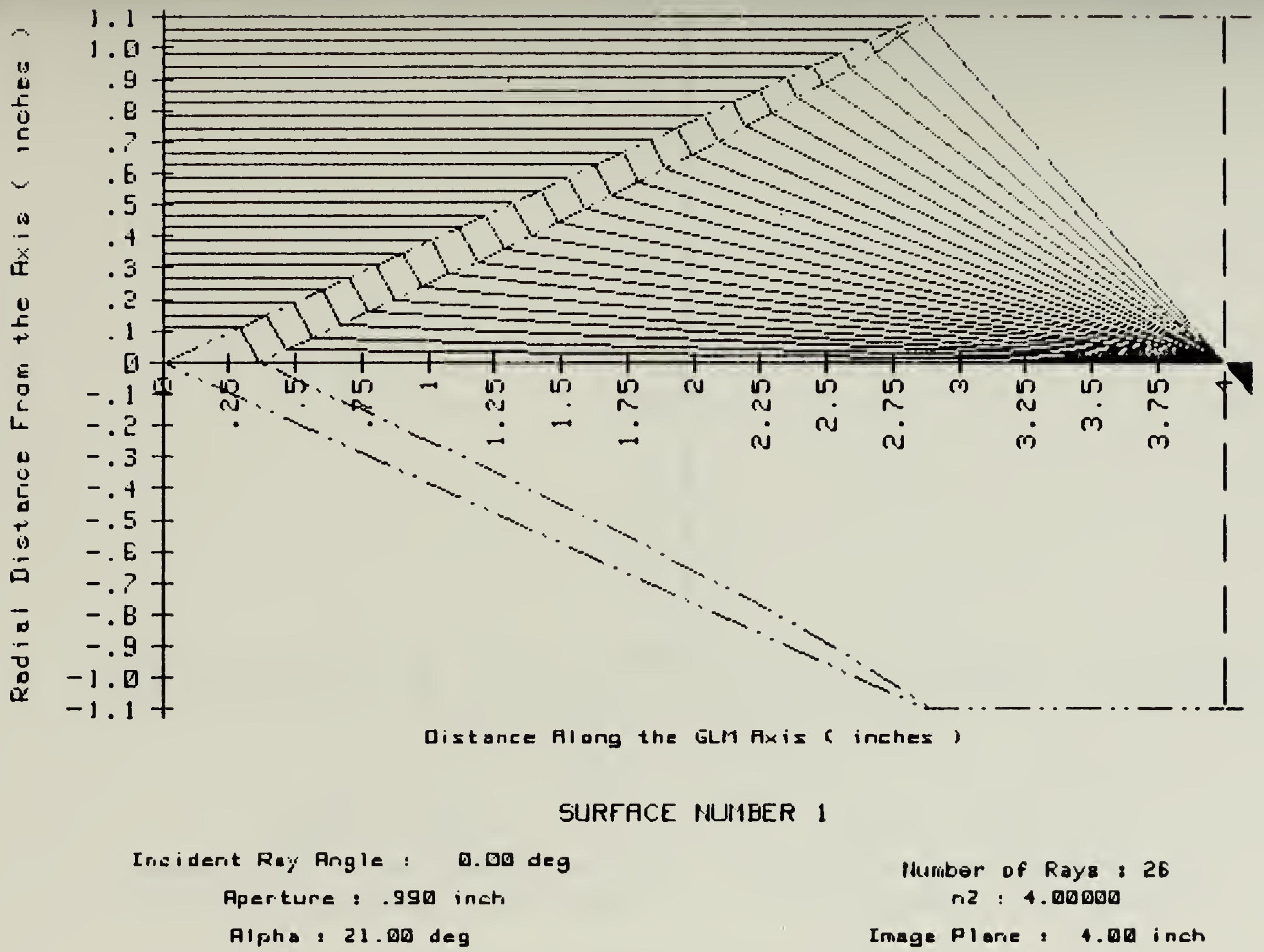
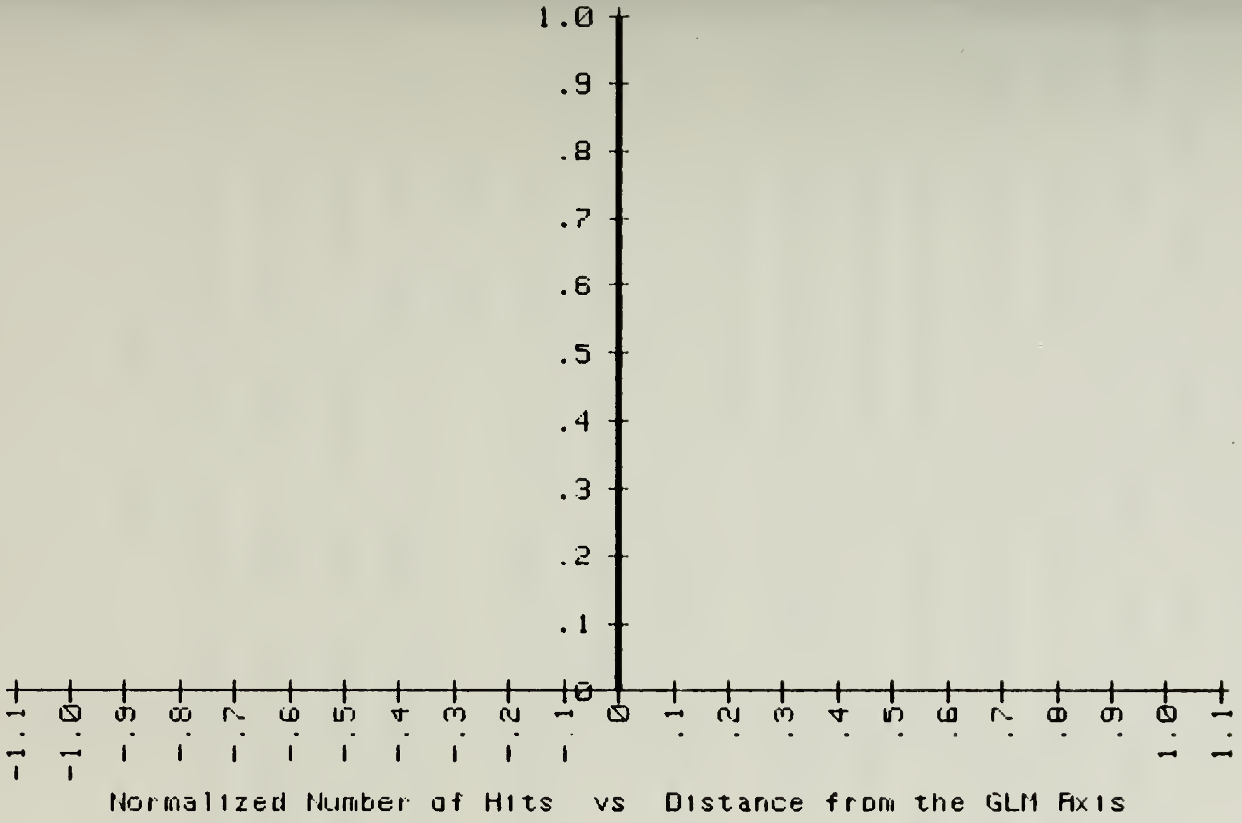


Figure 22. Ray Diagram Showing the Trajectories of the Light Rays Through the Final Lens Design.



SURFACE NUMBER 1

Number of Hits on the Image Plane : 26

Number of Increments [0,1.1] : 100

Number of Rays : 26

% of Rays to Image Plane : 100.00

Image Plane : 4.00 inches

Figure 23. Histogram Showing the Distribution of Rays on the Image Plane in Figure 22.

## IV. RECOMMENDATIONS FOR FUTURE WORK

This thesis investigated part of the properties of a conical lens. Future studies should investigate:

a) the effect of skew rays on the image.

b) the effect of total internal reflection on the image.

c) the calculation of monochromatic and chromatic aberrations for the conical lens.

d) the availability and applicability of materials to use for a lens.

e) the effect on the incident rays of the shock wave in the air ahead of the GLM.

f) the effect on the image of an object not on the lens axis.

g) the effect on the image when the object is no longer in the far field and the curvature of the ray front must be taken into account.

h) the ability of a conical lens to focus coherent and incoherent light.

# V. CONCLUSIONS

The completed design of figure 22 indicates that a conical lens can focus incident light which is parallel to the GLM axis. The conical lens design is compatible with the aerodynamic requirements of high pressure recovery and low drag. The ability to provide optical guidance information to the GLM powered by an integral rocket ramjet may provide the battle group commander with an inexpensive ASMD weapon for shipboard use.

## APPENDIX A

## CHART SAMPLE OUTPUT AND PROGRAM LISTING

This appendix contains the program listing for CHART. The subroutines listed in Table A-I were copied or derived from the HP-9845 utilities library, cassette number 2, Ser. No. 09845-10205, program REGPLT.

Subroutines DRIVER, POLYNOMIAL and PLOT-CUBIC are not implemented in the present version of CHART because of time constraints. They are designed to be used to fit a polynomial of degree 3, a cubic, between four points to estimate the shape of the second surface. The requirement to fit a curve stems from the fact that the trajectory of a ray cannot be calculated if the ray does not coincide with a known ray trajectory. Hence, two options are open. The first is to calculate an infinite number of points on the second surface which is not practical. The second option is to estimate the actual shape of the second surface with an analytic expression, a cubic in this application. The error of the estimate can be reduced by designing a second surface with a large number of points. CHART has been written to design a lens with a maximum number of 250 points.

CHART, as implemented on the HP-9845T is a slow program. The design procedure described in chapter III requires approximately fifteen minutes for an experienced operator.

Therefore, if the initial and final design charts used more or less rays then the computation time should be scaled accordingly.

## TABLE A-I

## SUBROUTINES DERIVED FROM THE HP-9845B UTILITIES LIBRARY

| Name | Line Number |
|---|---|
| Plot | 3710 |
| Laxes | 5280 |
| Driver | 13910 |
| Polynomial | 14400 |
| Plot-Cubic | 14910 |
| Min | 21040 |
| Max | 21160 |

## TABLE A-II

```
                              INITIAL PARAMETER VALUES

Alpha =  21.00 degrees                                 Rho-initial =  0.00
TAN(Alpha) =  .38                                      TAN(RHO-INITIAL) =  0.00

RHO1 = -55.50 degrees                                  Theta(critical) =  14.48 degrees

GLM Radius =  1.10 inches
Aperture =  1.10        Ya =  0.00 inch                Yb =  1.10 inches

n1 =  1.00000           n2 =  4.00000                  n3 1.00000

Number of Rays =  10

Design Focal Point =  4.00  inches
```

## TABLE A-III

CALCULATED DATA USED TO CONSTRUCT THE DESIGN CHART IN FIGURE 16.

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 1 | -1.697 | 13.317 | -68.820 | 0.000 | 21.180 | -1.697 |
| 1 | -1.697 | 13.326 | -68.829 | .007 | 21.171 | -1.614 |
| 1 | -1.697 | 13.335 | -68.838 | .013 | 21.162 | -1.530 |
| 1 | -1.697 | 13.344 | -68.847 | .020 | 21.153 | -1.447 |
| 1 | -1.697 | 13.354 | -68.856 | .027 | 21.144 | -1.363 |
| 1 | -1.697 | 13.363 | -68.866 | .033 | 21.134 | -1.279 |
| 1 | -1.697 | 13.372 | -68.875 | .040 | 21.125 | -1.195 |
| 1 | -1.697 | 13.381 | -68.884 | .047 | 21.116 | -1.111 |
| 1 | -1.697 | 13.390 | -68.893 | .053 | 21.107 | -1.027 |
| 1 | -1.697 | 13.399 | -68.902 | .060 | 21.098 | -.942 |
| 1 | -1.697 | 13.408 | -68.911 | .067 | 21.089 | -.857 |
| 1 | -1.697 | 13.417 | -68.920 | .073 | 21.080 | -.772 |
| 1 | -1.697 | 13.426 | -68.929 | .080 | 21.071 | -.687 |
| 1 | -1.697 | 13.435 | -68.938 | .087 | 21.062 | -.602 |
| 1 | -1.697 | 13.444 | -68.947 | .093 | 21.053 | -.517 |
| 1 | -1.697 | 13.453 | -68.956 | .100 | 21.044 | -.431 |
| 1 | -1.697 | 13.462 | -68.965 | .107 | 21.035 | -.345 |
| 1 | -1.697 | 13.471 | -68.974 | .113 | 21.026 | -.259 |
| 1 | -1.697 | 13.479 | -68.982 | .120 | 21.018 | -.173 |
| 1 | -1.697 | 13.488 | -68.991 | .127 | 21.009 | -.087 |
| 1 | -1.697 | 13.479 | -68.982 | .120 | 21.018 | -.173 |
| 1 | -1.697 | 13.480 | -68.983 | .121 | 21.017 | -.164 |
| 1 | -1.697 | 13.481 | -68.984 | .121 | 21.016 | -.156 |
| 1 | -1.697 | 13.482 | -68.985 | .122 | 21.015 | -.147 |
| 1 | -1.697 | 13.483 | -68.986 | .123 | 21.014 | -.138 |
| 1 | -1.697 | 13.484 | -68.987 | .123 | 21.013 | -.130 |
| 1 | -1.697 | 13.485 | -68.988 | .124 | 21.012 | -.121 |
| 1 | -1.697 | 13.486 | -68.989 | .125 | 21.011 | -.112 |
| 1 | -1.697 | 13.486 | -68.989 | .125 | 21.011 | -.104 |
| 1 | -1.697 | 13.487 | -68.990 | .126 | 21.010 | -.095 |
| 1 | -1.697 | 13.488 | -68.991 | .127 | 21.009 | -.087 |
| 1 | -1.697 | 13.489 | -68.992 | .127 | 21.008 | -.078 |
| 1 | -1.697 | 13.490 | -68.993 | .128 | 21.007 | -.069 |
| 1 | -1.697 | 13.491 | -68.994 | .129 | 21.006 | -.061 |
| 1 | -1.697 | 13.492 | -68.995 | .129 | 21.005 | -.052 |
| 1 | -1.697 | 13.493 | -68.996 | .130 | 21.004 | -.043 |
| 1 | -1.697 | 13.494 | -68.996 | .131 | 21.004 | -.035 |
| 1 | -1.697 | 13.494 | -68.997 | .131 | 21.003 | -.026 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.017 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.009 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.017 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.016 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.016 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.015 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.014 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.013 |

## TABLE A-III (CONT)

| RAY | BETA | THETA<RAY> | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.012 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.011 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.010 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.010 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.009 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.008 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.007 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.006 |
| 1 | -1.697 | 13.497 | -68.999 | .133 | 21.001 | -.005 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.004 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.003 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.003 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 2 | -3.673 | 13.087 | -68.589 | 0.000 | 21.411 | -3.673 |
| 2 | -3.673 | 13.108 | -68.611 | .013 | 21.389 | -3.498 |
| 2 | -3.673 | 13.129 | -68.632 | .027 | 21.368 | -3.321 |
| 2 | -3.673 | 13.150 | -68.653 | .040 | 21.347 | -3.144 |
| 2 | -3.673 | 13.171 | -68.674 | .053 | 21.326 | -2.966 |
| 2 | -3.673 | 13.192 | -68.695 | .067 | 21.305 | -2.787 |
| 2 | -3.673 | 13.213 | -68.716 | .080 | 21.284 | -2.608 |
| 2 | -3.673 | 13.234 | -68.737 | .093 | 21.263 | -2.427 |
| 2 | -3.673 | 13.255 | -68.758 | .107 | 21.242 | -2.245 |
| 2 | -3.673 | 13.276 | -68.779 | .120 | 21.221 | -2.063 |
| 2 | -3.673 | 13.296 | -68.799 | .133 | 21.201 | -1.880 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 2 | -3.673 | 13.317 | -68.820 | .147 | 21.180 | -1.696 |
| 2 | -3.673 | 13.337 | -68.840 | .160 | 21.160 | -1.511 |
| 2 | -3.673 | 13.358 | -68.861 | .174 | 21.139 | -1.325 |
| 2 | -3.673 | 13.378 | -68.881 | .187 | 21.119 | -1.139 |
| 2 | -3.673 | 13.398 | -68.901 | .200 | 21.099 | -.951 |
| 2 | -3.673 | 13.418 | -68.921 | .214 | 21.079 | -.763 |
| 2 | -3.673 | 13.438 | -68.941 | .227 | 21.059 | -.573 |
| 2 | -3.673 | 13.458 | -68.961 | .240 | 21.039 | -.383 |
| 2 | -3.673 | 13.477 | -68.980 | .254 | 21.020 | -.192 |
| 2 | -3.673 | 13.458 | -68.961 | .240 | 21.039 | -.383 |
| 2 | -3.673 | 13.460 | -68.963 | .242 | 21.037 | -.364 |
| 2 | -3.673 | 13.462 | -68.965 | .243 | 21.035 | -.345 |
| 2 | -3.673 | 13.464 | -68.967 | .244 | 21.033 | -.326 |
| 2 | -3.673 | 13.466 | -68.969 | .246 | 21.031 | -.307 |
| 2 | -3.673 | 13.468 | -68.971 | .247 | 21.029 | -.288 |
| 2 | -3.673 | 13.470 | -68.973 | .248 | 21.027 | -.268 |
| 2 | -3.673 | 13.472 | -68.975 | .250 | 21.025 | -.249 |
| 2 | -3.673 | 13.474 | -68.976 | .251 | 21.024 | -.230 |
| 2 | -3.673 | 13.475 | -68.978 | .252 | 21.022 | -.211 |
| 2 | -3.673 | 13.477 | -68.980 | .254 | 21.020 | -.192 |
| 2 | -3.673 | 13.479 | -68.982 | .255 | 21.018 | -.173 |
| 2 | -3.673 | 13.481 | -68.984 | .256 | 21.016 | -.154 |
| 2 | -3.673 | 13.483 | -68.986 | .258 | 21.014 | -.134 |
| 2 | -3.673 | 13.485 | -68.988 | .259 | 21.012 | -.115 |
| 2 | -3.673 | 13.487 | -68.990 | .260 | 21.010 | -.096 |
| 2 | -3.673 | 13.489 | -68.992 | .262 | 21.008 | -.077 |
| 2 | -3.673 | 13.491 | -68.994 | .263 | 21.006 | -.058 |
| 2 | -3.673 | 13.493 | -68.996 | .264 | 21.004 | -.038 |
| 2 | -3.673 | 13.495 | -68.998 | .266 | 21.002 | -.019 |
| 2 | -3.673 | 13.493 | -68.996 | .264 | 21.004 | -.038 |
| 2 | -3.673 | 13.493 | -68.996 | .264 | 21.004 | -.037 |
| 2 | -3.673 | 13.494 | -68.996 | .265 | 21.004 | -.035 |
| 2 | -3.673 | 13.494 | -68.997 | .265 | 21.003 | -.033 |
| 2 | -3.673 | 13.494 | -68.997 | .265 | 21.003 | -.031 |
| 2 | -3.673 | 13.494 | -68.997 | .265 | 21.003 | -.029 |
| 2 | -3.673 | 13.494 | -68.997 | .265 | 21.003 | -.027 |
| 2 | -3.673 | 13.494 | -68.997 | .265 | 21.003 | -.025 |
| 2 | -3.673 | 13.495 | -68.998 | .265 | 21.002 | -.023 |
| 2 | -3.673 | 13.495 | -68.998 | .265 | 21.002 | -.021 |
| 2 | -3.673 | 13.495 | -68.998 | .266 | 21.002 | -.019 |
| 2 | -3.673 | 13.495 | -68.998 | .266 | 21.002 | -.017 |
| 2 | -3.673 | 13.495 | -68.998 | .266 | 21.002 | -.015 |
| 2 | -3.673 | 13.496 | -68.999 | .266 | 21.001 | -.013 |
| 2 | -3.673 | 13.496 | -68.999 | .266 | 21.001 | -.012 |
| 2 | -3.673 | 13.496 | -68.999 | .266 | 21.001 | -.010 |
| 2 | -3.673 | 13.496 | -68.999 | .266 | 21.001 | -.008 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 2 | -3.673 | 13.496 | -68.999 | .267 | 21.001 | -.006 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.004 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.004 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.004 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.003 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.002 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.001 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.001 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.001 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.001 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.001 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 2 | -3.673 | 13.497 | -69.000 | .267 | 21.000 | -.000 |
| 3 | -5.999 | 12.787 | -68.290 | 0.000 | 21.710 | -5.999 |
| 3 | -5.999 | 12.825 | -68.327 | .020 | 21.673 | -5.722 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 3 | -5.999 | 12.862 | -68.365 | .040 | 21.635 | -5.442 |
| 3 | -5.999 | 12.899 | -68.402 | .060 | 21.598 | -5.160 |
| 3 | -5.999 | 12.936 | -68.439 | .080 | 21.561 | -4.876 |
| 3 | -5.999 | 12.972 | -68.475 | .100 | 21.525 | -4.589 |
| 3 | -5.999 | 13.009 | -68.512 | .120 | 21.488 | -4.300 |
| 3 | -5.999 | 13.045 | -68.548 | .140 | 21.452 | -4.009 |
| 3 | -5.999 | 13.081 | -68.584 | .160 | 21.416 | -3.715 |
| 3 | -5.999 | 13.117 | -68.620 | .180 | 21.380 | -3.419 |
| 3 | -5.999 | 13.153 | -68.656 | .200 | 21.344 | -3.120 |
| 3 | -5.999 | 13.189 | -68.692 | .220 | 21.308 | -2.819 |
| 3 | -5.999 | 13.224 | -68.727 | .240 | 21.273 | -2.516 |
| 3 | -5.999 | 13.259 | -68.762 | .260 | 21.238 | -2.210 |
| 3 | -5.999 | 13.294 | -68.797 | .280 | 21.203 | -1.902 |
| 3 | -5.999 | 13.329 | -68.831 | .300 | 21.169 | -1.591 |
| 3 | -5.999 | 13.363 | -68.866 | .320 | 21.134 | -1.278 |
| 3 | -5.999 | 13.397 | -68.900 | .340 | 21.100 | -.962 |
| 3 | -5.999 | 13.431 | -68.933 | .360 | 21.067 | -.644 |
| 3 | -5.999 | 13.464 | -68.967 | .380 | 21.033 | -.323 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.464 | -68.967 | .380 | 21.033 | -.323 |
| 3 | -5.999 | 13.467 | -68.970 | .382 | 21.030 | -.291 |
| 3 | -5.999 | 13.471 | -68.974 | .384 | 21.026 | -.259 |
| 3 | -5.999 | 13.474 | -68.977 | .386 | 21.023 | -.227 |
| 3 | -5.999 | 13.477 | -68.980 | .388 | 21.020 | -.194 |
| 3 | -5.999 | 13.481 | -68.983 | .390 | 21.017 | -.162 |
| 3 | -5.999 | 13.484 | -68.987 | .392 | 21.013 | -.130 |
| 3 | -5.999 | 13.487 | -68.990 | .394 | 21.010 | -.097 |
| 3 | -5.999 | 13.490 | -68.993 | .396 | 21.007 | -.065 |
| 3 | -5.999 | 13.494 | -68.997 | .398 | 21.003 | -.032 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.494 | -68.997 | .398 | 21.003 | -.032 |
| 3 | -5.999 | 13.494 | -68.997 | .399 | 21.003 | -.029 |
| 3 | -5.999 | 13.494 | -68.997 | .399 | 21.003 | -.026 |
| 3 | -5.999 | 13.495 | -68.998 | .399 | 21.002 | -.023 |
| 3 | -5.999 | 13.495 | -68.998 | .399 | 21.002 | -.019 |
| 3 | -5.999 | 13.495 | -68.998 | .399 | 21.002 | -.016 |
| 3 | -5.999 | 13.496 | -68.999 | .400 | 21.001 | -.013 |
| 3 | -5.999 | 13.496 | -68.999 | .400 | 21.001 | -.010 |
| 3 | -5.999 | 13.496 | -68.999 | .400 | 21.001 | -.006 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.003 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.003 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.003 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.003 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.002 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.002 |

# TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.002 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.001 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.001 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.001 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 3 | -5.999 | 13.497 | -69.000 | .400 | 21.000 | -.000 |
| 4 | -8.765 | 12.391 | -67.894 | 0.000 | 22.106 | -8.765 |
| 4 | -8.765 | 12.449 | -67.952 | .027 | 22.048 | -8.377 |
| 4 | -8.765 | 12.507 | -68.010 | .053 | 21.990 | -7.984 |
| 4 | -8.765 | 12.565 | -68.068 | .080 | 21.932 | -7.586 |
| 4 | -8.765 | 12.623 | -68.126 | .107 | 21.874 | -7.182 |
| 4 | -8.765 | 12.681 | -68.184 | .133 | 21.816 | -6.774 |
| 4 | -8.765 | 12.738 | -68.241 | .160 | 21.759 | -6.361 |
| 4 | -8.765 | 12.795 | -68.298 | .187 | 21.702 | -5.942 |
| 4 | -8.765 | 12.852 | -68.355 | .214 | 21.645 | -5.518 |
| 4 | -8.765 | 12.908 | -68.411 | .240 | 21.589 | -5.089 |
| 4 | -8.765 | 12.964 | -68.467 | .267 | 21.533 | -4.654 |
| 4 | -8.765 | 13.020 | -68.523 | .294 | 21.477 | -4.214 |
| 4 | -8.765 | 13.075 | -68.578 | .320 | 21.422 | -3.768 |
| 4 | -8.765 | 13.130 | -68.633 | .347 | 21.367 | -3.317 |
| 4 | -8.765 | 13.184 | -68.687 | .374 | 21.313 | -2.860 |
| 4 | -8.765 | 13.238 | -68.741 | .400 | 21.259 | -2.398 |
| 4 | -8.765 | 13.291 | -68.794 | .427 | 21.206 | -1.930 |
| 4 | -8.765 | 13.343 | -68.846 | .454 | 21.154 | -1.456 |
| 4 | -8.765 | 13.395 | -68.898 | .480 | 21.102 | -.976 |
| 4 | -8.765 | 13.447 | -68.949 | .507 | 21.051 | -.491 |
| 4 | -8.765 | 13.447 | -68.949 | .507 | 21.051 | -.491 |
| 4 | -8.765 | 13.452 | -68.955 | .510 | 21.045 | -.442 |
| 4 | -8.765 | 13.457 | -68.960 | .513 | 21.040 | -.393 |
| 4 | -8.765 | 13.462 | -68.965 | .515 | 21.035 | -.344 |
| 4 | -8.765 | 13.467 | -68.970 | .518 | 21.030 | -.295 |
| 4 | -8.765 | 13.472 | -68.975 | .521 | 21.025 | -.246 |
| 4 | -8.765 | 13.477 | -68.980 | .523 | 21.020 | -.197 |
| 4 | -8.765 | 13.482 | -68.985 | .526 | 21.015 | -.148 |

# TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 4 | -8.765 | 13.487 | -68.990 | .529 | 21.010 | -.099 |
| 4 | -8.765 | 13.492 | -68.995 | .531 | 21.005 | -.049 |
| 4 | -8.765 | 13.492 | -68.995 | .531 | 21.005 | -.049 |
| 4 | -8.765 | 13.493 | -68.995 | .531 | 21.005 | -.044 |
| 4 | -8.765 | 13.493 | -68.996 | .532 | 21.004 | -.040 |
| 4 | -8.765 | 13.494 | -68.996 | .532 | 21.004 | -.035 |
| 4 | -8.765 | 13.494 | -68.997 | .532 | 21.003 | -.030 |
| 4 | -8.765 | 13.495 | -68.997 | .533 | 21.003 | -.025 |
| 4 | -8.765 | 13.495 | -68.998 | .533 | 21.002 | -.020 |
| 4 | -8.765 | 13.496 | -68.998 | .533 | 21.002 | -.015 |
| 4 | -8.765 | 13.496 | -68.999 | .533 | 21.001 | -.010 |
| 4 | -8.765 | 13.497 | -68.999 | .534 | 21.001 | -.005 |
| 4 | -8.765 | 13.497 | -68.999 | .534 | 21.001 | -.005 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.004 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.004 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.003 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.003 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.002 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.002 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.001 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.001 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 4 | -8.765 | 13.497 | -69.000 | .534 | 21.000 | -.000 |
| 5 | -12.092 | 11.856 | -67.359 | 0.000 | 22.641 | -12.092 |
| 5 | -12.092 | 11.941 | -67.444 | .033 | 22.556 | -11.587 |
| 5 | -12.092 | 12.027 | -67.530 | .067 | 22.470 | -11.073 |
| 5 | -12.092 | 12.112 | -67.615 | .100 | 22.385 | -10.549 |
| 5 | -12.092 | 12.197 | -67.700 | .133 | 22.300 | -10.015 |
| 5 | -12.092 | 12.283 | -67.786 | .167 | 22.214 | -9.471 |
| 5 | -12.092 | 12.368 | -67.871 | .200 | 22.129 | -8.917 |
| 5 | -12.092 | 12.453 | -67.956 | .234 | 22.044 | -8.353 |
| 5 | -12.092 | 12.537 | -68.040 | .267 | 21.960 | -7.778 |
| 5 | -12.092 | 12.622 | -68.125 | .300 | 21.875 | -7.192 |
| 5 | -12.092 | 12.705 | -68.208 | .334 | 21.792 | -6.596 |
| 5 | -12.092 | 12.789 | -68.292 | .367 | 21.708 | -5.989 |
| 5 | -12.092 | 12.871 | -68.374 | .400 | 21.626 | -5.370 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 5 | -12.092 | 12.953 | -68.456 | .434 | 21.544 | -4.740 |
| 5 | -12.092 | 13.034 | -68.537 | .467 | 21.463 | -4.099 |
| 5 | -12.092 | 13.114 | -68.617 | .501 | 21.383 | -3.446 |
| 5 | -12.092 | 13.193 | -68.696 | .534 | 21.304 | -2.781 |
| 5 | -12.092 | 13.271 | -68.774 | .567 | 21.226 | -2.104 |
| 5 | -12.092 | 13.348 | -68.851 | .601 | 21.149 | -1.415 |
| 5 | -12.092 | 13.423 | -68.926 | .634 | 21.074 | -.714 |
| 5 | -12.092 | 13.348 | -68.851 | .601 | 21.149 | -1.415 |
| 5 | -12.092 | 13.355 | -68.858 | .604 | 21.142 | -1.345 |
| 5 | -12.092 | 13.363 | -68.866 | .607 | 21.134 | -1.275 |
| 5 | -12.092 | 13.371 | -68.874 | .611 | 21.126 | -1.206 |
| 5 | -12.092 | 13.378 | -68.881 | .614 | 21.119 | -1.136 |
| 5 | -12.092 | 13.386 | -68.889 | .617 | 21.111 | -1.066 |
| 5 | -12.092 | 13.393 | -68.896 | .621 | 21.104 | -.995 |
| 5 | -12.092 | 13.401 | -68.904 | .624 | 21.096 | -.925 |
| 5 | -12.092 | 13.408 | -68.911 | .627 | 21.089 | -.855 |
| 5 | -12.092 | 13.416 | -68.919 | .631 | 21.081 | -.784 |
| 5 | -12.092 | 13.423 | -68.926 | .634 | 21.074 | -.714 |
| 5 | -12.092 | 13.431 | -68.934 | .637 | 21.066 | -.643 |
| 5 | -12.092 | 13.438 | -68.941 | .641 | 21.059 | -.572 |
| 5 | -12.092 | 13.446 | -68.948 | .644 | 21.052 | -.501 |
| 5 | -12.092 | 13.453 | -68.956 | .647 | 21.044 | -.430 |
| 5 | -12.092 | 13.460 | -68.963 | .651 | 21.037 | -.358 |
| 5 | -12.092 | 13.468 | -68.971 | .654 | 21.029 | -.237 |
| 5 | -12.092 | 13.475 | -68.978 | .657 | 21.022 | -.215 |
| 5 | -12.092 | 13.482 | -68.985 | .661 | 21.015 | -.144 |
| 5 | -12.092 | 13.490 | -68.993 | .664 | 21.007 | -.072 |
| 5 | -12.092 | 13.482 | -68.985 | .661 | 21.015 | -.144 |
| 5 | -12.092 | 13.483 | -68.986 | .661 | 21.014 | -.137 |
| 5 | -12.092 | 13.484 | -68.987 | .661 | 21.013 | -.129 |
| 5 | -12.092 | 13.485 | -68.988 | .662 | 21.012 | -.122 |
| 5 | -12.092 | 13.485 | -68.988 | .662 | 21.012 | -.115 |
| 5 | -12.092 | 13.486 | -68.989 | .662 | 21.011 | -.108 |
| 5 | -12.092 | 13.487 | -68.990 | .663 | 21.010 | -.101 |
| 5 | -12.092 | 13.488 | -68.990 | .663 | 21.010 | -.093 |
| 5 | -12.092 | 13.488 | -68.991 | .663 | 21.009 | -.086 |
| 5 | -12.092 | 13.489 | -68.992 | .664 | 21.008 | -.079 |
| 5 | -12.092 | 13.490 | -68.993 | .664 | 21.007 | -.072 |
| 5 | -12.092 | 13.490 | -68.993 | .664 | 21.007 | -.065 |
| 5 | -12.092 | 13.491 | -68.994 | .665 | 21.006 | -.058 |
| 5 | -12.092 | 13.492 | -68.995 | .665 | 21.005 | -.050 |
| 5 | -12.092 | 13.493 | -68.996 | .665 | 21.004 | -.043 |
| 5 | -12.092 | 13.493 | -68.996 | .666 | 21.004 | -.036 |
| 5 | -12.092 | 13.494 | -68.997 | .666 | 21.003 | -.029 |
| 5 | -12.092 | 13.495 | -68.998 | .666 | 21.002 | -.022 |
| 5 | -12.092 | 13.496 | -68.999 | .667 | 21.001 | -.014 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 5 | -12.092 | 13.496 | -68.999 | .667 | 21.001 | -.007 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.496 | -68.999 | .667 | 21.001 | -.007 |
| 5 | -12.092 | 13.496 | -68.999 | .667 | 21.001 | -.006 |
| 5 | -12.092 | 13.496 | -68.999 | .667 | 21.001 | -.006 |
| 5 | -12.092 | 13.497 | -68.999 | .667 | 21.001 | -.005 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.004 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.004 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.003 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.002 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.001 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 5 | -12.092 | 13.497 | -69.000 | .667 | 21.000 | -.000 |
| 6 | -16.140 | 11.120 | -66.623 | 0.000 | 23.377 | -16.140 |
| 6 | -16.140 | 11.239 | -66.742 | .040 | 23.258 | -15.519 |
| 6 | -16.140 | 11.359 | -66.862 | .080 | 23.138 | -14.882 |
| 6 | -16.140 | 11.479 | -66.982 | .120 | 23.018 | -14.227 |
| 6 | -16.140 | 11.601 | -67.104 | .160 | 22.896 | -13.556 |
| 6 | -16.140 | 11.723 | -67.226 | .200 | 22.774 | -12.866 |
| 6 | -16.140 | 11.845 | -67.348 | .240 | 22.652 | -12.157 |
| 6 | -16.140 | 11.968 | -67.471 | .280 | 22.529 | -11.430 |
| 6 | -16.140 | 12.090 | -67.593 | .320 | 22.407 | -10.683 |
| 6 | -16.140 | 12.213 | -67.716 | .360 | 22.284 | -9.916 |
| 6 | -16.140 | 12.336 | -67.839 | .400 | 22.161 | -9.128 |
| 6 | -16.140 | 12.458 | -67.961 | .440 | 22.039 | -8.319 |
| 6 | -16.140 | 12.579 | -68.082 | .480 | 21.918 | -7.488 |
| 6 | -16.140 | 12.700 | -68.203 | .521 | 21.797 | -6.635 |
| 6 | -16.140 | 12.819 | -68.322 | .561 | 21.678 | -5.759 |
| 6 | -16.140 | 12.938 | -68.441 | .601 | 21.559 | -4.860 |
| 6 | -16.140 | 13.054 | -68.557 | .641 | 21.443 | -3.937 |
| 6 | -16.140 | 13.169 | -68.672 | .681 | 21.328 | -2.990 |
| 6 | -16.140 | 13.281 | -68.784 | .721 | 21.216 | -2.019 |
| 6 | -16.140 | 13.390 | -68.893 | .761 | 21.107 | -1.022 |
| 6 | -16.140 | 13.281 | -68.784 | .721 | 21.216 | -2.019 |

# TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 6 | -16.140 | 13.292 | -68.795 | .725 | 21.205 | -1.920 |
| 6 | -16.140 | 13.303 | -68.806 | .729 | 21.194 | -1.821 |
| 6 | -16.140 | 13.314 | -68.817 | .733 | 21.183 | -1.722 |
| 6 | -16.140 | 13.325 | -68.828 | .737 | 21.172 | -1.623 |
| 6 | -16.140 | 13.336 | -68.839 | .741 | 21.161 | -1.524 |
| 6 | -16.140 | 13.347 | -68.850 | .745 | 21.150 | -1.424 |
| 6 | -16.140 | 13.358 | -68.861 | .749 | 21.139 | -1.324 |
| 6 | -16.140 | 13.369 | -68.872 | .753 | 21.128 | -1.223 |
| 6 | -16.140 | 13.380 | -68.883 | .757 | 21.117 | -1.123 |
| 6 | -16.140 | 13.390 | -68.893 | .761 | 21.107 | -1.022 |
| 6 | -16.140 | 13.401 | -68.904 | .765 | 21.096 | -.921 |
| 6 | -16.140 | 13.412 | -68.915 | .769 | 21.085 | -.820 |
| 6 | -16.140 | 13.423 | -68.926 | .773 | 21.074 | -.718 |
| 6 | -16.140 | 13.433 | -68.936 | .777 | 21.064 | -.616 |
| 6 | -16.140 | 13.444 | -68.947 | .781 | 21.053 | -.514 |
| 6 | -16.140 | 13.455 | -68.958 | .785 | 21.042 | -.412 |
| 6 | -16.140 | 13.465 | -68.968 | .789 | 21.032 | -.309 |
| 6 | -16.140 | 13.476 | -68.979 | .793 | 21.021 | -.206 |
| 6 | -16.140 | 13.487 | -68.989 | .797 | 21.011 | -.103 |
| 6 | -16.140 | 13.476 | -68.979 | .793 | 21.021 | -.206 |
| 6 | -16.140 | 13.477 | -68.980 | .793 | 21.020 | -.196 |
| 6 | -16.140 | 13.478 | -68.981 | .794 | 21.019 | -.186 |
| 6 | -16.140 | 13.479 | -68.982 | .794 | 21.018 | -.176 |
| 6 | -16.140 | 13.480 | -68.983 | .794 | 21.017 | -.165 |
| 6 | -16.140 | 13.481 | -68.984 | .795 | 21.016 | -.155 |
| 6 | -16.140 | 13.482 | -68.985 | .795 | 21.015 | -.145 |
| 6 | -16.140 | 13.483 | -68.986 | .796 | 21.014 | -.134 |
| 6 | -16.140 | 13.484 | -68.987 | .796 | 21.013 | -.124 |
| 6 | -16.140 | 13.485 | -68.988 | .796 | 21.012 | -.114 |
| 6 | -16.140 | 13.487 | -68.989 | .797 | 21.011 | -.103 |
| 6 | -16.140 | 13.488 | -68.991 | .797 | 21.009 | -.093 |
| 6 | -16.140 | 13.489 | -68.992 | .798 | 21.008 | -.083 |
| 6 | -16.140 | 13.490 | -68.993 | .798 | 21.007 | -.072 |
| 6 | -16.140 | 13.491 | -68.994 | .798 | 21.006 | -.062 |
| 6 | -16.140 | 13.492 | -68.995 | .799 | 21.005 | -.052 |
| 6 | -16.140 | 13.493 | -68.996 | .799 | 21.004 | -.041 |
| 6 | -16.140 | 13.494 | -68.997 | .800 | 21.003 | -.031 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.021 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.010 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.021 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.020 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.019 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.018 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.017 |
| 6 | -16.140 | 13.495 | -68.998 | .800 | 21.002 | -.016 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.014 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.013 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.012 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.011 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.010 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.009 |
| 6 | -16.140 | 13.496 | -68.999 | .800 | 21.001 | -.008 |
| 6 | -16.140 | 13.496 | -68.999 | .801 | 21.001 | -.007 |
| 6 | -16.140 | 13.496 | -68.999 | .801 | 21.001 | -.006 |
| 6 | -16.140 | 13.497 | -68.999 | .801 | 21.001 | -.005 |
| 6 | -16.140 | 13.497 | -69.000 | .801 | 21.000 | -.004 |
| 6 | -16.140 | 13.497 | -69.000 | .801 | 21.000 | -.003 |
| 6 | -16.140 | 13.497 | -69.000 | .801 | 21.000 | -.002 |
| 6 | -16.140 | 13.497 | -69.000 | .801 | 21.000 | -.001 |
| 6 | -16.140 | 13.497 | -69.000 | .801 | 21.000 | -.000 |
| 7 | -21.114 | 10.088 | -65.591 | 0.000 | 24.409 | -21.114 |
| 7 | -21.114 | 10.246 | -65.749 | .047 | 24.251 | -20.394 |
| 7 | -21.114 | 10.406 | -65.909 | .093 | 24.091 | -19.647 |
| 7 | -21.114 | 10.570 | -66.073 | .140 | 23.927 | -18.872 |
| 7 | -21.114 | 10.736 | -66.239 | .187 | 23.761 | -18.068 |
| 7 | -21.114 | 10.905 | -66.408 | .234 | 23.592 | -17.233 |
| 7 | -21.114 | 11.076 | -66.579 | .280 | 23.421 | -16.366 |
| 7 | -21.114 | 11.249 | -66.752 | .327 | 23.248 | -15.466 |
| 7 | -21.114 | 11.424 | -66.926 | .374 | 23.074 | -14.531 |
| 7 | -21.114 | 11.600 | -67.103 | .420 | 22.897 | -13.560 |
| 7 | -21.114 | 11.778 | -67.280 | .467 | 22.720 | -12.550 |
| 7 | -21.114 | 11.956 | -67.459 | .514 | 22.541 | -11.500 |
| 7 | -21.114 | 12.135 | -67.637 | .561 | 22.363 | -10.410 |
| 7 | -21.114 | 12.313 | -67.816 | .607 | 22.184 | -9.276 |
| 7 | -21.114 | 12.491 | -67.994 | .654 | 22.006 | -8.097 |
| 7 | -21.114 | 12.667 | -68.170 | .701 | 21.830 | -6.872 |
| 7 | -21.114 | 12.841 | -68.344 | .747 | 21.656 | -5.599 |
| 7 | -21.114 | 13.012 | -68.515 | .794 | 21.485 | -4.277 |
| 7 | -21.114 | 13.179 | -68.682 | .841 | 21.318 | -2.904 |
| 7 | -21.114 | 13.341 | -68.844 | .888 | 21.156 | -1.479 |
| 7 | -21.114 | 13.179 | -68.682 | .841 | 21.318 | -2.904 |
| 7 | -21.114 | 13.195 | -68.698 | .846 | 21.302 | -2.764 |
| 7 | -21.114 | 13.212 | -68.715 | .850 | 21.285 | -2.623 |
| 7 | -21.114 | 13.228 | -68.731 | .855 | 21.269 | -2.482 |
| 7 | -21.114 | 13.244 | -68.747 | .860 | 21.253 | -2.340 |
| 7 | -21.114 | 13.260 | -68.763 | .864 | 21.237 | -2.198 |
| 7 | -21.114 | 13.277 | -68.780 | .869 | 21.220 | -2.055 |
| 7 | -21.114 | 13.293 | -68.796 | .874 | 21.204 | -1.912 |
| 7 | -21.114 | 13.309 | -68.812 | .878 | 21.188 | -1.768 |
| 7 | -21.114 | 13.325 | -68.828 | .883 | 21.172 | -1.624 |
| 7 | -21.114 | 13.341 | -68.844 | .888 | 21.156 | -1.479 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 7 | -21.114 | 13.357 | -68.860 | .892 | 21.140 | -1.333 |
| 7 | -21.114 | 13.373 | -68.876 | .897 | 21.124 | -1.187 |
| 7 | -21.114 | 13.388 | -68.891 | .902 | 21.109 | -1.041 |
| 7 | -21.114 | 13.404 | -68.907 | .906 | 21.093 | -.894 |
| 7 | -21.114 | 13.420 | -68.923 | .911 | 21.077 | -.746 |
| 7 | -21.114 | 13.435 | -68.938 | .916 | 21.062 | -.598 |
| 7 | -21.114 | 13.451 | -68.954 | .920 | 21.046 | -.449 |
| 7 | -21.114 | 13.466 | -68.969 | .925 | 21.031 | -.300 |
| 7 | -21.114 | 13.482 | -68.985 | .930 | 21.015 | -.150 |
| 7 | -21.114 | 13.497 | -69.000 | .934 | 21.000 | -.000 |
| 8 | -27.265 | 8.626 | -64.129 | 0.000 | 25.871 | -27.265 |
| 8 | -27.265 | 8.820 | -64.323 | .053 | 25.677 | -26.493 |
| 8 | -27.265 | 9.021 | -64.524 | .107 | 25.476 | -25.681 |
| 8 | -27.265 | 9.229 | -64.732 | .160 | 25.268 | -24.827 |
| 8 | -27.265 | 9.444 | -64.947 | .214 | 25.053 | -23.928 |
| 8 | -27.265 | 9.665 | -65.168 | .267 | 24.832 | -22.980 |
| 8 | -27.265 | 9.894 | -65.397 | .320 | 24.603 | -21.981 |
| 8 | -27.265 | 10.129 | -65.632 | .374 | 24.368 | -20.926 |
| 8 | -27.265 | 10.371 | -65.874 | .427 | 24.126 | -19.812 |
| 8 | -27.265 | 10.620 | -66.122 | .480 | 23.878 | -18.634 |
| 8 | -27.265 | 10.874 | -66.377 | .534 | 23.623 | -17.387 |
| 8 | -27.265 | 11.134 | -66.637 | .587 | 23.363 | -16.067 |
| 8 | -27.265 | 11.398 | -66.901 | .641 | 23.099 | -14.669 |
| 8 | -27.265 | 11.666 | -67.169 | .694 | 22.831 | -13.188 |
| 8 | -27.265 | 11.936 | -67.439 | .747 | 22.561 | -11.617 |
| 8 | -27.265 | 12.208 | -67.711 | .801 | 22.289 | -9.951 |
| 8 | -27.265 | 12.478 | -67.981 | .854 | 22.019 | -8.184 |
| 8 | -27.265 | 12.745 | -68.248 | .908 | 21.752 | -6.311 |
| 8 | -27.265 | 13.006 | -68.509 | .961 | 21.491 | -4.326 |
| 8 | -27.265 | 13.258 | -68.761 | 1.014 | 21.239 | -2.224 |
| 8 | -27.265 | 13.006 | -68.509 | .961 | 21.491 | -4.326 |
| 8 | -27.265 | 13.031 | -68.534 | .966 | 21.466 | -4.121 |
| 8 | -27.265 | 13.057 | -68.560 | .972 | 21.440 | -3.915 |
| 8 | -27.265 | 13.082 | -68.585 | .977 | 21.415 | -3.708 |
| 8 | -27.265 | 13.108 | -68.611 | .982 | 21.389 | -3.500 |
| 8 | -27.265 | 13.133 | -68.636 | .988 | 21.364 | -3.290 |
| 8 | -27.265 | 13.158 | -68.661 | .993 | 21.339 | -3.079 |
| 8 | -27.265 | 13.183 | -68.686 | .998 | 21.314 | -2.867 |
| 8 | -27.265 | 13.208 | -68.711 | 1.004 | 21.289 | -2.654 |
| 8 | -27.265 | 13.233 | -68.736 | 1.009 | 21.264 | -2.440 |
| 8 | -27.265 | 13.258 | -68.761 | 1.014 | 21.239 | -2.224 |
| 8 | -27.265 | 13.282 | -68.785 | 1.020 | 21.215 | -2.007 |
| 8 | -27.265 | 13.307 | -68.810 | 1.025 | 21.190 | -1.789 |
| 8 | -27.265 | 13.331 | -68.834 | 1.030 | 21.166 | -1.570 |
| 8 | -27.265 | 13.355 | -68.858 | 1.036 | 21.142 | -1.349 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 8 | -27.265 | 13.379 | -68.882 | 1.041 | 21.118 | -1.127 |
| 8 | -27.265 | 13.403 | -68.906 | 1.046 | 21.094 | -.904 |
| 8 | -27.265 | 13.427 | -68.930 | 1.052 | 21.070 | -.680 |
| 8 | -27.265 | 13.450 | -68.953 | 1.057 | 21.047 | -.455 |
| 8 | -27.265 | 13.474 | -68.977 | 1.062 | 21.023 | -.228 |
| 8 | -27.265 | 13.450 | -68.953 | 1.057 | 21.047 | -.455 |
| 8 | -27.265 | 13.453 | -68.956 | 1.058 | 21.044 | -.432 |
| 8 | -27.265 | 13.455 | -68.958 | 1.058 | 21.042 | -.409 |
| 8 | -27.265 | 13.457 | -68.960 | 1.059 | 21.040 | -.387 |
| 8 | -27.265 | 13.460 | -68.963 | 1.059 | 21.037 | -.364 |
| 8 | -27.265 | 13.462 | -68.965 | 1.060 | 21.035 | -.341 |
| 8 | -27.265 | 13.464 | -68.967 | 1.060 | 21.033 | -.319 |
| 8 | -27.265 | 13.467 | -68.970 | 1.061 | 21.030 | -.296 |
| 8 | -27.265 | 13.469 | -68.972 | 1.061 | 21.028 | -.273 |
| 8 | -27.265 | 13.471 | -68.974 | 1.062 | 21.026 | -.251 |
| 8 | -27.265 | 13.474 | -68.977 | 1.062 | 21.023 | -.228 |
| 8 | -27.265 | 13.476 | -68.979 | 1.063 | 21.021 | -.205 |
| 8 | -27.265 | 13.478 | -68.981 | 1.063 | 21.019 | -.182 |
| 8 | -27.265 | 13.481 | -68.984 | 1.064 | 21.016 | -.160 |
| 8 | -27.265 | 13.483 | -68.986 | 1.065 | 21.014 | -.137 |
| 8 | -27.265 | 13.485 | -68.988 | 1.065 | 21.012 | -.114 |
| 8 | -27.265 | 13.488 | -68.991 | 1.066 | 21.009 | -.091 |
| 8 | -27.265 | 13.490 | -68.993 | 1.066 | 21.007 | -.069 |
| 8 | -27.265 | 13.492 | -68.995 | 1.067 | 21.005 | -.046 |
| 8 | -27.265 | 13.495 | -68.998 | 1.067 | 21.002 | -.023 |
| 8 | -27.265 | 13.492 | -68.995 | 1.067 | 21.005 | -.046 |
| 8 | -27.265 | 13.493 | -68.996 | 1.067 | 21.004 | -.043 |
| 8 | -27.265 | 13.493 | -68.996 | 1.067 | 21.004 | -.041 |
| 8 | -27.265 | 13.493 | -68.996 | 1.067 | 21.004 | -.039 |
| 8 | -27.265 | 13.493 | -68.996 | 1.067 | 21.004 | -.037 |
| 8 | -27.265 | 13.494 | -68.997 | 1.067 | 21.003 | -.034 |
| 8 | -27.265 | 13.494 | -68.997 | 1.067 | 21.003 | -.032 |
| 8 | -27.265 | 13.494 | -68.997 | 1.067 | 21.003 | -.030 |
| 8 | -27.265 | 13.494 | -68.997 | 1.067 | 21.003 | -.027 |
| 8 | -27.265 | 13.494 | -68.997 | 1.067 | 21.003 | -.025 |
| 8 | -27.265 | 13.495 | -68.998 | 1.067 | 21.002 | -.023 |
| 8 | -27.265 | 13.495 | -68.998 | 1.067 | 21.002 | -.021 |
| 8 | -27.265 | 13.495 | -68.998 | 1.067 | 21.002 | -.018 |
| 8 | -27.265 | 13.495 | -68.998 | 1.067 | 21.002 | -.016 |
| 8 | -27.265 | 13.496 | -68.999 | 1.067 | 21.001 | -.014 |
| 8 | -27.265 | 13.496 | -68.999 | 1.067 | 21.001 | -.011 |
| 8 | -27.265 | 13.496 | -68.999 | 1.068 | 21.001 | -.009 |
| 8 | -27.265 | 13.496 | -68.999 | 1.068 | 21.001 | -.007 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.005 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.002 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.005 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.004 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.004 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.004 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.004 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.003 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.003 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.003 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.003 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.003 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.002 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.002 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.002 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.002 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.001 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.001 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.001 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.001 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.000 |
| 8 | -27.265 | 13.497 | -69.000 | 1.068 | 21.000 | -.000 |
| 9 | -34.865 | 6.562 | -62.065 | 0.000 | 27.935 | -34.865 |
| 9 | -34.865 | 6.769 | -62.272 | .060 | 27.728 | -34.142 |
| 9 | -34.865 | 6.989 | -62.492 | .120 | 27.508 | -33.368 |
| 9 | -34.865 | 7.221 | -62.724 | .180 | 27.276 | -32.539 |
| 9 | -34.865 | 7.467 | -62.970 | .240 | 27.030 | -31.649 |
| 9 | -34.865 | 7.728 | -63.231 | .300 | 26.769 | -30.693 |
| 9 | -34.865 | 8.004 | -63.507 | .360 | 26.493 | -29.661 |
| 9 | -34.865 | 8.296 | -63.799 | .420 | 26.201 | -28.547 |
| 9 | -34.865 | 8.606 | -64.109 | .480 | 25.891 | -27.341 |
| 9 | -34.865 | 8.935 | -64.437 | .541 | 25.563 | -26.032 |
| 9 | -34.865 | 9.282 | -64.785 | .601 | 25.215 | -24.608 |
| 9 | -34.865 | 9.648 | -65.151 | .661 | 24.849 | -23.054 |
| 9 | -34.865 | 10.034 | -65.537 | .721 | 24.463 | -21.356 |
| 9 | -34.865 | 10.439 | -65.942 | .781 | 24.058 | -19.495 |
| 9 | -34.865 | 10.861 | -66.364 | .841 | 23.636 | -17.452 |
| 9 | -34.865 | 11.298 | -66.801 | .901 | 23.199 | -15.204 |
| 9 | -34.865 | 11.747 | -67.250 | .961 | 22.750 | -12.727 |
| 9 | -34.865 | 12.201 | -67.704 | 1.021 | 22.296 | -9.994 |
| 9 | -34.865 | 12.652 | -68.155 | 1.081 | 21.845 | -6.980 |
| 9 | -34.865 | 13.089 | -68.592 | 1.141 | 21.408 | -3.656 |
| 9 | -34.865 | 12.652 | -68.155 | 1.081 | 21.845 | -6.980 |
| 9 | -34.865 | 12.696 | -68.199 | 1.087 | 21.801 | -6.662 |
| 9 | -34.865 | 12.741 | -68.244 | 1.093 | 21.756 | -6.341 |
| 9 | -34.865 | 12.785 | -68.288 | 1.099 | 21.712 | -6.016 |
| 9 | -34.865 | 12.829 | -68.332 | 1.105 | 21.668 | -5.689 |
| 9 | -34.865 | 12.873 | -68.376 | 1.111 | 21.624 | -5.358 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 9 | -34.865 | 12.916 | -68.419 | 1.117 | 21.581 | -5.024 |
| 9 | -34.865 | 12.960 | -68.463 | 1.123 | 21.537 | -4.687 |
| 9 | -34.865 | 13.003 | -68.506 | 1.129 | 21.494 | -4.347 |
| 9 | -34.865 | 13.046 | -68.549 | 1.135 | 21.451 | -4.003 |
| 9 | -34.865 | 13.089 | -68.592 | 1.141 | 21.408 | -3.656 |
| 9 | -34.865 | 13.131 | -68.634 | 1.147 | 21.366 | -3.306 |
| 9 | -34.865 | 13.173 | -68.676 | 1.153 | 21.324 | -2.952 |
| 9 | -34.865 | 13.215 | -68.718 | 1.159 | 21.282 | -2.595 |
| 9 | -34.865 | 13.256 | -68.759 | 1.165 | 21.241 | -2.235 |
| 9 | -34.865 | 13.297 | -68.800 | 1.171 | 21.200 | -1.871 |
| 9 | -34.865 | 13.338 | -68.841 | 1.177 | 21.159 | -1.504 |
| 9 | -34.865 | 13.378 | -68.881 | 1.183 | 21.119 | -1.133 |
| 9 | -34.865 | 13.418 | -68.921 | 1.189 | 21.079 | -.759 |
| 9 | -34.865 | 13.458 | -68.961 | 1.195 | 21.039 | -.381 |
| 9 | -34.865 | 13.418 | -68.921 | 1.189 | 21.079 | -.759 |
| 9 | -34.865 | 13.422 | -68.925 | 1.190 | 21.075 | -.721 |
| 9 | -34.865 | 13.426 | -68.929 | 1.190 | 21.071 | -.684 |
| 9 | -34.865 | 13.430 | -68.933 | 1.191 | 21.067 | -.646 |
| 9 | -34.865 | 13.434 | -68.937 | 1.192 | 21.063 | -.608 |
| 9 | -34.865 | 13.438 | -68.941 | 1.192 | 21.059 | -.570 |
| 9 | -34.865 | 13.442 | -68.945 | 1.193 | 21.055 | -.533 |
| 9 | -34.865 | 13.446 | -68.949 | 1.193 | 21.051 | -.495 |
| 9 | -34.865 | 13.450 | -68.953 | 1.194 | 21.047 | -.457 |
| 9 | -34.865 | 13.454 | -68.957 | 1.195 | 21.043 | -.419 |
| 9 | -34.865 | 13.458 | -68.961 | 1.195 | 21.039 | -.381 |
| 9 | -34.865 | 13.462 | -68.965 | 1.196 | 21.035 | -.343 |
| 9 | -34.865 | 13.466 | -68.969 | 1.196 | 21.031 | -.305 |
| 9 | -34.865 | 13.470 | -68.973 | 1.197 | 21.027 | -.267 |
| 9 | -34.865 | 13.474 | -68.977 | 1.198 | 21.023 | -.229 |
| 9 | -34.865 | 13.478 | -68.981 | 1.198 | 21.019 | -.191 |
| 9 | -34.865 | 13.481 | -68.984 | 1.199 | 21.016 | -.153 |
| 9 | -34.865 | 13.485 | -68.988 | 1.199 | 21.012 | -.115 |
| 9 | -34.865 | 13.489 | -68.992 | 1.200 | 21.008 | -.077 |
| 9 | -34.865 | 13.493 | -68.996 | 1.201 | 21.004 | -.038 |
| 9 | -34.865 | 13.489 | -68.992 | 1.200 | 21.008 | -.077 |
| 9 | -34.865 | 13.490 | -68.993 | 1.200 | 21.007 | -.073 |
| 9 | -34.865 | 13.490 | -68.993 | 1.200 | 21.007 | -.069 |
| 9 | -34.865 | 13.490 | -68.993 | 1.200 | 21.007 | -.065 |
| 9 | -34.865 | 13.491 | -68.994 | 1.200 | 21.006 | -.061 |
| 9 | -34.865 | 13.491 | -68.994 | 1.200 | 21.006 | -.057 |
| 9 | -34.865 | 13.492 | -68.995 | 1.200 | 21.005 | -.054 |
| 9 | -34.865 | 13.492 | -68.995 | 1.200 | 21.005 | -.050 |
| 9 | -34.865 | 13.492 | -68.995 | 1.201 | 21.005 | -.046 |
| 9 | -34.865 | 13.493 | -68.996 | 1.201 | 21.004 | -.042 |
| 9 | -34.865 | 13.493 | -68.996 | 1.201 | 21.004 | -.038 |
| 9 | -34.865 | 13.494 | -68.997 | 1.201 | 21.003 | -.034 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 9 | -34.865 | 13.494 | -68.997 | 1.201 | 21.003 | -.031 |
| 9 | -34.865 | 13.494 | -68.997 | 1.201 | 21.003 | -.027 |
| 9 | -34.865 | 13.495 | -68.998 | 1.201 | 21.002 | -.023 |
| 9 | -34.865 | 13.495 | -68.998 | 1.201 | 21.002 | -.019 |
| 9 | -34.865 | 13.495 | -68.998 | 1.201 | 21.002 | -.015 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.011 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.008 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.004 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.008 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.007 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.007 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.007 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.006 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.006 |
| 9 | -34.865 | 13.496 | -68.999 | 1.201 | 21.001 | -.005 |
| 9 | -34.865 | 13.497 | -68.999 | 1.201 | 21.001 | -.005 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.005 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.004 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.004 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.003 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.003 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.003 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.002 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.002 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.002 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.001 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.001 |
| 9 | -34.865 | 13.497 | -69.000 | 1.201 | 21.000 | -.000 |
| 10 | -44.118 | 3.740 | -59.243 | 0.000 | 30.757 | -44.118 |
| 10 | -44.118 | 3.899 | -59.402 | .067 | 30.598 | -43.620 |
| 10 | -44.118 | 4.071 | -59.574 | .133 | 30.426 | -43.076 |
| 10 | -44.118 | 4.259 | -59.762 | .200 | 30.238 | -42.482 |
| 10 | -44.118 | 4.463 | -59.966 | .267 | 30.034 | -41.829 |
| 10 | -44.118 | 4.688 | -60.191 | .334 | 29.809 | -41.109 |
| 10 | -44.118 | 4.935 | -60.438 | .400 | 29.562 | -40.310 |
| 10 | -44.118 | 5.208 | -60.711 | .467 | 29.289 | -39.420 |
| 10 | -44.118 | 5.511 | -61.014 | .534 | 28.986 | -38.423 |
| 10 | -44.118 | 5.848 | -61.351 | .601 | 28.649 | -37.298 |
| 10 | -44.118 | 6.226 | -61.729 | .667 | 28.271 | -36.021 |
| 10 | -44.118 | 6.650 | -62.153 | .734 | 27.847 | -34.559 |
| 10 | -44.118 | 7.128 | -62.631 | .801 | 27.369 | -32.373 |
| 10 | -44.118 | 7.669 | -63.172 | .868 | 26.328 | -30.909 |
| 10 | -44.118 | 8.283 | -63.786 | .934 | 26.214 | -28.600 |
| 10 | -44.118 | 8.978 | -64.481 | 1.001 | 25.519 | -25.856 |
| 10 | -44.118 | 9.763 | -65.266 | 1.068 | 24.734 | -22.556 |

# TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 10 | -44.118 | 10.638 | -66.141 | 1.134 | 23.859 | -18.544 |
| 10 | -44.118 | 11.590 | -67.092 | 1.201 | 22.908 | -13.618 |
| 10 | -44.118 | 12.574 | -68.077 | 1.268 | 21.923 | -7.527 |
| 10 | -44.118 | 11.590 | -67.092 | 1.201 | 22.908 | -13.618 |
| 10 | -44.118 | 11.688 | -67.191 | 1.208 | 22.809 | -13.066 |
| 10 | -44.118 | 11.786 | -67.289 | 1.215 | 22.711 | -12.501 |
| 10 | -44.118 | 11.884 | -67.387 | 1.221 | 22.613 | -11.925 |
| 10 | -44.118 | 11.983 | -67.486 | 1.228 | 22.514 | -11.336 |
| 10 | -44.118 | 12.082 | -67.585 | 1.235 | 22.415 | -10.735 |
| 10 | -44.118 | 12.181 | -67.684 | 1.241 | 22.316 | -10.120 |
| 10 | -44.118 | 12.279 | -67.782 | 1.248 | 22.218 | -9.492 |
| 10 | -44.118 | 12.378 | -67.881 | 1.255 | 22.119 | -8.851 |
| 10 | -44.118 | 12.476 | -67.979 | 1.261 | 22.021 | -8.196 |
| 10 | -44.118 | 12.574 | -68.077 | 1.268 | 21.923 | -7.527 |
| 10 | -44.118 | 12.671 | -68.174 | 1.275 | 21.826 | -6.843 |
| 10 | -44.118 | 12.767 | -68.270 | 1.281 | 21.730 | -6.145 |
| 10 | -44.118 | 12.863 | -68.366 | 1.288 | 21.634 | -5.431 |
| 10 | -44.118 | 12.958 | -68.461 | 1.295 | 21.539 | -4.703 |
| 10 | -44.118 | 13.051 | -68.554 | 1.301 | 21.446 | -3.959 |
| 10 | -44.118 | 13.144 | -68.647 | 1.308 | 21.353 | -3.199 |
| 10 | -44.118 | 13.235 | -68.738 | 1.315 | 21.262 | -2.424 |
| 10 | -44.118 | 13.324 | -68.827 | 1.321 | 21.173 | -1.632 |
| 10 | -44.118 | 13.411 | -68.914 | 1.328 | 21.086 | -.824 |
| 10 | -44.118 | 13.324 | -68.827 | 1.321 | 21.173 | -1.632 |
| 10 | -44.118 | 13.333 | -68.836 | 1.322 | 21.164 | -1.552 |
| 10 | -44.118 | 13.342 | -68.845 | 1.323 | 21.155 | -1.472 |
| 10 | -44.118 | 13.350 | -68.853 | 1.323 | 21.147 | -1.392 |
| 10 | -44.118 | 13.359 | -68.862 | 1.324 | 21.138 | -1.311 |
| 10 | -44.118 | 13.368 | -68.871 | 1.325 | 21.129 | -1.230 |
| 10 | -44.118 | 13.377 | -68.880 | 1.325 | 21.120 | -1.149 |
| 10 | -44.118 | 13.385 | -68.888 | 1.326 | 21.112 | -1.068 |
| 10 | -44.118 | 13.394 | -68.897 | 1.327 | 21.103 | -.987 |
| 10 | -44.118 | 13.403 | -68.906 | 1.327 | 21.094 | -.906 |
| 10 | -44.118 | 13.411 | -68.914 | 1.328 | 21.086 | -.824 |
| 10 | -44.118 | 13.420 | -68.923 | 1.329 | 21.077 | -.743 |
| 10 | -44.118 | 13.429 | -68.932 | 1.329 | 21.068 | -.661 |
| 10 | -44.118 | 13.437 | -68.940 | 1.330 | 21.060 | -.579 |
| 10 | -44.118 | 13.446 | -68.949 | 1.331 | 21.051 | -.497 |
| 10 | -44.118 | 13.455 | -68.957 | 1.331 | 21.043 | -.414 |
| 10 | -44.118 | 13.463 | -68.966 | 1.332 | 21.034 | -.332 |
| 10 | -44.118 | 13.472 | -68.975 | 1.333 | 21.025 | -.249 |
| 10 | -44.118 | 13.480 | -68.983 | 1.333 | 21.017 | -.166 |
| 10 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.083 |
| 10 | -44.118 | 13.480 | -68.983 | 1.333 | 21.017 | -.166 |
| 10 | -44.118 | 13.481 | -68.984 | 1.333 | 21.016 | -.158 |
| 10 | -44.118 | 13.482 | -68.985 | 1.333 | 21.015 | -.150 |

## TABLE A-III (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 10 | -44.118 | 13.483 | -68.986 | 1.334 | 21.014 | -.141 |
| 10 | -44.118 | 13.483 | -68.986 | 1.334 | 21.014 | -.133 |
| 10 | -44.118 | 13.484 | -68.987 | 1.334 | 21.013 | -.125 |
| 10 | -44.118 | 13.485 | -68.988 | 1.334 | 21.012 | -.116 |
| 10 | -44.118 | 13.486 | -68.989 | 1.334 | 21.011 | -.108 |
| 10 | -44.118 | 13.487 | -68.990 | 1.334 | 21.010 | -.100 |
| 10 | -44.118 | 13.488 | -68.991 | 1.334 | 21.009 | -.091 |
| 10 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.083 |
| 10 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.075 |
| 10 | -44.118 | 13.490 | -68.993 | 1.334 | 21.007 | -.067 |
| 10 | -44.118 | 13.491 | -68.994 | 1.334 | 21.006 | -.058 |
| 10 | -44.118 | 13.492 | -68.995 | 1.334 | 21.005 | -.050 |
| 10 | -44.118 | 13.493 | -68.996 | 1.334 | 21.004 | -.042 |
| 10 | -44.118 | 13.494 | -68.997 | 1.334 | 21.003 | -.033 |
| 10 | -44.118 | 13.495 | -68.997 | 1.334 | 21.003 | -.025 |
| 10 | -44.118 | 13.495 | -68.998 | 1.335 | 21.002 | -.017 |
| 10 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.008 |
| 10 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.008 |
| 10 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.007 |
| 10 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.007 |
| 10 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.006 |
| 10 | -44.118 | 13.497 | -68.999 | 1.335 | 21.001 | -.005 |
| 10 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.004 |
| 10 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.003 |
| 10 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.002 |
| 10 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.002 |
| 10 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.001 |

## TABLE A-IV

SECOND SURFACE COORDINATE (X,Y) VALUES, S, ALPHA2 AND RHO2 FOR THE FAMILY OF SURFACES IN FIGURES 17 AND 18. THE COLUMN LABELED BEGIN RAY CORRESPONDS TO THE FAMILY MEMBER.

| Begin RAY | Next RAY | Xsurf (next) | Ysurf (next) | S (next) | Alpha2 (next) | RHO 2 (next) |
|---|---|---|---|---|---|---|
| 1 | 1 | .362 | .000 | .133 | 21.000 | -.000 |
| 1 | 2 | .649 | .110 | .133 | 21.201 | -1.880 |
| 1 | 3 | .935 | .221 | .132 | 21.466 | -4.122 |
| 1 | 4 | 1.220 | .333 | .130 | 21.324 | -6.831 |
| 1 | 5 | 1.504 | .447 | .125 | 22.321 | -10.147 |
| 1 | 6 | 1.786 | .563 | .118 | 23.024 | -14.262 |
| 1 | 7 | 2.067 | .682 | .107 | 24.043 | -19.425 |
| 1 | 8 | 2.344 | .805 | .090 | 25.538 | -25.934 |
| 1 | 9 | 2.616 | .936 | .066 | 27.707 | -34.070 |
| 1 | 10 | 2.882 | 1.076 | .030 | 30.688 | -43.901 |
| 2 | 2 | .724 | .000 | .267 | 21.201 | -.000 |
| 2 | 3 | 1.010 | .111 | .266 | 21.228 | -2.125 |
| 2 | 4 | 1.296 | .222 | .265 | 21.538 | -4.693 |
| 2 | 5 | 1.581 | .334 | .262 | 21.973 | -7.870 |
| 2 | 6 | 1.365 | .449 | .256 | 22.603 | -11.868 |
| 2 | 7 | 2.146 | .566 | .248 | 23.542 | -16.978 |
| 2 | 8 | 2.425 | .687 | .234 | 24.971 | -23.577 |
| 2 | 9 | 2.699 | .815 | .212 | 27.147 | -32.074 |
| 2 | 10 | 2.967 | .953 | .179 | 30.300 | -42.677 |
| 3 | 3 | 1.086 | .000 | .400 | 21.228 | -.000 |
| 3 | 4 | 1.372 | .111 | .399 | 21.262 | -2.420 |
| 3 | 5 | 1.658 | .222 | .398 | 21.633 | -5.420 |
| 3 | 6 | 1.943 | .335 | .394 | 22.180 | -9.251 |
| 3 | 7 | 2.226 | .450 | .388 | 23.021 | -14.243 |
| 3 | 8 | 2.506 | .570 | .377 | 24.354 | -20.865 |
| 3 | 9 | 2.782 | .695 | .358 | 26.502 | -29.695 |
| 3 | 10 | 3.052 | .829 | .329 | 29.826 | -41.164 |
| 4 | 4 | 1.449 | .000 | .534 | 21.262 | -.000 |
| 4 | 5 | 1.734 | .111 | .532 | 21.307 | -2.810 |
| 4 | 6 | 2.020 | .223 | .531 | 21.766 | -6.414 |
| 4 | 7 | 2.304 | .336 | .527 | 22.493 | -11.208 |
| 4 | 8 | 2.586 | .453 | .518 | 23.698 | -17.757 |
| 4 | 9 | 2.864 | .575 | .504 | 25.766 | -26.849 |
| 4 | 10 | 3.136 | .706 | .478 | 29.242 | -39.268 |
| 5 | 5 | 1.811 | .000 | .667 | 21.307 | -.000 |
| 5 | 6 | 2.096 | .111 | .666 | 21.371 | -3.349 |
| 5 | 7 | 2.382 | .223 | .664 | 21.970 | -7.849 |
| 5 | 8 | 2.665 | .337 | .658 | 23.011 | -14.189 |
| 5 | 9 | 2.946 | .457 | .647 | 24.932 | -23.412 |
| 5 | 10 | 3.220 | .584 | .626 | 28.510 | -36.832 |
| 6 | 6 | 2.173 | .000 | .801 | 21.371 | -.000 |

## TABLE A-IV (CONT)

| Begin Surf | Next Surf | Xsurf (next) | Ysurf (next) | S (next) | Alpha2 (next) | RHO 2 (next) |
|---|---|---|---|---|---|---|
| 6 | 7 | 2.458 | .112 | .799 | 21.469 | -4.143 |
| 6 | 8 | 2.743 | .224 | .796 | 22.313 | -10.099 |
| 6 | 9 | 3.026 | .340 | .789 | 24.002 | -19.227 |
| 6 | 10 | 3.303 | .463 | .773 | 27.578 | -33.615 |
| 7 | 7 | 2.535 | .000 | .934 | 21.469 | -.000 |
| 7 | 8 | 2.820 | .112 | .932 | 21.634 | -5.429 |
| 7 | 9 | 3.105 | .225 | .928 | 22.996 | -14.107 |
| 7 | 10 | 3.385 | .344 | .917 | 26.378 | -29.225 |
| 8 | 8 | 2.897 | .000 | 1.068 | 21.634 | -.000 |
| 8 | 9 | 3.182 | .113 | 1.064 | 21.971 | -7.855 |
| 8 | 10 | 3.465 | .227 | 1.059 | 24.843 | -23.029 |
| 9 | 9 | 3.259 | .000 | 1.201 | 21.971 | -.000 |
| 9 | 10 | 3.543 | .114 | 1.196 | 22.986 | -14.051 |
| 10 | 10 | 3.622 | .000 | 1.335 | 22.986 | -.001 |

TABLE A-V

COMPARISON OF THE MAXIMUM ALLOWED LENGTH OF THE RAY IN THE LENS QA AND THE ACTUAL VALUE USED $S_{MAX}$, FOR EACH SURFACE IN FIGURES 17 AND 18. ALPHA2 AND RHO2 ARE THE VALUES AT $S_{MAX}$.

| RAY | QA | Smax | Alpha2 | RHO 2 |
|---|---|---|---|---|
| 1 | .133 | .133 | 21.000 | -.000 |
| 2 | .267 | .267 | 21.201 | -.000 |
| 3 | .400 | .400 | 21.228 | -.000 |
| 4 | .534 | .534 | 21.262 | -.000 |
| 5 | .667 | .667 | 21.307 | -.000 |
| 6 | .801 | .801 | 21.371 | -.000 |
| 7 | .934 | .934 | 21.469 | -.000 |
| 8 | 1.068 | 1.068 | 21.634 | -.000 |
| 9 | 1.201 | 1.201 | 21.971 | -.000 |
| 10 | 1.335 | 1.335 | 22.986 | -.001 |

# TABLE A-VI

CALCULATED VALUES OF S AND ALPHA2 USED TO CONSTRUCT THE DESIGN CHART IN FIGURE 19 FOR THE FINAL DESIGN OF SURFACE NUMBER 1 in FIGURE 20.

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 1 | -1.697 | 13.317 | -68.820 | 0.000 | 21.180 | -1.697 |
| 1 | -1.697 | 13.326 | -68.829 | .007 | 21.171 | -1.614 |
| 1 | -1.697 | 13.335 | -68.838 | .013 | 21.162 | -1.530 |
| 1 | -1.697 | 13.344 | -68.847 | .020 | 21.153 | -1.447 |
| 1 | -1.697 | 13.354 | -68.856 | .027 | 21.144 | -1.363 |
| 1 | -1.697 | 13.363 | -68.866 | .033 | 21.134 | -1.279 |
| 1 | -1.697 | 13.372 | -68.875 | .040 | 21.125 | -1.195 |
| 1 | -1.697 | 13.381 | -68.884 | .047 | 21.116 | -1.111 |
| 1 | -1.697 | 13.390 | -68.893 | .053 | 21.107 | -1.027 |
| 1 | -1.697 | 13.399 | -68.902 | .060 | 21.098 | -.942 |
| 1 | -1.697 | 13.408 | -68.911 | .067 | 21.089 | -.857 |
| 1 | -1.697 | 13.417 | -68.920 | .073 | 21.080 | -.772 |
| 1 | -1.697 | 13.426 | -68.929 | .080 | 21.071 | -.687 |
| 1 | -1.697 | 13.435 | -68.938 | .087 | 21.062 | -.602 |
| 1 | -1.697 | 13.444 | -68.947 | .093 | 21.053 | -.517 |
| 1 | -1.697 | 13.453 | -68.956 | .100 | 21.044 | -.431 |
| 1 | -1.697 | 13.462 | -68.965 | .107 | 21.035 | -.345 |
| 1 | -1.697 | 13.471 | -68.974 | .113 | 21.026 | -.259 |
| 1 | -1.697 | 13.479 | -68.982 | .120 | 21.018 | -.173 |
| 1 | -1.697 | 13.488 | -68.991 | .127 | 21.009 | -.087 |
| 1 | -1.697 | 13.479 | -68.982 | .120 | 21.018 | -.173 |
| 1 | -1.697 | 13.480 | -68.983 | .121 | 21.017 | -.164 |
| 1 | -1.697 | 13.481 | -68.984 | .121 | 21.016 | -.156 |
| 1 | -1.697 | 13.482 | -68.985 | .122 | 21.015 | -.147 |
| 1 | -1.697 | 13.483 | -68.986 | .123 | 21.014 | -.138 |
| 1 | -1.697 | 13.484 | -68.987 | .123 | 21.013 | -.130 |
| 1 | -1.697 | 13.485 | -68.988 | .124 | 21.012 | -.121 |
| 1 | -1.697 | 13.486 | -68.989 | .125 | 21.011 | -.112 |
| 1 | -1.697 | 13.486 | -68.989 | .125 | 21.011 | -.104 |
| 1 | -1.697 | 13.487 | -68.990 | .126 | 21.010 | -.095 |
| 1 | -1.697 | 13.488 | -68.991 | .127 | 21.009 | -.087 |
| 1 | -1.697 | 13.489 | -68.992 | .127 | 21.008 | -.078 |
| 1 | -1.697 | 13.490 | -68.993 | .128 | 21.007 | -.069 |
| 1 | -1.697 | 13.491 | -68.994 | .129 | 21.006 | -.061 |
| 1 | -1.697 | 13.492 | -68.995 | .129 | 21.005 | -.052 |
| 1 | -1.697 | 13.493 | -68.996 | .130 | 21.004 | -.043 |
| 1 | -1.697 | 13.494 | -68.996 | .131 | 21.004 | -.035 |
| 1 | -1.697 | 13.494 | -68.997 | .131 | 21.003 | -.026 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.017 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.009 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.017 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.016 |
| 1 | -1.697 | 13.495 | -68.998 | .132 | 21.002 | -.016 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.015 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.014 |
| 1 | -1.697 | 13.496 | -68.999 | .132 | 21.001 | -.013 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.012 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.011 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.010 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.010 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.009 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.008 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.007 |
| 1 | -1.697 | 13.496 | -68.999 | .133 | 21.001 | -.006 |
| 1 | -1.697 | 13.497 | -68.999 | .133 | 21.001 | -.005 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.004 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.003 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.003 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.002 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.001 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 1 | -1.697 | 13.497 | -69.000 | .133 | 21.000 | -.000 |
| 2 | -2.373 | 13.241 | -68.743 | 0.000 | 21.257 | -2.373 |
| 2 | -2.373 | 13.254 | -68.757 | .009 | 21.243 | -2.258 |
| 2 | -2.373 | 13.267 | -68.770 | .018 | 21.230 | -2.142 |
| 2 | -2.373 | 13.280 | -68.783 | .027 | 21.217 | -2.026 |
| 2 | -2.373 | 13.293 | -68.796 | .036 | 21.204 | -1.910 |
| 2 | -2.373 | 13.306 | -68.809 | .045 | 21.191 | -1.793 |
| 2 | -2.373 | 13.319 | -68.822 | .054 | 21.178 | -1.676 |
| 2 | -2.373 | 13.332 | -68.835 | .064 | 21.165 | -1.558 |
| 2 | -2.373 | 13.345 | -68.848 | .073 | 21.152 | -1.441 |
| 2 | -2.373 | 13.358 | -68.861 | .082 | 21.139 | -1.323 |
| 2 | -2.373 | 13.371 | -68.874 | .091 | 21.126 | -1.204 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 2 | -2.373 | 13.384 | -68.887 | .100 | 21.113 | -1.085 |
| 2 | -2.373 | 13.396 | -68.899 | .109 | 21.101 | -.966 |
| 2 | -2.373 | 13.409 | -68.912 | .118 | 21.088 | -.847 |
| 2 | -2.373 | 13.422 | -68.925 | .127 | 21.075 | -.727 |
| 2 | -2.373 | 13.434 | -68.937 | .136 | 21.063 | -.606 |
| 2 | -2.373 | 13.447 | -68.950 | .145 | 21.050 | -.486 |
| 2 | -2.373 | 13.460 | -68.963 | .154 | 21.037 | -.365 |
| 2 | -2.373 | 13.472 | -68.975 | .163 | 21.025 | -.244 |
| 2 | -2.373 | 13.485 | -68.988 | .172 | 21.012 | -.122 |
| 2 | -2.373 | 13.497 | -69.000 | .182 | 21.000 | -.000 |
| 2 | -2.373 | 13.485 | -68.988 | .172 | 21.012 | -.122 |
| 2 | -2.373 | 13.486 | -68.989 | .173 | 21.011 | -.110 |
| 2 | -2.373 | 13.487 | -68.990 | .174 | 21.010 | -.098 |
| 2 | -2.373 | 13.488 | -68.991 | .175 | 21.009 | -.085 |
| 2 | -2.373 | 13.490 | -68.993 | .176 | 21.007 | -.073 |
| 2 | -2.373 | 13.491 | -68.994 | .177 | 21.006 | -.061 |
| 2 | -2.373 | 13.492 | -68.995 | .178 | 21.005 | -.049 |
| 2 | -2.373 | 13.493 | -68.996 | .179 | 21.004 | -.037 |
| 2 | -2.373 | 13.495 | -68.998 | .180 | 21.002 | -.024 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.012 |
| 2 | -2.373 | 13.497 | -69.000 | .182 | 21.000 | -.000 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.012 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.011 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.010 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.009 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.007 |
| 2 | -2.373 | 13.496 | -68.999 | .181 | 21.001 | -.006 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.005 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.004 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.002 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .182 | 21.000 | -.000 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.001 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.000 |
| 2 | -2.373 | 13.497 | -69.000 | .181 | 21.000 | -.000 |
| 2 | -2.373 | 13.497 | -69.000 | .182 | 21.000 | -.000 |
| 2 | -2.373 | 13.497 | -69.000 | .182 | 21.000 | -.000 |
| 3 | -3.088 | 13.157 | -68.660 | 0.000 | 21.340 | -3.088 |
| 3 | -3.088 | 13.175 | -68.678 | .011 | 21.322 | -2.939 |
| 3 | -3.088 | 13.192 | -68.695 | .023 | 21.305 | -2.790 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 3 | -3.088 | 13.210 | -68.713 | .034 | 21.287 | -2.640 |
| 3 | -3.088 | 13.227 | -68.730 | .046 | 21.270 | -2.490 |
| 3 | -3.088 | 13.244 | -68.747 | .057 | 21.253 | -2.339 |
| 3 | -3.088 | 13.262 | -68.765 | .069 | 21.235 | -2.187 |
| 3 | -3.088 | 13.279 | -68.782 | .080 | 21.218 | -2.035 |
| 3 | -3.088 | 13.296 | -68.799 | .092 | 21.201 | -1.882 |
| 3 | -3.088 | 13.313 | -68.816 | .103 | 21.184 | -1.728 |
| 3 | -3.088 | 13.330 | -68.833 | .115 | 21.167 | -1.574 |
| 3 | -3.088 | 13.347 | -68.850 | .126 | 21.150 | -1.420 |
| 3 | -3.088 | 13.364 | -68.867 | .138 | 21.133 | -1.264 |
| 3 | -3.088 | 13.381 | -68.884 | .149 | 21.116 | -1.108 |
| 3 | -3.088 | 13.398 | -68.901 | .161 | 21.099 | -.952 |
| 3 | -3.088 | 13.415 | -68.918 | .172 | 21.082 | -.795 |
| 3 | -3.088 | 13.431 | -68.934 | .184 | 21.066 | -.637 |
| 3 | -3.088 | 13.448 | -68.951 | .195 | 21.049 | -.479 |
| 3 | -3.088 | 13.464 | -68.967 | .207 | 21.033 | -.320 |
| 3 | -3.088 | 13.481 | -68.984 | .218 | 21.016 | -.160 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.481 | -68.984 | .218 | 21.016 | -.160 |
| 3 | -3.088 | 13.482 | -68.985 | .219 | 21.015 | -.144 |
| 3 | -3.088 | 13.484 | -68.987 | .220 | 21.013 | -.128 |
| 3 | -3.088 | 13.486 | -68.989 | .222 | 21.011 | -.112 |
| 3 | -3.088 | 13.487 | -68.990 | .223 | 21.010 | -.096 |
| 3 | -3.088 | 13.489 | -68.992 | .224 | 21.008 | -.080 |
| 3 | -3.088 | 13.491 | -68.993 | .225 | 21.007 | -.064 |
| 3 | -3.088 | 13.492 | -68.995 | .226 | 21.005 | -.048 |
| 3 | -3.088 | 13.494 | -68.997 | .227 | 21.003 | -.032 |
| 3 | -3.088 | 13.495 | -68.998 | .228 | 21.002 | -.016 |
| 3 | -3.088 | 13.495 | -68.998 | .228 | 21.002 | -.016 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.014 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.013 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.011 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.010 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.008 |
| 3 | -3.088 | 13.496 | -68.999 | .229 | 21.001 | -.006 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.005 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.003 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.002 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.002 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.001 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.001 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.001 |
| 3 | -3.088 | 13.497 | -69.000 | .229 | 21.000 | -.001 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.001 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.001 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 3 | -3.088 | 13.497 | -69.000 | .230 | 21.000 | -.000 |
| 4 | -3.845 | 13.065 | -68.568 | 0.000 | 21.432 | -3.845 |
| 4 | -3.845 | 13.088 | -68.591 | .014 | 21.409 | -3.662 |
| 4 | -3.845 | 13.110 | -68.613 | .028 | 21.387 | -3.478 |
| 4 | -3.845 | 13.133 | -68.636 | .042 | 21.364 | -3.293 |
| 4 | -3.845 | 13.155 | -68.658 | .056 | 21.342 | -3.107 |
| 4 | -3.845 | 13.177 | -68.680 | .069 | 21.320 | -2.920 |
| 4 | -3.845 | 13.199 | -68.702 | .083 | 21.298 | -2.732 |
| 4 | -3.845 | 13.221 | -68.724 | .097 | 21.276 | -2.543 |
| 4 | -3.845 | 13.243 | -68.746 | .111 | 21.254 | -2.353 |
| 4 | -3.845 | 13.265 | -68.768 | .125 | 21.232 | -2.162 |
| 4 | -3.845 | 13.286 | -68.789 | .139 | 21.211 | -1.970 |
| 4 | -3.845 | 13.308 | -68.811 | .153 | 21.189 | -1.778 |
| 4 | -3.845 | 13.329 | -68.832 | .167 | 21.168 | -1.584 |
| 4 | -3.845 | 13.351 | -68.854 | .180 | 21.146 | -1.389 |
| 4 | -3.845 | 13.372 | -68.875 | .194 | 21.125 | -1.194 |
| 4 | -3.845 | 13.393 | -68.896 | .208 | 21.104 | -.997 |
| 4 | -3.845 | 13.414 | -68.917 | .222 | 21.083 | -.800 |
| 4 | -3.845 | 13.435 | -68.938 | .236 | 21.062 | -.601 |
| 4 | -3.845 | 13.456 | -68.959 | .250 | 21.041 | -.402 |
| 4 | -3.845 | 13.476 | -68.979 | .264 | 21.021 | -.201 |
| 4 | -3.845 | 13.476 | -68.979 | .264 | 21.021 | -.201 |
| 4 | -3.845 | 13.479 | -68.982 | .265 | 21.018 | -.181 |
| 4 | -3.845 | 13.481 | -68.984 | .267 | 21.016 | -.161 |
| 4 | -3.845 | 13.483 | -68.986 | .268 | 21.014 | -.141 |
| 4 | -3.845 | 13.485 | -68.988 | .269 | 21.012 | -.121 |
| 4 | -3.845 | 13.487 | -68.990 | .271 | 21.010 | -.101 |
| 4 | -3.845 | 13.489 | -68.992 | .272 | 21.008 | -.081 |
| 4 | -3.845 | 13.491 | -68.994 | .273 | 21.006 | -.061 |
| 4 | -3.845 | 13.493 | -68.996 | .275 | 21.004 | -.040 |
| 4 | -3.845 | 13.495 | -68.998 | .276 | 21.002 | -.020 |
| 4 | -3.845 | 13.495 | -68.998 | .276 | 21.002 | -.020 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 4 | -3.845 | 13.495 | -68.998 | .276 | 21.002 | -.018 |
| 4 | -3.845 | 13.495 | -68.998 | .277 | 21.002 | -.016 |
| 4 | -3.845 | 13.496 | -68.999 | .277 | 21.001 | -.014 |
| 4 | -3.845 | 13.496 | -68.999 | .277 | 21.001 | -.012 |
| 4 | -3.845 | 13.496 | -68.999 | .277 | 21.001 | -.010 |
| 4 | -3.845 | 13.496 | -68.999 | .277 | 21.001 | -.008 |
| 4 | -3.845 | 13.496 | -68.999 | .277 | 21.001 | -.006 |
| 4 | -3.845 | 13.497 | -69.000 | .277 | 21.000 | -.004 |
| 4 | -3.845 | 13.497 | -69.000 | .277 | 21.000 | -.002 |
| 4 | -3.845 | 13.497 | -69.000 | .277 | 21.000 | -.002 |
| 4 | -3.845 | 13.497 | -69.000 | .277 | 21.000 | -.002 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.002 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.001 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.001 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.001 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.001 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.001 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 4 | -3.845 | 13.497 | -69.000 | .278 | 21.000 | -.000 |
| 5 | -4.649 | 12.965 | -68.468 | 0.000 | 21.532 | -4.649 |
| 5 | -4.649 | 12.993 | -68.496 | .016 | 21.504 | -4.430 |
| 5 | -4.649 | 13.020 | -68.523 | .033 | 21.477 | -4.209 |
| 5 | -4.649 | 13.048 | -68.551 | .049 | 21.449 | -3.987 |
| 5 | -4.649 | 13.075 | -68.578 | .065 | 21.422 | -3.764 |
| 5 | -4.649 | 13.103 | -68.606 | .081 | 21.394 | -3.539 |
| 5 | -4.649 | 13.130 | -68.633 | .098 | 21.367 | -3.313 |
| 5 | -4.649 | 13.157 | -68.660 | .114 | 21.340 | -3.086 |
| 5 | -4.649 | 13.184 | -68.687 | .130 | 21.313 | -2.857 |
| 5 | -4.649 | 13.211 | -68.714 | .147 | 21.286 | -2.627 |
| 5 | -4.649 | 13.238 | -68.741 | .163 | 21.259 | -2.395 |
| 5 | -4.649 | 13.265 | -68.768 | .179 | 21.232 | -2.162 |
| 5 | -4.649 | 13.291 | -68.794 | .195 | 21.206 | -1.927 |
| 5 | -4.649 | 13.317 | -68.820 | .212 | 21.180 | -1.692 |
| 5 | -4.649 | 13.344 | -68.847 | .228 | 21.153 | -1.454 |
| 5 | -4.649 | 13.370 | -68.873 | .244 | 21.127 | -1.216 |

# TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 5 | -4.649 | 13.395 | -68.898 | .261 | 21.102 | -.975 |
| 5 | -4.649 | 13.421 | -68.924 | .277 | 21.076 | -.734 |
| 5 | -4.649 | 13.447 | -68.950 | .293 | 21.050 | -.491 |
| 5 | -4.649 | 13.472 | -68.975 | .309 | 21.025 | -.246 |
| 5 | -4.649 | 13.472 | -68.975 | .309 | 21.025 | -.246 |
| 5 | -4.649 | 13.474 | -68.977 | .311 | 21.023 | -.221 |
| 5 | -4.649 | 13.477 | -68.980 | .313 | 21.020 | -.197 |
| 5 | -4.649 | 13.479 | -68.982 | .314 | 21.018 | -.172 |
| 5 | -4.649 | 13.482 | -68.985 | .316 | 21.015 | -.148 |
| 5 | -4.649 | 13.484 | -68.987 | .318 | 21.013 | -.123 |
| 5 | -4.649 | 13.487 | -68.990 | .319 | 21.010 | -.099 |
| 5 | -4.649 | 13.490 | -68.992 | .321 | 21.008 | -.074 |
| 5 | -4.649 | 13.492 | -68.995 | .322 | 21.005 | -.049 |
| 5 | -4.649 | 13.495 | -68.997 | .324 | 21.003 | -.025 |
| 5 | -4.649 | 13.492 | -68.995 | .322 | 21.005 | -.049 |
| 5 | -4.649 | 13.492 | -68.995 | .323 | 21.005 | -.047 |
| 5 | -4.649 | 13.493 | -68.995 | .323 | 21.005 | -.044 |
| 5 | -4.649 | 13.493 | -68.996 | .323 | 21.004 | -.042 |
| 5 | -4.649 | 13.493 | -68.996 | .323 | 21.004 | -.039 |
| 5 | -4.649 | 13.493 | -68.996 | .323 | 21.004 | -.037 |
| 5 | -4.649 | 13.494 | -68.996 | .323 | 21.004 | -.035 |
| 5 | -4.649 | 13.494 | -68.997 | .324 | 21.003 | -.032 |
| 5 | -4.649 | 13.494 | -68.997 | .324 | 21.003 | -.030 |
| 5 | -4.649 | 13.494 | -68.997 | .324 | 21.003 | -.027 |
| 5 | -4.649 | 13.495 | -68.997 | .324 | 21.003 | -.025 |
| 5 | -4.649 | 13.495 | -68.998 | .324 | 21.002 | -.022 |
| 5 | -4.649 | 13.495 | -68.998 | .324 | 21.002 | -.020 |
| 5 | -4.649 | 13.495 | -68.998 | .325 | 21.002 | -.017 |
| 5 | -4.649 | 13.496 | -68.998 | .325 | 21.002 | -.015 |
| 5 | -4.649 | 13.496 | -68.999 | .325 | 21.001 | -.012 |
| 5 | -4.649 | 13.496 | -68.999 | .325 | 21.001 | -.010 |
| 5 | -4.649 | 13.496 | -68.999 | .325 | 21.001 | -.007 |
| 5 | -4.649 | 13.497 | -68.999 | .325 | 21.001 | -.005 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.002 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.002 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.002 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.002 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.002 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.001 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.001 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.001 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.001 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 5 | -4.649 | 13.497 | -69.000 | .326 | 21.000 | -.000 |
| 6 | -5.502 | 12.854 | -68.357 | 0.000 | 21.643 | -5.502 |
| 6 | -5.502 | 12.888 | -68.390 | .019 | 21.610 | -5.246 |
| 6 | -5.502 | 12.921 | -68.424 | .037 | 21.576 | -4.987 |
| 6 | -5.502 | 12.955 | -68.458 | .056 | 21.542 | -4.727 |
| 6 | -5.502 | 12.988 | -68.491 | .075 | 21.509 | -4.465 |
| 6 | -5.502 | 13.021 | -68.524 | .093 | 21.476 | -4.201 |
| 6 | -5.502 | 13.054 | -68.557 | .112 | 21.443 | -3.935 |
| 6 | -5.502 | 13.087 | -68.590 | .131 | 21.410 | -3.667 |
| 6 | -5.502 | 13.120 | -68.623 | .149 | 21.377 | -3.397 |
| 6 | -5.502 | 13.153 | -68.656 | .168 | 21.344 | -3.125 |
| 6 | -5.502 | 13.185 | -68.688 | .187 | 21.312 | -2.851 |
| 6 | -5.502 | 13.217 | -68.720 | .206 | 21.280 | -2.576 |
| 6 | -5.502 | 13.249 | -68.752 | .224 | 21.248 | -2.298 |
| 6 | -5.502 | 13.281 | -68.784 | .243 | 21.216 | -2.018 |
| 6 | -5.502 | 13.313 | -68.815 | .262 | 21.185 | -1.736 |
| 6 | -5.502 | 13.344 | -68.847 | .280 | 21.153 | -1.451 |
| 6 | -5.502 | 13.375 | -68.878 | .299 | 21.122 | -1.165 |
| 6 | -5.502 | 13.406 | -68.909 | .318 | 21.091 | -.877 |
| 6 | -5.502 | 13.437 | -68.939 | .336 | 21.061 | -.587 |
| 6 | -5.502 | 13.467 | -68.970 | .355 | 21.030 | -.294 |
| 6 | -5.502 | 13.437 | -68.939 | .336 | 21.061 | -.587 |
| 6 | -5.502 | 13.440 | -68.943 | .338 | 21.057 | -.558 |
| 6 | -5.502 | 13.443 | -68.946 | .340 | 21.054 | -.529 |
| 6 | -5.502 | 13.446 | -68.949 | .342 | 21.051 | -.499 |
| 6 | -5.502 | 13.449 | -68.952 | .344 | 21.048 | -.470 |
| 6 | -5.502 | 13.452 | -68.955 | .346 | 21.045 | -.441 |
| 6 | -5.502 | 13.455 | -68.958 | .348 | 21.042 | -.412 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 6 | -5.502 | 13.458 | -68.961 | .349 | 21.039 | -.382 |
| 6 | -5.502 | 13.461 | -68.964 | .351 | 21.036 | -.353 |
| 6 | -5.502 | 13.464 | -68.967 | .353 | 21.033 | -.324 |
| 6 | -5.502 | 13.467 | -68.970 | .355 | 21.030 | -.294 |
| 6 | -5.502 | 13.470 | -68.973 | .357 | 21.027 | -.265 |
| 6 | -5.502 | 13.473 | -68.976 | .359 | 21.024 | -.236 |
| 6 | -5.502 | 13.476 | -68.979 | .361 | 21.021 | -.206 |
| 6 | -5.502 | 13.479 | -68.982 | .363 | 21.018 | -.177 |
| 6 | -5.502 | 13.482 | -68.985 | .364 | 21.015 | -.147 |
| 6 | -5.502 | 13.485 | -68.988 | .366 | 21.012 | -.118 |
| 6 | -5.502 | 13.488 | -68.991 | .368 | 21.009 | -.089 |
| 6 | -5.502 | 13.491 | -68.994 | .370 | 21.006 | -.059 |
| 6 | -5.502 | 13.494 | -68.997 | .372 | 21.003 | -.030 |
| 6 | -5.502 | 13.491 | -68.994 | .370 | 21.006 | -.059 |
| 6 | -5.502 | 13.491 | -68.994 | .370 | 21.006 | -.056 |
| 6 | -5.502 | 13.492 | -68.995 | .370 | 21.005 | -.053 |
| 6 | -5.502 | 13.492 | -68.995 | .371 | 21.005 | -.050 |
| 6 | -5.502 | 13.492 | -68.995 | .371 | 21.005 | -.047 |
| 6 | -5.502 | 13.493 | -68.995 | .371 | 21.005 | -.044 |
| 6 | -5.502 | 13.493 | -68.996 | .371 | 21.004 | -.041 |
| 6 | -5.502 | 13.493 | -68.996 | .371 | 21.004 | -.038 |
| 6 | -5.502 | 13.493 | -68.996 | .371 | 21.004 | -.035 |
| 6 | -5.502 | 13.494 | -68.997 | .372 | 21.003 | -.032 |
| 6 | -5.502 | 13.494 | -68.997 | .372 | 21.003 | -.030 |
| 6 | -5.502 | 13.494 | -68.997 | .372 | 21.003 | -.027 |
| 6 | -5.502 | 13.495 | -68.998 | .372 | 21.002 | -.024 |
| 6 | -5.502 | 13.495 | -68.998 | .372 | 21.002 | -.021 |
| 6 | -5.502 | 13.495 | -68.998 | .373 | 21.002 | -.018 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.015 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.012 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.009 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.006 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.003 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.006 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.006 |
| 6 | -5.502 | 13.496 | -68.999 | .373 | 21.001 | -.005 |
| 6 | -5.502 | 13.497 | -68.999 | .373 | 21.001 | -.005 |
| 6 | -5.502 | 13.497 | -69.000 | .373 | 21.000 | -.005 |
| 6 | -5.502 | 13.497 | -69.000 | .373 | 21.000 | -.004 |
| 6 | -5.502 | 13.497 | -69.000 | .373 | 21.000 | -.004 |
| 6 | -5.502 | 13.497 | -69.000 | .373 | 21.000 | -.004 |
| 6 | -5.502 | 13.497 | -69.000 | .373 | 21.000 | -.004 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.003 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.003 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.003 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.002 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.002 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.002 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.001 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 6 | -5.502 | 13.497 | -69.000 | .374 | 21.000 | -.000 |
| 7 | -6.409 | 12.731 | -68.234 | 0.000 | 21.766 | -6.409 |
| 7 | -6.409 | 12.772 | -68.274 | .021 | 21.726 | -6.115 |
| 7 | -6.409 | 12.812 | -68.315 | .042 | 21.685 | -5.817 |
| 7 | -6.409 | 12.852 | -68.355 | .063 | 21.645 | -5.518 |
| 7 | -6.409 | 12.892 | -68.394 | .084 | 21.606 | -5.215 |
| 7 | -6.409 | 12.931 | -68.434 | .105 | 21.566 | -4.910 |
| 7 | -6.409 | 12.971 | -68.474 | .127 | 21.526 | -4.602 |
| 7 | -6.409 | 13.010 | -68.513 | .148 | 21.487 | -4.292 |
| 7 | -6.409 | 13.049 | -68.552 | .169 | 21.448 | -3.978 |
| 7 | -6.409 | 13.088 | -68.591 | .190 | 21.409 | -3.662 |
| 7 | -6.409 | 13.126 | -68.629 | .211 | 21.371 | -3.343 |
| 7 | -6.409 | 13.165 | -68.668 | .232 | 21.332 | -3.022 |
| 7 | -6.409 | 13.203 | -68.706 | .253 | 21.294 | -2.697 |
| 7 | -6.409 | 13.241 | -68.744 | .274 | 21.256 | -2.370 |
| 7 | -6.409 | 13.278 | -68.781 | .295 | 21.219 | -2.040 |
| 7 | -6.409 | 13.316 | -68.819 | .316 | 21.181 | -1.707 |
| 7 | -6.409 | 13.353 | -68.856 | .337 | 21.144 | -1.372 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 7 | -6.409 | 13.389 | -68.892 | .359 | 21.108 | -1.033 |
| 7 | -6.409 | 13.426 | -68.928 | .380 | 21.072 | -.692 |
| 7 | -6.409 | 13.461 | -68.964 | .401 | 21.036 | -.347 |
| 7 | -6.409 | 13.426 | -68.928 | .380 | 21.072 | -.692 |
| 7 | -6.409 | 13.429 | -68.932 | .382 | 21.068 | -.657 |
| 7 | -6.409 | 13.433 | -68.936 | .384 | 21.064 | -.623 |
| 7 | -6.409 | 13.436 | -68.939 | .386 | 21.061 | -.589 |
| 7 | -6.409 | 13.440 | -68.943 | .388 | 21.057 | -.554 |
| 7 | -6.409 | 13.444 | -68.946 | .390 | 21.054 | -.520 |
| 7 | -6.409 | 13.447 | -68.950 | .392 | 21.050 | -.485 |
| 7 | -6.409 | 13.451 | -68.954 | .394 | 21.046 | -.451 |
| 7 | -6.409 | 13.454 | -68.957 | .396 | 21.043 | -.416 |
| 7 | -6.409 | 13.458 | -68.961 | .399 | 21.039 | -.382 |
| 7 | -6.409 | 13.461 | -68.964 | .401 | 21.036 | -.347 |
| 7 | -6.409 | 13.465 | -68.968 | .403 | 21.032 | -.313 |
| 7 | -6.409 | 13.469 | -68.972 | .405 | 21.028 | -.278 |
| 7 | -6.409 | 13.472 | -68.975 | .407 | 21.025 | -.243 |
| 7 | -6.409 | 13.476 | -68.979 | .409 | 21.021 | -.209 |
| 7 | -6.409 | 13.479 | -68.982 | .411 | 21.018 | -.174 |
| 7 | -6.409 | 13.483 | -68.986 | .413 | 21.014 | -.139 |
| 7 | -6.409 | 13.486 | -68.989 | .415 | 21.011 | -.104 |
| 7 | -6.409 | 13.490 | -68.993 | .418 | 21.007 | -.070 |
| 7 | -6.409 | 13.493 | -68.996 | .420 | 21.004 | -.035 |
| 7 | -6.409 | 13.490 | -68.993 | .418 | 21.007 | -.070 |
| 7 | -6.409 | 13.490 | -68.993 | .418 | 21.007 | -.066 |
| 7 | -6.409 | 13.491 | -68.994 | .418 | 21.006 | -.063 |
| 7 | -6.409 | 13.491 | -68.994 | .418 | 21.006 | -.059 |
| 7 | -6.409 | 13.491 | -68.994 | .418 | 21.006 | -.056 |
| 7 | -6.409 | 13.492 | -68.995 | .419 | 21.005 | -.052 |
| 7 | -6.409 | 13.492 | -68.995 | .419 | 21.005 | -.049 |
| 7 | -6.409 | 13.492 | -68.995 | .419 | 21.005 | -.045 |
| 7 | -6.409 | 13.493 | -68.996 | .419 | 21.004 | -.042 |
| 7 | -6.409 | 13.493 | -68.996 | .419 | 21.004 | -.038 |
| 7 | -6.409 | 13.493 | -68.996 | .420 | 21.004 | -.035 |
| 7 | -6.409 | 13.494 | -68.997 | .420 | 21.003 | -.031 |
| 7 | -6.409 | 13.494 | -68.997 | .420 | 21.003 | -.028 |
| 7 | -6.409 | 13.495 | -68.998 | .420 | 21.002 | -.024 |
| 7 | -6.409 | 13.495 | -68.998 | .420 | 21.002 | -.021 |
| 7 | -6.409 | 13.495 | -68.998 | .421 | 21.002 | -.017 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.014 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.010 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.007 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.003 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.007 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.007 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.006 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.006 |
| 7 | -6.409 | 13.496 | -68.999 | .421 | 21.001 | -.006 |
| 7 | -6.409 | 13.497 | -68.999 | .421 | 21.001 | -.005 |
| 7 | -6.409 | 13.497 | -69.000 | .421 | 21.000 | -.005 |
| 7 | -6.409 | 13.497 | -69.000 | .421 | 21.000 | -.005 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.004 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.004 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.003 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.003 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.003 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.002 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.002 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.002 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.001 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.001 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.001 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 7 | -6.409 | 13.497 | -69.000 | .422 | 21.000 | -.000 |
| 8 | -7.375 | 12.595 | -68.098 | 0.000 | 21.902 | -7.375 |
| 8 | -7.375 | 12.643 | -68.146 | .023 | 21.854 | -7.041 |
| 8 | -7.375 | 12.690 | -68.193 | .047 | 21.807 | -6.704 |
| 8 | -7.375 | 12.738 | -68.241 | .070 | 21.759 | -6.363 |
| 8 | -7.375 | 12.785 | -68.288 | .094 | 21.712 | -6.018 |
| 8 | -7.375 | 12.831 | -68.334 | .117 | 21.666 | -5.670 |
| 8 | -7.375 | 12.878 | -68.381 | .141 | 21.619 | -5.319 |
| 8 | -7.375 | 12.924 | -68.427 | .164 | 21.573 | -4.963 |
| 8 | -7.375 | 12.970 | -68.473 | .188 | 21.527 | -4.604 |
| 8 | -7.375 | 13.016 | -68.519 | .211 | 21.481 | -4.241 |
| 8 | -7.375 | 13.062 | -68.565 | .235 | 21.435 | -3.875 |
| 8 | -7.375 | 13.107 | -68.610 | .258 | 21.390 | -3.505 |
| 8 | -7.375 | 13.152 | -68.655 | .282 | 21.345 | -3.131 |
| 8 | -7.375 | 13.196 | -68.699 | .305 | 21.301 | -2.753 |
| 8 | -7.375 | 13.241 | -68.744 | .329 | 21.256 | -2.371 |
| 8 | -7.375 | 13.285 | -68.787 | .352 | 21.213 | -1.986 |
| 8 | -7.375 | 13.328 | -68.831 | .376 | 21.169 | -1.597 |
| 8 | -7.375 | 13.371 | -68.874 | .399 | 21.126 | -1.203 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 8 | -7.375 | 13.413 | -68.916 | .423 | 21.084 | -.806 |
| 8 | -7.375 | 13.455 | -68.958 | .446 | 21.042 | -.405 |
| 8 | -7.375 | 13.413 | -68.916 | .423 | 21.084 | -.806 |
| 8 | -7.375 | 13.418 | -68.921 | .425 | 21.079 | -.766 |
| 8 | -7.375 | 13.422 | -68.925 | .428 | 21.075 | -.726 |
| 8 | -7.375 | 13.426 | -68.929 | .430 | 21.071 | -.686 |
| 8 | -7.375 | 13.430 | -68.933 | .432 | 21.067 | -.646 |
| 8 | -7.375 | 13.435 | -68.937 | .435 | 21.063 | -.606 |
| 8 | -7.375 | 13.439 | -68.942 | .437 | 21.058 | -.566 |
| 8 | -7.375 | 13.443 | -68.946 | .439 | 21.054 | -.526 |
| 8 | -7.375 | 13.447 | -68.950 | .442 | 21.050 | -.486 |
| 8 | -7.375 | 13.451 | -68.954 | .444 | 21.046 | -.445 |
| 8 | -7.375 | 13.455 | -68.958 | .446 | 21.042 | -.405 |
| 8 | -7.375 | 13.460 | -68.963 | .449 | 21.037 | -.365 |
| 8 | -7.375 | 13.464 | -68.967 | .451 | 21.033 | -.324 |
| 8 | -7.375 | 13.468 | -68.971 | .453 | 21.029 | -.284 |
| 8 | -7.375 | 13.472 | -68.975 | .456 | 21.025 | -.244 |
| 8 | -7.375 | 13.476 | -68.979 | .458 | 21.021 | -.203 |
| 8 | -7.375 | 13.480 | -68.983 | .460 | 21.017 | -.162 |
| 8 | -7.375 | 13.485 | -68.988 | .463 | 21.012 | -.122 |
| 8 | -7.375 | 13.489 | -68.992 | .465 | 21.008 | -.081 |
| 8 | -7.375 | 13.493 | -68.996 | .467 | 21.004 | -.041 |
| 8 | -7.375 | 13.489 | -68.992 | .465 | 21.008 | -.081 |
| 8 | -7.375 | 13.489 | -68.992 | .465 | 21.008 | -.077 |
| 8 | -7.375 | 13.490 | -68.993 | .466 | 21.007 | -.073 |
| 8 | -7.375 | 13.490 | -68.993 | .466 | 21.007 | -.069 |
| 8 | -7.375 | 13.490 | -68.993 | .466 | 21.007 | -.065 |
| 8 | -7.375 | 13.491 | -68.994 | .466 | 21.006 | -.061 |
| 8 | -7.375 | 13.491 | -68.994 | .467 | 21.006 | -.057 |
| 8 | -7.375 | 13.492 | -68.995 | .467 | 21.005 | -.053 |
| 8 | -7.375 | 13.492 | -68.995 | .467 | 21.005 | -.049 |
| 8 | -7.375 | 13.492 | -68.995 | .467 | 21.005 | -.045 |
| 8 | -7.375 | 13.493 | -68.996 | .467 | 21.004 | -.041 |
| 8 | -7.375 | 13.493 | -68.996 | .468 | 21.004 | -.037 |
| 8 | -7.375 | 13.494 | -68.997 | .468 | 21.003 | -.033 |
| 8 | -7.375 | 13.494 | -68.997 | .468 | 21.003 | -.028 |
| 8 | -7.375 | 13.495 | -68.998 | .468 | 21.002 | -.024 |
| 8 | -7.375 | 13.495 | -68.998 | .469 | 21.002 | -.020 |
| 8 | -7.375 | 13.495 | -68.998 | .469 | 21.002 | -.016 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.012 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.008 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.004 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.008 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.008 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.007 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.007 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.007 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.006 |
| 8 | -7.375 | 13.496 | -68.999 | .469 | 21.001 | -.006 |
| 8 | -7.375 | 13.497 | -68.999 | .470 | 21.001 | -.005 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.005 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.004 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.004 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.004 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.003 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.003 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.002 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.002 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.002 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.001 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 8 | -7.375 | 13.497 | -69.000 | .470 | 21.000 | -.000 |
| 9 | -8.406 | 12.445 | -67.948 | 0.000 | 22.052 | -8.406 |
| 9 | -8.406 | 12.500 | -68.003 | .026 | 21.997 | -8.032 |
| 9 | -8.406 | 12.556 | -68.059 | .052 | 21.941 | -7.653 |
| 9 | -8.406 | 12.611 | -68.114 | .078 | 21.886 | -7.269 |
| 9 | -8.406 | 12.666 | -68.169 | .104 | 21.831 | -6.881 |
| 9 | -8.406 | 12.720 | -68.223 | .129 | 21.777 | -6.488 |
| 9 | -8.406 | 12.775 | -68.278 | .155 | 21.722 | -6.090 |
| 9 | -8.406 | 12.829 | -68.332 | .181 | 21.668 | -5.688 |
| 9 | -8.406 | 12.883 | -68.386 | .207 | 21.614 | -5.280 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 9 | -8.406 | 12.937 | -68.440 | .233 | 21.560 | -4.863 |
| 9 | -8.406 | 12.990 | -68.493 | .259 | 21.507 | -4.451 |
| 9 | -8.406 | 13.043 | -68.546 | .285 | 21.454 | -4.029 |
| 9 | -8.406 | 13.095 | -68.598 | .311 | 21.402 | -3.602 |
| 9 | -8.406 | 13.147 | -68.650 | .337 | 21.350 | -3.170 |
| 9 | -8.406 | 13.199 | -68.702 | .363 | 21.298 | -2.732 |
| 9 | -8.406 | 13.250 | -68.753 | .388 | 21.247 | -2.290 |
| 9 | -8.406 | 13.301 | -68.804 | .414 | 21.196 | -1.842 |
| 9 | -8.406 | 13.351 | -68.854 | .440 | 21.146 | -1.390 |
| 9 | -8.406 | 13.400 | -68.903 | .466 | 21.097 | -.932 |
| 9 | -8.406 | 13.449 | -68.952 | .492 | 21.048 | -.469 |
| 9 | -8.406 | 13.449 | -68.952 | .492 | 21.048 | -.469 |
| 9 | -8.406 | 13.454 | -68.957 | .495 | 21.043 | -.422 |
| 9 | -8.406 | 13.459 | -68.962 | .497 | 21.038 | -.375 |
| 9 | -8.406 | 13.463 | -68.966 | .500 | 21.034 | -.329 |
| 9 | -8.406 | 13.468 | -68.971 | .502 | 21.029 | -.282 |
| 9 | -8.406 | 13.473 | -68.976 | .505 | 21.024 | -.235 |
| 9 | -8.406 | 13.478 | -68.981 | .508 | 21.019 | -.188 |
| 9 | -8.406 | 13.483 | -68.986 | .510 | 21.014 | -.141 |
| 9 | -8.406 | 13.487 | -68.990 | .513 | 21.010 | -.094 |
| 9 | -8.406 | 13.492 | -68.995 | .515 | 21.005 | -.047 |
| 9 | -8.406 | 13.492 | -68.995 | .515 | 21.005 | -.047 |
| 9 | -8.406 | 13.493 | -68.996 | .516 | 21.004 | -.042 |
| 9 | -8.406 | 13.493 | -68.996 | .516 | 21.004 | -.038 |
| 9 | -8.406 | 13.494 | -68.997 | .516 | 21.003 | -.033 |
| 9 | -8.406 | 13.494 | -68.997 | .516 | 21.003 | -.028 |
| 9 | -8.406 | 13.495 | -68.998 | .517 | 21.002 | -.024 |
| 9 | -8.406 | 13.495 | -68.998 | .517 | 21.002 | -.019 |
| 9 | -8.406 | 13.496 | -68.999 | .517 | 21.001 | -.014 |
| 9 | -8.406 | 13.496 | -68.999 | .517 | 21.001 | -.009 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.005 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.005 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.004 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.004 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.003 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.003 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.002 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.002 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.001 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.001 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 9 | -8.406 | 13.497 | -69.000 | .518 | 21.000 | -.000 |
| 10 | -9.507 | 12.277 | -67.780 | 0.000 | 22.220 | -9.507 |
| 10 | -9.507 | 12.341 | -67.844 | .028 | 22.156 | -9.091 |
| 10 | -9.507 | 12.405 | -67.908 | .057 | 22.092 | -8.670 |
| 10 | -9.507 | 12.469 | -67.972 | .085 | 22.028 | -8.242 |
| 10 | -9.507 | 12.533 | -68.036 | .113 | 21.964 | -7.808 |
| 10 | -9.507 | 12.596 | -68.099 | .141 | 21.901 | -7.369 |
| 10 | -9.507 | 12.660 | -68.163 | .170 | 21.837 | -6.923 |
| 10 | -9.507 | 12.723 | -68.226 | .198 | 21.774 | -6.471 |
| 10 | -9.507 | 12.785 | -68.288 | .226 | 21.712 | -6.013 |
| 10 | -9.507 | 12.848 | -68.351 | .255 | 21.649 | -5.548 |
| 10 | -9.507 | 12.910 | -68.412 | .283 | 21.588 | -5.077 |
| 10 | -9.507 | 12.971 | -68.474 | .311 | 21.526 | -4.600 |
| 10 | -9.507 | 13.032 | -68.535 | .340 | 21.465 | -4.116 |
| 10 | -9.507 | 13.092 | -68.595 | .368 | 21.405 | -3.625 |
| 10 | -9.507 | 13.152 | -68.655 | .396 | 21.345 | -3.128 |
| 10 | -9.507 | 13.212 | -68.715 | .424 | 21.285 | -2.624 |
| 10 | -9.507 | 13.270 | -68.773 | .453 | 21.227 | -2.113 |
| 10 | -9.507 | 13.328 | -68.831 | .481 | 21.169 | -1.595 |
| 10 | -9.507 | 13.385 | -68.888 | .509 | 21.112 | -1.070 |
| 10 | -9.507 | 13.442 | -68.945 | .538 | 21.055 | -.539 |
| 10 | -9.507 | 13.385 | -68.888 | .509 | 21.112 | -1.070 |
| 10 | -9.507 | 13.391 | -68.894 | .512 | 21.106 | -1.017 |
| 10 | -9.507 | 13.397 | -68.900 | .515 | 21.100 | -.965 |
| 10 | -9.507 | 13.402 | -68.905 | .518 | 21.095 | -.912 |
| 10 | -9.507 | 13.408 | -68.911 | .521 | 21.089 | -.858 |
| 10 | -9.507 | 13.414 | -68.916 | .523 | 21.084 | -.805 |
| 10 | -9.507 | 13.419 | -68.922 | .526 | 21.078 | -.752 |
| 10 | -9.507 | 13.425 | -68.928 | .529 | 21.072 | -.699 |
| 10 | -9.507 | 13.430 | -68.933 | .532 | 21.067 | -.646 |
| 10 | -9.507 | 13.436 | -68.939 | .535 | 21.061 | -.592 |
| 10 | -9.507 | 13.442 | -68.945 | .538 | 21.055 | -.539 |
| 10 | -9.507 | 13.447 | -68.950 | .540 | 21.050 | -.485 |
| 10 | -9.507 | 13.453 | -68.956 | .543 | 21.044 | -.431 |
| 10 | -9.507 | 13.458 | -68.961 | .546 | 21.039 | -.378 |
| 10 | -9.507 | 13.464 | -68.967 | .549 | 21.033 | -.324 |
| 10 | -9.507 | 13.469 | -68.972 | .552 | 21.028 | -.270 |
| 10 | -9.507 | 13.475 | -68.978 | .555 | 21.022 | -.216 |
| 10 | -9.507 | 13.480 | -68.983 | .557 | 21.017 | -.162 |
| 10 | -9.507 | 13.486 | -68.989 | .560 | 21.011 | -.108 |
| 10 | -9.507 | 13.492 | -68.994 | .563 | 21.006 | -.054 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 10 | -9.507 | 13.492 | -68.994 | .563 | 21.006 | -.054 |
| 10 | -9.507 | 13.492 | -68.995 | .563 | 21.005 | -.049 |
| 10 | -9.507 | 13.493 | -68.996 | .564 | 21.004 | -.043 |
| 10 | -9.507 | 13.493 | -68.996 | .564 | 21.004 | -.038 |
| 10 | -9.507 | 13.494 | -68.997 | .564 | 21.003 | -.033 |
| 10 | -9.507 | 13.494 | -68.997 | .564 | 21.003 | -.027 |
| 10 | -9.507 | 13.495 | -68.998 | .565 | 21.002 | -.022 |
| 10 | -9.507 | 13.495 | -68.998 | .565 | 21.002 | -.016 |
| 10 | -9.507 | 13.496 | -68.999 | .565 | 21.001 | -.011 |
| 10 | -9.507 | 13.496 | -68.999 | .566 | 21.001 | -.005 |
| 10 | -9.507 | 13.496 | -68.999 | .566 | 21.001 | -.005 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.005 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.004 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.004 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.003 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.003 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.002 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.002 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.001 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.001 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.001 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 10 | -9.507 | 13.497 | -69.000 | .566 | 21.000 | -.000 |
| 11 | -10.685 | 12.090 | -67.593 | 0.000 | 22.407 | -10.685 |
| 11 | -10.685 | 12.164 | -67.667 | .031 | 22.333 | -10.227 |
| 11 | -10.685 | 12.237 | -67.740 | .061 | 22.260 | -9.762 |
| 11 | -10.685 | 12.311 | -67.814 | .092 | 22.186 | -9.289 |
| 11 | -10.685 | 12.384 | -67.887 | .123 | 22.113 | -8.809 |
| 11 | -10.685 | 12.458 | -67.961 | .153 | 22.039 | -8.320 |
| 11 | -10.685 | 12.531 | -68.034 | .184 | 21.966 | -7.825 |
| 11 | -10.685 | 12.603 | -68.106 | .215 | 21.894 | -7.321 |
| 11 | -10.685 | 12.676 | -68.179 | .246 | 21.821 | -6.809 |
| 11 | -10.685 | 12.748 | -68.251 | .276 | 21.749 | -6.289 |
| 11 | -10.685 | 12.819 | -68.322 | .307 | 21.678 | -5.760 |
| 11 | -10.685 | 12.890 | -68.393 | .338 | 21.607 | -5.224 |
| 11 | -10.685 | 12.961 | -68.464 | .368 | 21.536 | -4.679 |
| 11 | -10.685 | 13.031 | -68.534 | .399 | 21.466 | -4.125 |
| 11 | -10.685 | 13.100 | -68.603 | .430 | 21.397 | -3.562 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 11 | -10.685 | 13.169 | -68.671 | .460 | 21.329 | -2.991 |
| 11 | -10.685 | 13.236 | -68.739 | .491 | 21.261 | -2.411 |
| 11 | -10.685 | 13.303 | -68.806 | .522 | 21.194 | -1.822 |
| 11 | -10.685 | 13.369 | -68.872 | .553 | 21.128 | -1.224 |
| 11 | -10.685 | 13.433 | -68.936 | .583 | 21.064 | -.616 |
| 11 | -10.685 | 13.369 | -68.872 | .553 | 21.128 | -1.224 |
| 11 | -10.685 | 13.375 | -68.878 | .556 | 21.122 | -1.163 |
| 11 | -10.685 | 13.382 | -68.885 | .559 | 21.115 | -1.103 |
| 11 | -10.685 | 13.388 | -68.891 | .562 | 21.109 | -1.042 |
| 11 | -10.685 | 13.395 | -68.898 | .565 | 21.102 | -.982 |
| 11 | -10.685 | 13.401 | -68.904 | .568 | 21.096 | -.921 |
| 11 | -10.685 | 13.408 | -68.911 | .571 | 21.089 | -.860 |
| 11 | -10.685 | 13.414 | -68.917 | .574 | 21.083 | -.800 |
| 11 | -10.685 | 13.421 | -68.924 | .577 | 21.076 | -.739 |
| 11 | -10.685 | 13.427 | -68.930 | .580 | 21.070 | -.678 |
| 11 | -10.685 | 13.433 | -68.936 | .583 | 21.064 | -.616 |
| 11 | -10.685 | 13.440 | -68.943 | .586 | 21.057 | -.555 |
| 11 | -10.685 | 13.446 | -68.949 | .589 | 21.051 | -.494 |
| 11 | -10.685 | 13.453 | -68.956 | .592 | 21.044 | -.432 |
| 11 | -10.685 | 13.459 | -68.962 | .596 | 21.038 | -.371 |
| 11 | -10.685 | 13.465 | -68.968 | .599 | 21.032 | -.309 |
| 11 | -10.685 | 13.472 | -68.975 | .602 | 21.025 | -.248 |
| 11 | -10.685 | 13.478 | -68.981 | .605 | 21.019 | -.186 |
| 11 | -10.685 | 13.484 | -68.987 | .608 | 21.013 | -.124 |
| 11 | -10.685 | 13.491 | -68.994 | .611 | 21.006 | -.062 |
| 11 | -10.685 | 13.484 | -68.987 | .608 | 21.013 | -.124 |
| 11 | -10.685 | 13.485 | -68.988 | .608 | 21.012 | -.118 |
| 11 | -10.685 | 13.486 | -68.989 | .608 | 21.011 | -.112 |
| 11 | -10.685 | 13.486 | -68.989 | .609 | 21.011 | -.105 |
| 11 | -10.685 | 13.487 | -68.990 | .609 | 21.010 | -.099 |
| 11 | -10.685 | 13.488 | -68.991 | .609 | 21.009 | -.093 |
| 11 | -10.685 | 13.488 | -68.991 | .610 | 21.009 | -.087 |
| 11 | -10.685 | 13.489 | -68.992 | .610 | 21.008 | -.081 |
| 11 | -10.685 | 13.489 | -68.992 | .610 | 21.008 | -.074 |
| 11 | -10.685 | 13.490 | -68.993 | .611 | 21.007 | -.068 |
| 11 | -10.685 | 13.491 | -68.994 | .611 | 21.006 | -.062 |
| 11 | -10.685 | 13.491 | -68.994 | .611 | 21.006 | -.056 |
| 11 | -10.685 | 13.492 | -68.995 | .612 | 21.005 | -.050 |
| 11 | -10.685 | 13.493 | -68.996 | .612 | 21.004 | -.043 |
| 11 | -10.685 | 13.493 | -68.996 | .612 | 21.004 | -.037 |
| 11 | -10.685 | 13.494 | -68.997 | .612 | 21.003 | -.031 |
| 11 | -10.685 | 13.495 | -68.997 | .613 | 21.003 | -.025 |
| 11 | -10.685 | 13.495 | -68.998 | .613 | 21.002 | -.019 |
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.012 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.006 |
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.012 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.012 |
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.011 |
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.011 |
| 11 | -10.685 | 13.496 | -68.999 | .613 | 21.001 | -.010 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.009 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.009 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.008 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.007 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.007 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.006 |
| 11 | -10.685 | 13.496 | -68.999 | .614 | 21.001 | -.006 |
| 11 | -10.685 | 13.497 | -68.999 | .614 | 21.001 | -.005 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.004 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.004 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.003 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.002 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.002 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.001 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 11 | -10.685 | 13.497 | -69.000 | .614 | 21.000 | -.000 |
| 12 | -11.947 | 11.881 | -67.384 | 0.000 | 22.616 | -11.947 |
| 12 | -11.947 | 11.965 | -67.468 | .033 | 22.532 | -11.446 |
| 12 | -11.947 | 12.049 | -67.552 | .066 | 22.448 | -10.937 |
| 12 | -11.947 | 12.133 | -67.636 | .099 | 22.364 | -10.418 |
| 12 | -11.947 | 12.217 | -67.720 | .132 | 22.280 | -9.889 |
| 12 | -11.947 | 12.301 | -67.804 | .166 | 22.196 | -9.351 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 12 | -11.947 | 12.385 | -67.888 | .199 | 22.112 | -8.803 |
| 12 | -11.947 | 12.469 | -67.972 | .232 | 22.028 | -8.245 |
| 12 | -11.947 | 12.552 | -68.055 | .265 | 21.945 | -7.677 |
| 12 | -11.947 | 12.635 | -68.138 | .298 | 21.862 | -7.098 |
| 12 | -11.947 | 12.717 | -68.220 | .331 | 21.780 | -6.508 |
| 12 | -11.947 | 12.799 | -68.302 | .364 | 21.698 | -5.908 |
| 12 | -11.947 | 12.881 | -68.384 | .397 | 21.616 | -5.297 |
| 12 | -11.947 | 12.961 | -68.464 | .430 | 21.536 | -4.675 |
| 12 | -11.947 | 13.041 | -68.544 | .463 | 21.456 | -4.042 |
| 12 | -11.947 | 13.120 | -68.623 | .497 | 21.377 | -3.398 |
| 12 | -11.947 | 13.198 | -68.701 | .530 | 21.299 | -2.742 |
| 12 | -11.947 | 13.275 | -68.778 | .563 | 21.222 | -2.074 |
| 12 | -11.947 | 13.350 | -68.853 | .596 | 21.147 | -1.395 |
| 12 | -11.947 | 13.424 | -68.927 | .629 | 21.073 | -.703 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.424 | -68.927 | .629 | 21.073 | -.703 |
| 12 | -11.947 | 13.432 | -68.935 | .632 | 21.065 | -.633 |
| 12 | -11.947 | 13.439 | -68.942 | .636 | 21.058 | -.564 |
| 12 | -11.947 | 13.446 | -68.949 | .639 | 21.051 | -.494 |
| 12 | -11.947 | 13.454 | -68.957 | .642 | 21.043 | -.423 |
| 12 | -11.947 | 13.461 | -68.964 | .645 | 21.036 | -.353 |
| 12 | -11.947 | 13.468 | -68.971 | .649 | 21.029 | -.283 |
| 12 | -11.947 | 13.475 | -68.978 | .652 | 21.022 | -.212 |
| 12 | -11.947 | 13.483 | -68.986 | .655 | 21.014 | -.142 |
| 12 | -11.947 | 13.490 | -68.993 | .659 | 21.007 | -.071 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.490 | -68.993 | .659 | 21.007 | -.071 |
| 12 | -11.947 | 13.491 | -68.994 | .659 | 21.006 | -.064 |
| 12 | -11.947 | 13.491 | -68.994 | .659 | 21.006 | -.057 |
| 12 | -11.947 | 13.492 | -68.995 | .660 | 21.005 | -.050 |
| 12 | -11.947 | 13.493 | -68.996 | .660 | 21.004 | -.043 |
| 12 | -11.947 | 13.493 | -68.996 | .660 | 21.004 | -.035 |
| 12 | -11.947 | 13.494 | -68.997 | .661 | 21.003 | -.028 |
| 12 | -11.947 | 13.495 | -68.998 | .661 | 21.002 | -.021 |
| 12 | -11.947 | 13.496 | -68.999 | .661 | 21.001 | -.014 |
| 12 | -11.947 | 13.496 | -68.999 | .662 | 21.001 | -.007 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.496 | -68.999 | .662 | 21.001 | -.007 |
| 12 | -11.947 | 13.496 | -68.999 | .662 | 21.001 | -.006 |
| 12 | -11.947 | 13.496 | -68.999 | .662 | 21.001 | -.006 |
| 12 | -11.947 | 13.497 | -68.999 | .662 | 21.001 | -.005 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.004 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.004 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.003 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.002 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.001 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.001 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.001 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.001 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.001 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 12 | -11.947 | 13.497 | -69.000 | .662 | 21.000 | -.000 |
| 13 | -13.300 | 11.646 | -67.149 | 0.000 | 22.851 | -13.300 |
| 13 | -13.300 | 11.741 | -67.244 | .036 | 22.756 | -12.758 |
| 13 | -13.300 | 11.837 | -67.340 | .071 | 22.660 | -12.203 |
| 13 | -13.300 | 11.933 | -67.436 | .107 | 22.564 | -11.638 |
| 13 | -13.300 | 12.029 | -67.532 | .142 | 22.468 | -11.060 |
| 13 | -13.300 | 12.125 | -67.628 | .178 | 22.372 | -10.470 |
| 13 | -13.300 | 12.221 | -67.724 | .213 | 22.276 | -9.868 |
| 13 | -13.300 | 12.316 | -67.819 | .249 | 22.181 | -9.253 |
| 13 | -13.300 | 12.412 | -67.915 | .284 | 22.085 | -8.626 |
| 13 | -13.300 | 12.507 | -68.010 | .320 | 21.990 | -7.985 |
| 13 | -13.300 | 12.602 | -68.105 | .355 | 21.895 | -7.331 |
| 13 | -13.300 | 12.696 | -68.199 | .391 | 21.801 | -6.663 |
| 13 | -13.300 | 12.790 | -68.293 | .426 | 21.707 | -5.981 |
| 13 | -13.300 | 12.882 | -68.385 | .462 | 21.615 | -5.285 |
| 13 | -13.300 | 12.974 | -68.477 | .497 | 21.523 | -4.575 |
| 13 | -13.300 | 13.065 | -68.568 | .533 | 21.432 | -3.850 |
| 13 | -13.300 | 13.154 | -68.657 | .568 | 21.343 | -3.110 |
| 13 | -13.300 | 13.242 | -68.745 | .604 | 21.255 | -2.356 |
| 13 | -13.300 | 13.329 | -68.832 | .639 | 21.168 | -1.586 |
| 13 | -13.300 | 13.414 | -68.917 | .675 | 21.083 | -.801 |
| 13 | -13.300 | 13.329 | -68.832 | .639 | 21.168 | -1.586 |
| 13 | -13.300 | 13.338 | -68.841 | .643 | 21.159 | -1.508 |
| 13 | -13.300 | 13.346 | -68.849 | .646 | 21.151 | -1.430 |
| 13 | -13.300 | 13.355 | -68.858 | .650 | 21.142 | -1.352 |
| 13 | -13.300 | 13.363 | -68.866 | .653 | 21.134 | -1.274 |
| 13 | -13.300 | 13.372 | -68.875 | .657 | 21.125 | -1.195 |
| 13 | -13.300 | 13.380 | -68.883 | .660 | 21.117 | -1.117 |
| 13 | -13.300 | 13.389 | -68.892 | .664 | 21.108 | -1.038 |
| 13 | -13.300 | 13.397 | -68.900 | .667 | 21.100 | -.959 |
| 13 | -13.300 | 13.406 | -68.909 | .671 | 21.091 | -.880 |
| 13 | -13.300 | 13.414 | -68.917 | .675 | 21.083 | -.801 |
| 13 | -13.300 | 13.422 | -68.925 | .678 | 21.075 | -.721 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 13 | -13.300 | 13.431 | -68.934 | .682 | 21.066 | -.642 |
| 13 | -13.300 | 13.439 | -68.942 | .685 | 21.058 | -.562 |
| 13 | -13.300 | 13.447 | -68.950 | .689 | 21.050 | -.482 |
| 13 | -13.300 | 13.456 | -68.959 | .692 | 21.041 | -.402 |
| 13 | -13.300 | 13.464 | -68.967 | .696 | 21.033 | -.322 |
| 13 | -13.300 | 13.472 | -68.975 | .699 | 21.025 | -.242 |
| 13 | -13.300 | 13.481 | -68.984 | .703 | 21.016 | -.161 |
| 13 | -13.300 | 13.489 | -68.992 | .707 | 21.008 | -.081 |
| 13 | -13.300 | 13.481 | -68.984 | .703 | 21.016 | -.161 |
| 13 | -13.300 | 13.481 | -68.984 | .703 | 21.016 | -.153 |
| 13 | -13.300 | 13.482 | -68.985 | .704 | 21.015 | -.145 |
| 13 | -13.300 | 13.483 | -68.986 | .704 | 21.014 | -.137 |
| 13 | -13.300 | 13.484 | -68.987 | .704 | 21.013 | -.129 |
| 13 | -13.300 | 13.485 | -68.988 | .705 | 21.012 | -.121 |
| 13 | -13.300 | 13.486 | -68.988 | .705 | 21.012 | -.113 |
| 13 | -13.300 | 13.486 | -68.989 | .705 | 21.011 | -.105 |
| 13 | -13.300 | 13.487 | -68.990 | .706 | 21.010 | -.097 |
| 13 | -13.300 | 13.488 | -68.991 | .706 | 21.009 | -.089 |
| 13 | -13.300 | 13.489 | -68.992 | .707 | 21.008 | -.081 |
| 13 | -13.300 | 13.490 | -68.993 | .707 | 21.007 | -.073 |
| 13 | -13.300 | 13.490 | -68.993 | .707 | 21.007 | -.065 |
| 13 | -13.300 | 13.491 | -68.994 | .708 | 21.006 | -.057 |
| 13 | -13.300 | 13.492 | -68.995 | .708 | 21.005 | -.048 |
| 13 | -13.300 | 13.493 | -68.996 | .708 | 21.004 | -.040 |
| 13 | -13.300 | 13.494 | -68.997 | .709 | 21.003 | -.032 |
| 13 | -13.300 | 13.495 | -68.998 | .709 | 21.002 | -.024 |
| 13 | -13.300 | 13.495 | -68.998 | .709 | 21.002 | -.016 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.008 |
| 13 | -13.300 | 13.495 | -68.998 | .709 | 21.002 | -.016 |
| 13 | -13.300 | 13.495 | -68.998 | .709 | 21.002 | -.015 |
| 13 | -13.300 | 13.496 | -68.999 | .709 | 21.001 | -.015 |
| 13 | -13.300 | 13.496 | -68.999 | .709 | 21.001 | -.014 |
| 13 | -13.300 | 13.496 | -68.999 | .709 | 21.001 | -.013 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.012 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.011 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.011 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.010 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.009 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.008 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.007 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.006 |
| 13 | -13.300 | 13.496 | -68.999 | .710 | 21.001 | -.006 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.005 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.004 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.003 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.002 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.002 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.002 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.002 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.001 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 13 | -13.300 | 13.497 | -69.000 | .710 | 21.000 | -.000 |
| 14 | -14.755 | 11.382 | -66.885 | 0.000 | 23.115 | -14.755 |
| 14 | -14.755 | 11.490 | -66.993 | .038 | 23.007 | -14.170 |
| 14 | -14.755 | 11.598 | -67.101 | .076 | 22.899 | -13.572 |
| 14 | -14.755 | 11.706 | -67.209 | .114 | 22.791 | -12.959 |
| 14 | -14.755 | 11.815 | -67.318 | .152 | 22.682 | -12.331 |
| 14 | -14.755 | 11.924 | -67.427 | .190 | 22.573 | -11.689 |
| 14 | -14.755 | 12.034 | -67.537 | .227 | 22.463 | -11.031 |
| 14 | -14.755 | 12.143 | -67.646 | .265 | 22.354 | -10.357 |
| 14 | -14.755 | 12.252 | -67.755 | .303 | 22.245 | -9.668 |
| 14 | -14.755 | 12.361 | -67.864 | .341 | 22.136 | -8.962 |
| 14 | -14.755 | 12.470 | -67.973 | .379 | 22.027 | -8.238 |
| 14 | -14.755 | 12.578 | -68.081 | .417 | 21.919 | -7.498 |
| 14 | -14.755 | 12.685 | -68.188 | .455 | 21.812 | -6.740 |
| 14 | -14.755 | 12.792 | -68.295 | .493 | 21.705 | -5.964 |
| 14 | -14.755 | 12.897 | -68.400 | .531 | 21.600 | -5.170 |
| 14 | -14.755 | 13.002 | -68.505 | .569 | 21.495 | -4.357 |
| 14 | -14.755 | 13.105 | -68.608 | .606 | 21.392 | -3.524 |
| 14 | -14.755 | 13.206 | -68.709 | .644 | 21.291 | -2.673 |
| 14 | -14.755 | 13.305 | -68.808 | .682 | 21.192 | -1.802 |
| 14 | -14.755 | 13.402 | -68.905 | .720 | 21.095 | -.911 |
| 14 | -14.755 | 13.402 | -68.905 | .720 | 21.095 | -.911 |
| 14 | -14.755 | 13.412 | -68.915 | .724 | 21.085 | -.821 |
| 14 | -14.755 | 13.421 | -68.924 | .728 | 21.076 | -.730 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 14 | -14.755 | 13.431 | -68.934 | .732 | 21.066 | -.640 |
| 14 | -14.755 | 13.440 | -68.943 | .735 | 21.057 | -.549 |
| 14 | -14.755 | 13.450 | -68.953 | .739 | 21.047 | -.458 |
| 14 | -14.755 | 13.459 | -68.962 | .743 | 21.038 | -.367 |
| 14 | -14.755 | 13.469 | -68.972 | .747 | 21.028 | -.275 |
| 14 | -14.755 | 13.478 | -68.981 | .751 | 21.019 | -.184 |
| 14 | -14.755 | 13.488 | -68.991 | .754 | 21.009 | -.092 |
| 14 | -14.755 | 13.488 | -68.991 | .754 | 21.009 | -.092 |
| 14 | -14.755 | 13.489 | -68.992 | .755 | 21.008 | -.083 |
| 14 | -14.755 | 13.490 | -68.993 | .755 | 21.007 | -.074 |
| 14 | -14.755 | 13.490 | -68.993 | .755 | 21.007 | -.064 |
| 14 | -14.755 | 13.491 | -68.994 | .756 | 21.006 | -.055 |
| 14 | -14.755 | 13.492 | -68.995 | .756 | 21.005 | -.046 |
| 14 | -14.755 | 13.493 | -68.996 | .757 | 21.004 | -.037 |
| 14 | -14.755 | 13.494 | -68.997 | .757 | 21.003 | -.028 |
| 14 | -14.755 | 13.495 | -68.998 | .757 | 21.002 | -.018 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.009 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.009 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.008 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.007 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.006 |
| 14 | -14.755 | 13.496 | -68.999 | .758 | 21.001 | -.006 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.005 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.004 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.003 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.002 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.001 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.000 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.000 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.000 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.000 |
| 14 | -14.755 | 13.497 | -69.000 | .758 | 21.000 | -.000 |
| 15 | -16.320 | 11.085 | -66.588 | 0.000 | 23.412 | -16.320 |
| 15 | -16.320 | 11.205 | -66.708 | .040 | 23.292 | -15.694 |
| 15 | -16.320 | 11.327 | -66.830 | .081 | 23.170 | -15.052 |
| 15 | -16.320 | 11.449 | -66.952 | .121 | 23.048 | -14.393 |
| 15 | -16.320 | 11.572 | -67.075 | .161 | 22.925 | -13.715 |
| 15 | -16.320 | 11.696 | -67.199 | .202 | 22.801 | -13.019 |
| 15 | -16.320 | 11.820 | -67.323 | .242 | 22.677 | -12.305 |
| 15 | -16.320 | 11.944 | -67.447 | .282 | 22.553 | -11.570 |

TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 15 | -16.320 | 12.069 | -67.572 | .322 | 22.428 | -10.816 |
| 15 | -16.320 | 12.193 | -67.696 | .363 | 22.304 | -10.041 |
| 15 | -16.320 | 12.318 | -67.821 | .403 | 22.179 | -9.245 |
| 15 | -16.320 | 12.442 | -67.945 | .443 | 22.055 | -8.427 |
| 15 | -16.320 | 12.565 | -68.068 | .484 | 21.932 | -7.587 |
| 15 | -16.320 | 12.688 | -68.191 | .524 | 21.809 | -6.724 |
| 15 | -16.320 | 12.809 | -68.312 | .564 | 21.688 | -5.838 |
| 15 | -16.320 | 12.929 | -68.432 | .605 | 21.568 | -4.927 |
| 15 | -16.320 | 13.047 | -68.550 | .645 | 21.450 | -3.992 |
| 15 | -16.320 | 13.164 | -68.667 | .685 | 21.333 | -3.033 |
| 15 | -16.320 | 13.278 | -68.780 | .726 | 21.220 | -2.048 |
| 15 | -16.320 | 13.389 | -68.892 | .766 | 21.108 | -1.037 |
| 15 | -16.320 | 13.278 | -68.780 | .726 | 21.220 | -2.048 |
| 15 | -16.320 | 13.289 | -68.792 | .730 | 21.208 | -1.948 |
| 15 | -16.320 | 13.300 | -68.803 | .734 | 21.197 | -1.848 |
| 15 | -16.320 | 13.311 | -68.814 | .738 | 21.186 | -1.747 |
| 15 | -16.320 | 13.322 | -68.825 | .742 | 21.175 | -1.646 |
| 15 | -16.320 | 13.334 | -68.836 | .746 | 21.164 | -1.546 |
| 15 | -16.320 | 13.345 | -68.848 | .750 | 21.152 | -1.444 |
| 15 | -16.320 | 13.356 | -68.859 | .754 | 21.141 | -1.343 |
| 15 | -16.320 | 13.367 | -68.870 | .758 | 21.130 | -1.241 |
| 15 | -16.320 | 13.378 | -68.881 | .762 | 21.119 | -1.139 |
| 15 | -16.320 | 13.389 | -68.892 | .766 | 21.108 | -1.037 |
| 15 | -16.320 | 13.400 | -68.903 | .770 | 21.097 | -.934 |
| 15 | -16.320 | 13.411 | -68.914 | .774 | 21.086 | -.832 |
| 15 | -16.320 | 13.422 | -68.925 | .778 | 21.075 | -.729 |
| 15 | -16.320 | 13.432 | -68.935 | .782 | 21.065 | -.625 |
| 15 | -16.320 | 13.443 | -68.946 | .786 | 21.054 | -.522 |
| 15 | -16.320 | 13.454 | -68.957 | .790 | 21.043 | -.418 |
| 15 | -16.320 | 13.465 | -68.968 | .794 | 21.032 | -.314 |
| 15 | -16.320 | 13.476 | -68.979 | .798 | 21.021 | -.209 |
| 15 | -16.320 | 13.486 | -68.989 | .802 | 21.011 | -.105 |
| 15 | -16.320 | 13.486 | -68.989 | .802 | 21.011 | -.105 |
| 15 | -16.320 | 13.487 | -68.990 | .803 | 21.010 | -.094 |
| 15 | -16.320 | 13.489 | -68.991 | .803 | 21.009 | -.084 |
| 15 | -16.320 | 13.490 | -68.993 | .803 | 21.007 | -.073 |
| 15 | -16.320 | 13.491 | -68.994 | .804 | 21.006 | -.063 |
| 15 | -16.320 | 13.492 | -68.995 | .804 | 21.005 | -.052 |
| 15 | -16.320 | 13.493 | -68.996 | .805 | 21.004 | -.042 |
| 15 | -16.320 | 13.494 | -68.997 | .805 | 21.003 | -.031 |
| 15 | -16.320 | 13.495 | -68.998 | .805 | 21.002 | -.021 |
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.010 |
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.010 |
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.009 |
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.008 |
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.007 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 15 | -16.320 | 13.496 | -68.999 | .806 | 21.001 | -.006 |
| 15 | -16.320 | 13.497 | -68.999 | .806 | 21.001 | -.005 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.004 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.003 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.002 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.001 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.000 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.000 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.000 |
| 15 | -16.320 | 13.497 | -69.000 | .806 | 21.000 | -.000 |
| 16 | -18.005 | 10.749 | -66.252 | 0.000 | 23.748 | -18.005 |
| 16 | -18.005 | 10.883 | -66.386 | .043 | 23.614 | -17.341 |
| 16 | -18.005 | 11.019 | -66.522 | .085 | 23.478 | -16.657 |
| 16 | -18.005 | 11.156 | -66.659 | .128 | 23.341 | -15.952 |
| 16 | -18.005 | 11.294 | -66.797 | .171 | 23.203 | -15.226 |
| 16 | -18.005 | 11.434 | -66.937 | .214 | 23.063 | -14.477 |
| 16 | -18.005 | 11.574 | -67.077 | .256 | 22.923 | -13.704 |
| 16 | -18.005 | 11.715 | -67.218 | .299 | 22.782 | -12.908 |
| 16 | -18.005 | 11.857 | -67.360 | .342 | 22.640 | -12.087 |
| 16 | -18.005 | 11.999 | -67.502 | .384 | 22.498 | -11.240 |
| 16 | -18.005 | 12.141 | -67.644 | .427 | 22.356 | -10.367 |
| 16 | -18.005 | 12.283 | -67.786 | .470 | 22.214 | -9.467 |
| 16 | -18.005 | 12.425 | -67.928 | .513 | 22.072 | -8.538 |
| 16 | -18.005 | 12.566 | -68.069 | .555 | 21.931 | -7.580 |
| 16 | -18.005 | 12.706 | -68.209 | .598 | 21.791 | -6.593 |
| 16 | -18.005 | 12.844 | -68.347 | .641 | 21.653 | -5.575 |
| 16 | -18.005 | 12.980 | -68.483 | .683 | 21.517 | -4.525 |
| 16 | -18.005 | 13.114 | -68.617 | .726 | 21.383 | -3.444 |
| 16 | -18.005 | 13.245 | -68.748 | .769 | 21.252 | -2.329 |
| 16 | -18.005 | 13.373 | -68.876 | .811 | 21.124 | -1.182 |
| 16 | -18.005 | 13.245 | -68.748 | .769 | 21.252 | -2.329 |
| 16 | -18.005 | 13.258 | -68.761 | .773 | 21.239 | -2.216 |
| 16 | -18.005 | 13.271 | -68.774 | .777 | 21.226 | -2.103 |
| 16 | -18.005 | 13.284 | -68.787 | .782 | 21.213 | -1.989 |
| 16 | -18.005 | 13.297 | -68.800 | .786 | 21.200 | -1.874 |
| 16 | -18.005 | 13.310 | -68.813 | .790 | 21.187 | -1.760 |
| 16 | -18.005 | 13.323 | -68.826 | .794 | 21.174 | -1.645 |
| 16 | -18.005 | 13.335 | -68.838 | .799 | 21.162 | -1.530 |
| 16 | -18.005 | 13.348 | -68.851 | .803 | 21.149 | -1.414 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 16 | -18.005 | 13.361 | -68.864 | .807 | 21.136 | -1.298 |
| 16 | -18.005 | 13.373 | -68.876 | .811 | 21.124 | -1.182 |
| 16 | -18.005 | 13.386 | -68.889 | .816 | 21.111 | -1.065 |
| 16 | -18.005 | 13.398 | -68.901 | .820 | 21.099 | -.948 |
| 16 | -18.005 | 13.411 | -68.914 | .824 | 21.086 | -.831 |
| 16 | -18.005 | 13.423 | -68.926 | .829 | 21.074 | -.713 |
| 16 | -18.005 | 13.436 | -68.939 | .833 | 21.061 | -.595 |
| 16 | -18.005 | 13.448 | -68.951 | .837 | 21.049 | -.477 |
| 16 | -18.005 | 13.460 | -68.963 | .841 | 21.037 | -.358 |
| 16 | -18.005 | 13.473 | -68.976 | .846 | 21.024 | -.239 |
| 16 | -18.005 | 13.485 | -68.988 | .850 | 21.012 | -.120 |
| 16 | -18.005 | 13.473 | -68.976 | .846 | 21.024 | -.239 |
| 16 | -18.005 | 13.474 | -68.977 | .846 | 21.023 | -.227 |
| 16 | -18.005 | 13.475 | -68.978 | .847 | 21.022 | -.215 |
| 16 | -18.005 | 13.476 | -68.979 | .847 | 21.021 | -.203 |
| 16 | -18.005 | 13.478 | -68.980 | .847 | 21.020 | -.191 |
| 16 | -18.005 | 13.479 | -68.982 | .848 | 21.018 | -.179 |
| 16 | -18.005 | 13.480 | -68.983 | .848 | 21.017 | -.168 |
| 16 | -18.005 | 13.481 | -68.984 | .849 | 21.016 | -.156 |
| 16 | -18.005 | 13.482 | -68.985 | .849 | 21.015 | -.144 |
| 16 | -18.005 | 13.484 | -68.987 | .850 | 21.013 | -.132 |
| 16 | -18.005 | 13.485 | -68.988 | .850 | 21.012 | -.120 |
| 16 | -18.005 | 13.486 | -68.989 | .850 | 21.011 | -.108 |
| 16 | -18.005 | 13.487 | -68.990 | .851 | 21.010 | -.096 |
| 16 | -18.005 | 13.489 | -68.991 | .851 | 21.009 | -.084 |
| 16 | -18.005 | 13.490 | -68.993 | .852 | 21.007 | -.072 |
| 16 | -18.005 | 13.491 | -68.994 | .852 | 21.006 | -.060 |
| 16 | -18.005 | 13.492 | -68.995 | .852 | 21.005 | -.048 |
| 16 | -18.005 | 13.493 | -68.996 | .853 | 21.004 | -.036 |
| 16 | -18.005 | 13.495 | -68.998 | .853 | 21.002 | -.024 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.012 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.012 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.011 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.010 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.008 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.007 |
| 16 | -18.005 | 13.496 | -68.999 | .854 | 21.001 | -.006 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.005 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.004 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.002 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.001 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.000 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.000 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.000 |
| 16 | -18.005 | 13.497 | -69.000 | .854 | 21.000 | -.000 |
| 17 | -19.823 | 10.369 | -65.872 | 0.000 | 24.128 | -19.823 |
| 17 | -19.823 | 10.517 | -66.020 | .045 | 23.980 | -19.124 |
| 17 | -19.823 | 10.668 | -66.171 | .090 | 23.829 | -18.400 |
| 17 | -19.823 | 10.821 | -66.323 | .135 | 23.677 | -17.652 |
| 17 | -19.823 | 10.975 | -66.478 | .180 | 23.522 | -16.878 |
| 17 | -19.823 | 11.132 | -66.635 | .226 | 23.365 | -16.077 |
| 17 | -19.823 | 11.290 | -66.793 | .271 | 23.207 | -15.248 |
| 17 | -19.823 | 11.450 | -66.953 | .316 | 23.047 | -14.389 |
| 17 | -19.823 | 11.611 | -67.114 | .361 | 22.886 | -13.500 |
| 17 | -19.823 | 11.773 | -67.275 | .406 | 22.725 | -12.578 |
| 17 | -19.823 | 11.935 | -67.438 | .451 | 22.562 | -11.624 |
| 17 | -19.823 | 12.098 | -67.601 | .496 | 22.399 | -10.636 |
| 17 | -19.823 | 12.261 | -67.764 | .541 | 22.236 | -9.612 |
| 17 | -19.823 | 12.423 | -67.926 | .586 | 22.074 | -8.552 |
| 17 | -19.823 | 12.584 | -68.087 | .632 | 21.913 | -7.454 |
| 17 | -19.823 | 12.744 | -68.247 | .677 | 21.753 | -6.316 |
| 17 | -19.823 | 12.902 | -68.405 | .722 | 21.595 | -5.138 |
| 17 | -19.823 | 13.056 | -68.559 | .767 | 21.441 | -3.918 |
| 17 | -19.823 | 13.208 | -68.711 | .812 | 21.289 | -2.656 |
| 17 | -19.823 | 13.355 | -68.858 | .857 | 21.142 | -1.350 |
| 17 | -19.823 | 13.208 | -68.711 | .812 | 21.289 | -2.656 |
| 17 | -19.823 | 13.223 | -68.726 | .817 | 21.274 | -2.527 |
| 17 | -19.823 | 13.238 | -68.741 | .821 | 21.259 | -2.398 |
| 17 | -19.823 | 13.252 | -68.755 | .826 | 21.245 | -2.269 |
| 17 | -19.823 | 13.267 | -68.770 | .830 | 21.230 | -2.139 |
| 17 | -19.823 | 13.282 | -68.785 | .835 | 21.215 | -2.009 |
| 17 | -19.823 | 13.297 | -68.800 | .839 | 21.200 | -1.878 |
| 17 | -19.823 | 13.311 | -68.814 | .844 | 21.186 | -1.747 |
| 17 | -19.823 | 13.326 | -68.829 | .848 | 21.171 | -1.615 |
| 17 | -19.823 | 13.340 | -68.843 | .853 | 21.157 | -1.483 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 17 | -19.823 | 13.355 | -68.858 | .857 | 21.142 | -1.350 |
| 17 | -19.823 | 13.369 | -68.872 | .862 | 21.128 | -1.217 |
| 17 | -19.823 | 13.384 | -68.887 | .866 | 21.113 | -1.084 |
| 17 | -19.823 | 13.398 | -68.901 | .871 | 21.099 | -.950 |
| 17 | -19.823 | 13.412 | -68.915 | .875 | 21.085 | -.816 |
| 17 | -19.823 | 13.427 | -68.930 | .880 | 21.070 | -.681 |
| 17 | -19.823 | 13.441 | -68.944 | .884 | 21.056 | -.545 |
| 17 | -19.823 | 13.455 | -68.958 | .889 | 21.042 | -.410 |
| 17 | -19.823 | 13.469 | -68.972 | .893 | 21.028 | -.274 |
| 17 | -19.823 | 13.483 | -68.986 | .898 | 21.014 | -.137 |
| 17 | -19.823 | 13.497 | -69.000 | .902 | 21.000 | -.000 |
| 18 | -21.784 | 9.938 | -65.441 | 0.000 | 24.559 | -21.784 |
| 18 | -21.784 | 10.101 | -65.604 | .048 | 24.396 | -21.055 |
| 18 | -21.784 | 10.267 | -65.770 | .095 | 24.230 | -20.297 |
| 18 | -21.784 | 10.436 | -65.939 | .143 | 24.061 | -19.509 |
| 18 | -21.784 | 10.608 | -66.111 | .190 | 23.889 | -18.691 |
| 18 | -21.784 | 10.782 | -66.285 | .238 | 23.715 | -17.840 |
| 18 | -21.784 | 10.960 | -66.463 | .285 | 23.537 | -16.955 |
| 18 | -21.784 | 11.140 | -66.643 | .333 | 23.357 | -16.035 |
| 18 | -21.784 | 11.322 | -66.825 | .380 | 23.175 | -15.077 |
| 18 | -21.784 | 11.506 | -67.009 | .428 | 22.991 | -14.080 |
| 18 | -21.784 | 11.692 | -67.195 | .475 | 22.805 | -13.042 |
| 18 | -21.784 | 11.878 | -67.381 | .523 | 22.619 | -11.961 |
| 18 | -21.784 | 12.066 | -67.569 | .570 | 22.431 | -10.835 |
| 18 | -21.784 | 12.253 | -67.756 | .618 | 22.244 | -9.663 |
| 18 | -21.784 | 12.439 | -67.942 | .665 | 22.058 | -8.443 |
| 18 | -21.784 | 12.625 | -68.127 | .713 | 21.873 | -7.172 |
| 18 | -21.784 | 12.807 | -68.310 | .760 | 21.690 | -5.849 |
| 18 | -21.784 | 12.987 | -68.490 | .808 | 21.510 | -4.472 |
| 18 | -21.784 | 13.163 | -68.666 | .855 | 21.334 | -3.039 |
| 18 | -21.784 | 13.333 | -68.836 | .903 | 21.164 | -1.549 |
| 18 | -21.784 | 13.333 | -68.836 | .903 | 21.164 | -1.549 |
| 18 | -21.784 | 13.350 | -68.853 | .908 | 21.147 | -1.397 |
| 18 | -21.784 | 13.367 | -68.869 | .912 | 21.131 | -1.244 |
| 18 | -21.784 | 13.383 | -68.886 | .917 | 21.114 | -1.090 |
| 18 | -21.784 | 13.400 | -68.903 | .922 | 21.097 | -.936 |
| 18 | -21.784 | 13.416 | -68.919 | .927 | 21.081 | -.782 |
| 18 | -21.784 | 13.432 | -68.935 | .931 | 21.065 | -.627 |
| 18 | -21.784 | 13.449 | -68.952 | .936 | 21.048 | -.471 |
| 18 | -21.784 | 13.465 | -68.968 | .941 | 21.032 | -.314 |
| 18 | -21.784 | 13.481 | -68.984 | .946 | 21.016 | -.158 |
| 18 | -21.784 | 13.481 | -68.984 | .946 | 21.016 | -.158 |
| 18 | -21.784 | 13.483 | -68.986 | .946 | 21.014 | -.142 |
| 18 | -21.784 | 13.484 | -68.987 | .947 | 21.013 | -.126 |
| 18 | -21.784 | 13.486 | -68.989 | .947 | 21.011 | -.110 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 18 | -21.784 | 13.487 | -68.990 | .947 | 21.010 | -.095 |
| 18 | -21.784 | 13.489 | -68.992 | .948 | 21.008 | -.079 |
| 18 | -21.784 | 13.491 | -68.994 | .948 | 21.006 | -.063 |
| 18 | -21.784 | 13.492 | -68.995 | .949 | 21.005 | -.047 |
| 18 | -21.784 | 13.494 | -68.997 | .949 | 21.003 | -.032 |
| 18 | -21.784 | 13.495 | -68.998 | .950 | 21.002 | -.016 |
| 18 | -21.784 | 13.495 | -68.998 | .950 | 21.002 | -.016 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.014 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.013 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.011 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.009 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.008 |
| 18 | -21.784 | 13.496 | -68.999 | .950 | 21.001 | -.006 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.005 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.003 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.002 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.002 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.001 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.000 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.000 |
| 18 | -21.784 | 13.497 | -69.000 | .950 | 21.000 | -.000 |
| 19 | -23.903 | 9.450 | -64.953 | 0.000 | 25.047 | -23.903 |
| 19 | -23.903 | 9.626 | -65.129 | .050 | 24.871 | -23.149 |
| 19 | -23.903 | 9.807 | -65.310 | .100 | 24.690 | -22.362 |
| 19 | -23.903 | 9.993 | -65.496 | .150 | 24.504 | -21.541 |
| 19 | -23.903 | 10.182 | -65.685 | .200 | 24.315 | -20.684 |
| 19 | -23.903 | 10.376 | -65.879 | .250 | 24.121 | -19.788 |
| 19 | -23.903 | 10.574 | -66.077 | .300 | 23.923 | -18.851 |
| 19 | -23.903 | 10.776 | -66.279 | .349 | 23.721 | -17.872 |
| 19 | -23.903 | 10.981 | -66.484 | .399 | 23.516 | -16.847 |
| 19 | -23.903 | 11.190 | -66.693 | .449 | 23.307 | -15.773 |
| 19 | -23.903 | 11.402 | -66.905 | .499 | 23.095 | -14.649 |
| 19 | -23.903 | 11.616 | -67.118 | .549 | 22.882 | -13.472 |
| 19 | -23.903 | 11.831 | -67.334 | .599 | 22.666 | -12.239 |
| 19 | -23.903 | 12.047 | -67.550 | .649 | 22.450 | -10.946 |
| 19 | -23.903 | 12.264 | -67.767 | .699 | 22.233 | -9.591 |
| 19 | -23.903 | 12.480 | -67.983 | .749 | 22.017 | -8.172 |
| 19 | -23.903 | 12.693 | -68.196 | .799 | 21.804 | -6.684 |
| 19 | -23.903 | 12.903 | -68.406 | .849 | 21.594 | -5.126 |
| 19 | -23.903 | 13.108 | -68.611 | .899 | 21.389 | -3.494 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 19 | -23.903 | 13.307 | -68.810 | .948 | 21.190 | -1.786 |
| 19 | -23.903 | 13.497 | -69.000 | .998 | 21.000 | -.000 |
| 20 | -26.190 | 8.895 | -64.398 | 0.000 | 25.602 | -26.190 |
| 20 | -26.190 | 9.085 | -64.588 | .052 | 25.412 | -25.421 |
| 20 | -26.190 | 9.280 | -64.783 | .105 | 25.217 | -24.615 |
| 20 | -26.190 | 9.481 | -64.984 | .157 | 25.016 | -23.769 |
| 20 | -26.190 | 9.688 | -65.191 | .209 | 24.809 | -22.880 |
| 20 | -26.190 | 9.902 | -65.405 | .262 | 24.595 | -21.946 |
| 20 | -26.190 | 10.121 | -65.624 | .314 | 24.376 | -20.964 |
| 20 | -26.190 | 10.346 | -65.849 | .366 | 24.151 | -19.931 |
| 20 | -26.190 | 10.576 | -66.079 | .419 | 23.921 | -18.842 |
| 20 | -26.190 | 10.812 | -66.315 | .471 | 23.685 | -17.695 |
| 20 | -26.190 | 11.052 | -66.555 | .523 | 23.445 | -16.486 |
| 20 | -26.190 | 11.297 | -66.800 | .576 | 23.200 | -15.210 |
| 20 | -26.190 | 11.545 | -67.048 | .628 | 22.952 | -13.863 |
| 20 | -26.190 | 11.796 | -67.299 | .680 | 22.701 | -12.441 |
| 20 | -26.190 | 12.049 | -67.552 | .732 | 22.448 | -10.940 |
| 20 | -26.190 | 12.301 | -67.804 | .785 | 22.196 | -9.354 |
| 20 | -26.190 | 12.552 | -68.055 | .837 | 21.945 | -7.679 |
| 20 | -26.190 | 12.799 | -68.302 | .889 | 21.698 | -5.910 |
| 20 | -26.190 | 13.041 | -68.544 | .942 | 21.456 | -4.043 |
| 20 | -26.190 | 13.274 | -68.777 | .994 | 21.223 | -2.075 |
| 20 | -26.190 | 13.497 | -69.000 | 1.046 | 21.000 | -.000 |
| 21 | -28.661 | 8.267 | -63.770 | 0.000 | 26.230 | -28.661 |
| 21 | -28.661 | 8.467 | -63.970 | .055 | 26.030 | -27.888 |
| 21 | -28.661 | 8.674 | -64.177 | .109 | 25.823 | -27.073 |
| 21 | -28.661 | 8.890 | -64.393 | .164 | 25.607 | -26.213 |
| 21 | -28.661 | 9.113 | -64.616 | .219 | 25.384 | -25.305 |
| 21 | -28.661 | 9.345 | -64.848 | .274 | 25.152 | -24.344 |
| 21 | -28.661 | 9.585 | -65.088 | .328 | 24.912 | -23.326 |
| 21 | -28.661 | 9.833 | -65.336 | .383 | 24.664 | -22.248 |
| 21 | -28.661 | 10.090 | -65.593 | .438 | 24.407 | -21.104 |
| 21 | -28.661 | 10.355 | -65.858 | .493 | 24.142 | -19.889 |
| 21 | -28.661 | 10.627 | -66.130 | .547 | 23.870 | -18.598 |
| 21 | -28.661 | 10.906 | -66.409 | .602 | 23.591 | -17.224 |
| 21 | -28.661 | 11.192 | -66.695 | .657 | 23.305 | -15.762 |
| 21 | -28.661 | 11.484 | -66.987 | .711 | 23.013 | -14.204 |
| 21 | -28.661 | 11.779 | -67.282 | .766 | 22.718 | -12.543 |
| 21 | -28.661 | 12.076 | -67.579 | .821 | 22.421 | -10.771 |
| 21 | -28.661 | 12.373 | -67.876 | .876 | 22.124 | -8.882 |
| 21 | -28.661 | 12.667 | -68.170 | .930 | 21.830 | -6.868 |
| 21 | -28.661 | 12.956 | -68.459 | .985 | 21.541 | -4.721 |
| 21 | -28.661 | 13.234 | -68.737 | 1.040 | 21.263 | -2.433 |
| 21 | -28.661 | 12.956 | -68.459 | .985 | 21.541 | -4.721 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 21 | -28.661 | 12.984 | -68.487 | .990 | 21.513 | -4.498 |
| 21 | -28.661 | 13.012 | -68.515 | .996 | 21.485 | -4.274 |
| 21 | -28.661 | 13.040 | -68.543 | 1.001 | 21.457 | -4.049 |
| 21 | -28.661 | 13.068 | -68.571 | 1.007 | 21.429 | -3.823 |
| 21 | -28.661 | 13.096 | -68.599 | 1.012 | 21.401 | -3.595 |
| 21 | -28.661 | 13.124 | -68.627 | 1.018 | 21.373 | -3.365 |
| 21 | -28.661 | 13.151 | -68.654 | 1.023 | 21.346 | -3.134 |
| 21 | -28.661 | 13.179 | -68.682 | 1.029 | 21.318 | -2.902 |
| 21 | -28.661 | 13.206 | -68.709 | 1.034 | 21.291 | -2.663 |
| 21 | -28.661 | 13.234 | -68.737 | 1.040 | 21.263 | -2.433 |
| 21 | -28.661 | 13.261 | -68.764 | 1.045 | 21.236 | -2.197 |
| 21 | -28.661 | 13.288 | -68.791 | 1.051 | 21.209 | -1.958 |
| 21 | -28.661 | 13.314 | -68.817 | 1.056 | 21.183 | -1.719 |
| 21 | -28.661 | 13.341 | -68.844 | 1.062 | 21.156 | -1.478 |
| 21 | -28.661 | 13.367 | -68.870 | 1.067 | 21.130 | -1.235 |
| 21 | -28.661 | 13.394 | -68.897 | 1.073 | 21.103 | -.991 |
| 21 | -28.661 | 13.420 | -68.923 | 1.078 | 21.077 | -.746 |
| 21 | -28.661 | 13.446 | -68.949 | 1.084 | 21.051 | -.499 |
| 21 | -28.661 | 13.471 | -68.974 | 1.089 | 21.026 | -.250 |
| 21 | -28.661 | 13.471 | -68.974 | 1.089 | 21.026 | -.250 |
| 21 | -28.661 | 13.474 | -68.977 | 1.090 | 21.023 | -.225 |
| 21 | -28.661 | 13.477 | -68.980 | 1.090 | 21.020 | -.200 |
| 21 | -28.661 | 13.479 | -68.982 | 1.091 | 21.018 | -.175 |
| 21 | -28.661 | 13.482 | -68.985 | 1.091 | 21.015 | -.150 |
| 21 | -28.661 | 13.484 | -68.987 | 1.092 | 21.013 | -.125 |
| 21 | -28.661 | 13.487 | -68.990 | 1.092 | 21.010 | -.100 |
| 21 | -28.661 | 13.489 | -68.992 | 1.093 | 21.008 | -.075 |
| 21 | -28.661 | 13.492 | -68.995 | 1.093 | 21.005 | -.050 |
| 21 | -28.661 | 13.494 | -68.997 | 1.094 | 21.003 | -.025 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.000 |
| 21 | -28.661 | 13.494 | -68.997 | 1.094 | 21.003 | -.025 |
| 21 | -28.661 | 13.495 | -68.998 | 1.094 | 21.002 | -.023 |
| 21 | -28.661 | 13.495 | -68.998 | 1.094 | 21.002 | -.020 |
| 21 | -28.661 | 13.495 | -68.998 | 1.094 | 21.002 | -.018 |
| 21 | -28.661 | 13.496 | -68.998 | 1.094 | 21.002 | -.015 |
| 21 | -28.661 | 13.496 | -68.999 | 1.094 | 21.001 | -.013 |
| 21 | -28.661 | 13.496 | -68.999 | 1.094 | 21.001 | -.010 |
| 21 | -28.661 | 13.496 | -68.999 | 1.094 | 21.001 | -.008 |
| 21 | -28.661 | 13.497 | -68.999 | 1.094 | 21.001 | -.005 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.003 |
| 21 | -28.661 | 13.497 | -68.999 | 1.094 | 21.001 | -.005 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.005 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.005 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.004 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.004 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.004 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.004 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.003 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.003 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.003 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.003 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.002 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.002 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.002 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.002 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.001 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.001 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.001 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.001 |
| 21 | -28.661 | 13.497 | -69.000 | 1.094 | 21.000 | -.000 |
| 22 | -31.326 | 7.556 | -63.058 | 0.000 | 26.942 | -31.326 |
| 22 | -31.326 | 7.762 | -63.265 | .057 | 26.735 | -30.564 |
| 22 | -31.326 | 7.979 | -63.482 | .114 | 26.518 | -29.756 |
| 22 | -31.326 | 8.205 | -63.708 | .171 | 26.292 | -28.898 |
| 22 | -31.326 | 8.442 | -63.945 | .229 | 26.055 | -27.985 |
| 22 | -31.326 | 8.690 | -64.193 | .286 | 25.807 | -27.012 |
| 22 | -31.326 | 8.949 | -64.452 | .343 | 25.548 | -25.974 |
| 22 | -31.326 | 9.219 | -64.722 | .400 | 25.278 | -24.866 |
| 22 | -31.326 | 9.502 | -65.005 | .457 | 24.995 | -23.680 |
| 22 | -31.326 | 9.796 | -65.299 | .514 | 24.701 | -22.410 |
| 22 | -31.326 | 10.103 | -65.606 | .571 | 24.394 | -21.046 |
| 22 | -31.326 | 10.420 | -65.923 | .628 | 24.077 | -19.582 |
| 22 | -31.326 | 10.749 | -66.252 | .686 | 23.748 | -18.005 |
| 22 | -31.326 | 11.087 | -66.590 | .743 | 23.410 | -16.308 |
| 22 | -31.326 | 11.434 | -66.937 | .800 | 23.063 | -14.477 |
| 22 | -31.326 | 11.786 | -67.289 | .857 | 22.711 | -12.501 |
| 22 | -31.326 | 12.141 | -67.644 | .914 | 22.356 | -10.367 |
| 22 | -31.326 | 12.496 | -67.999 | .971 | 22.001 | -8.063 |
| 22 | -31.326 | 12.844 | -68.347 | 1.028 | 21.653 | -5.575 |
| 22 | -31.326 | 13.180 | -68.683 | 1.085 | 21.317 | -2.891 |
| 22 | -31.326 | 12.844 | -68.347 | 1.028 | 21.653 | -5.575 |
| 22 | -31.326 | 12.878 | -68.381 | 1.034 | 21.619 | -5.315 |
| 22 | -31.326 | 12.913 | -68.415 | 1.040 | 21.585 | -5.054 |
| 22 | -31.326 | 12.947 | -68.450 | 1.045 | 21.550 | -4.791 |
| 22 | -31.326 | 12.980 | -68.483 | 1.051 | 21.517 | -4.525 |
| 22 | -31.326 | 13.014 | -68.517 | 1.057 | 21.483 | -4.258 |
| 22 | -31.326 | 13.048 | -68.551 | 1.063 | 21.449 | -3.989 |
| 22 | -31.326 | 13.081 | -68.584 | 1.068 | 21.416 | -3.717 |
| 22 | -31.326 | 13.114 | -68.617 | 1.074 | 21.383 | -3.444 |
| 22 | -31.326 | 13.147 | -68.650 | 1.080 | 21.350 | -3.168 |
| 22 | -31.326 | 13.180 | -68.683 | 1.085 | 21.317 | -2.891 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 22 | -31.326 | 13.213 | -68.716 | 1.091 | 21.284 | -2.611 |
| 22 | -31.326 | 13.245 | -68.748 | 1.097 | 21.252 | -2.330 |
| 22 | -31.326 | 13.278 | -68.781 | 1.103 | 21.219 | -2.046 |
| 22 | -31.326 | 13.310 | -68.813 | 1.108 | 21.187 | -1.760 |
| 22 | -31.326 | 13.342 | -68.845 | 1.114 | 21.155 | -1.472 |
| 22 | -31.326 | 13.373 | -68.876 | 1.120 | 21.124 | -1.182 |
| 22 | -31.326 | 13.405 | -68.908 | 1.125 | 21.092 | -.890 |
| 22 | -31.326 | 13.436 | -68.939 | 1.131 | 21.061 | -.595 |
| 22 | -31.326 | 13.466 | -68.969 | 1.137 | 21.031 | -.299 |
| 22 | -31.326 | 13.436 | -68.939 | 1.131 | 21.061 | -.595 |
| 22 | -31.326 | 13.439 | -68.942 | 1.132 | 21.058 | -.566 |
| 22 | -31.326 | 13.442 | -68.945 | 1.132 | 21.055 | -.536 |
| 22 | -31.326 | 13.445 | -68.948 | 1.133 | 21.052 | -.506 |
| 22 | -31.326 | 13.448 | -68.951 | 1.133 | 21.049 | -.477 |
| 22 | -31.326 | 13.451 | -68.954 | 1.134 | 21.046 | -.447 |
| 22 | -31.326 | 13.454 | -68.957 | 1.135 | 21.043 | -.418 |
| 22 | -31.326 | 13.457 | -68.960 | 1.135 | 21.040 | -.388 |
| 22 | -31.326 | 13.460 | -68.963 | 1.136 | 21.037 | -.358 |
| 22 | -31.326 | 13.463 | -68.966 | 1.136 | 21.034 | -.328 |
| 22 | -31.326 | 13.466 | -68.969 | 1.137 | 21.031 | -.299 |
| 22 | -31.326 | 13.470 | -68.973 | 1.137 | 21.027 | -.269 |
| 22 | -31.326 | 13.473 | -68.976 | 1.138 | 21.024 | -.239 |
| 22 | -31.326 | 13.476 | -68.979 | 1.139 | 21.021 | -.209 |
| 22 | -31.326 | 13.479 | -68.982 | 1.139 | 21.018 | -.179 |
| 22 | -31.326 | 13.482 | -68.985 | 1.140 | 21.015 | -.150 |
| 22 | -31.326 | 13.485 | -68.988 | 1.140 | 21.012 | -.120 |
| 22 | -31.326 | 13.488 | -68.991 | 1.141 | 21.009 | -.090 |
| 22 | -31.326 | 13.491 | -68.994 | 1.141 | 21.006 | -.060 |
| 22 | -31.326 | 13.494 | -68.997 | 1.142 | 21.003 | -.030 |
| 22 | -31.326 | 13.491 | -68.994 | 1.141 | 21.006 | -.060 |
| 22 | -31.326 | 13.491 | -68.994 | 1.141 | 21.006 | -.057 |
| 22 | -31.326 | 13.492 | -68.995 | 1.141 | 21.005 | -.054 |
| 22 | -31.326 | 13.492 | -68.995 | 1.142 | 21.005 | -.051 |
| 22 | -31.326 | 13.492 | -68.995 | 1.142 | 21.005 | -.048 |
| 22 | -31.326 | 13.492 | -68.995 | 1.142 | 21.005 | -.045 |
| 22 | -31.326 | 13.493 | -68.996 | 1.142 | 21.004 | -.042 |
| 22 | -31.326 | 13.493 | -68.996 | 1.142 | 21.004 | -.039 |
| 22 | -31.326 | 13.493 | -68.996 | 1.142 | 21.004 | -.036 |
| 22 | -31.326 | 13.494 | -68.997 | 1.142 | 21.003 | -.033 |
| 22 | -31.326 | 13.494 | -68.997 | 1.142 | 21.003 | -.030 |
| 22 | -31.326 | 13.494 | -68.997 | 1.142 | 21.003 | -.027 |
| 22 | -31.326 | 13.495 | -68.998 | 1.142 | 21.002 | -.024 |
| 22 | -31.326 | 13.495 | -68.998 | 1.142 | 21.002 | -.021 |
| 22 | -31.326 | 13.495 | -68.998 | 1.142 | 21.002 | -.018 |
| 22 | -31.326 | 13.496 | -68.998 | 1.142 | 21.002 | -.015 |
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.012 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.009 |
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.006 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.003 |
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.006 |
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.006 |
| 22 | -31.326 | 13.496 | -68.999 | 1.142 | 21.001 | -.005 |
| 22 | -31.326 | 13.497 | -68.999 | 1.142 | 21.001 | -.005 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.005 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.004 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.004 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.004 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.004 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.003 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.003 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.003 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.002 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.002 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.002 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.001 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.001 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.001 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.001 |
| 22 | -31.326 | 13.497 | -69.000 | 1.142 | 21.000 | -.000 |
| 23 | -34.198 | 6.753 | -62.256 | 0.000 | 27.744 | -34.198 |
| 23 | -34.198 | 6.961 | -62.464 | .060 | 27.536 | -33.465 |
| 23 | -34.198 | 7.181 | -62.684 | .119 | 27.316 | -32.683 |
| 23 | -34.198 | 7.413 | -62.916 | .179 | 27.084 | -31.846 |
| 23 | -34.198 | 7.658 | -63.161 | .238 | 26.839 | -30.949 |
| 23 | -34.198 | 7.917 | -63.420 | .298 | 26.580 | -29.986 |
| 23 | -34.198 | 8.191 | -63.694 | .357 | 26.306 | -28.951 |
| 23 | -34.198 | 8.480 | -63.983 | .417 | 26.017 | -27.835 |
| 23 | -34.198 | 8.786 | -64.289 | .476 | 25.711 | -26.629 |
| 23 | -34.198 | 9.109 | -64.612 | .536 | 25.388 | -25.323 |
| 23 | -34.198 | 9.449 | -64.952 | .595 | 25.048 | -23.907 |
| 23 | -34.198 | 9.806 | -65.309 | .655 | 24.691 | -22.367 |
| 23 | -34.198 | 10.181 | -65.684 | .714 | 24.316 | -20.688 |
| 23 | -34.198 | 10.573 | -66.076 | .774 | 23.924 | -18.855 |
| 23 | -34.198 | 10.981 | -66.484 | .833 | 23.516 | -16.850 |
| 23 | -34.198 | 11.401 | -66.904 | .893 | 23.096 | -14.653 |
| 23 | -34.198 | 11.831 | -67.333 | .952 | 22.667 | -12.242 |
| 23 | -34.198 | 12.264 | -67.767 | 1.012 | 22.233 | -9.594 |
| 23 | -34.198 | 12.693 | -68.196 | 1.071 | 21.804 | -6.686 |
| 23 | -34.198 | 13.108 | -68.611 | 1.131 | 21.389 | -3.495 |
| 23 | -34.198 | 13.497 | -69.000 | 1.191 | 21.000 | -.000 |

# TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 24 | -37.285 | 5.852 | -61.355 | 0.000 | 28.645 | -37.285 |
| 24 | -37.285 | 6.055 | -61.558 | .062 | 28.442 | -36.603 |
| 24 | -37.285 | 6.270 | -61.773 | .124 | 28.227 | -35.870 |
| 24 | -37.285 | 6.500 | -62.003 | .186 | 27.997 | -35.080 |
| 24 | -37.285 | 6.745 | -62.248 | .248 | 27.752 | -34.227 |
| 24 | -37.285 | 7.007 | -62.510 | .310 | 27.490 | -33.304 |
| 24 | -37.285 | 7.287 | -62.790 | .372 | 27.210 | -32.301 |
| 24 | -37.285 | 7.587 | -63.090 | .434 | 26.910 | -31.209 |
| 24 | -37.285 | 7.909 | -63.412 | .495 | 26.588 | -30.017 |
| 24 | -37.285 | 8.254 | -63.757 | .557 | 26.243 | -28.710 |
| 24 | -37.285 | 8.623 | -64.126 | .619 | 25.874 | -27.274 |
| 24 | -37.285 | 9.019 | -64.522 | .681 | 25.478 | -25.690 |
| 24 | -37.285 | 9.441 | -64.944 | .743 | 25.056 | -23.937 |
| 24 | -37.285 | 9.892 | -65.395 | .805 | 24.605 | -21.990 |
| 24 | -37.285 | 10.369 | -65.872 | .867 | 24.128 | -19.820 |
| 24 | -37.285 | 10.872 | -66.375 | .929 | 23.625 | -17.395 |
| 24 | -37.285 | 11.397 | -66.900 | .991 | 23.100 | -14.676 |
| 24 | -37.285 | 11.935 | -67.438 | 1.053 | 22.562 | -11.623 |
| 24 | -37.285 | 12.477 | -67.980 | 1.115 | 22.020 | -8.189 |
| 24 | -37.285 | 13.005 | -68.508 | 1.177 | 21.492 | -4.329 |
| 24 | -37.285 | 12.477 | -67.980 | 1.115 | 22.020 | -8.189 |
| 24 | -37.285 | 12.531 | -68.034 | 1.121 | 21.966 | -7.823 |
| 24 | -37.285 | 12.584 | -68.087 | 1.127 | 21.913 | -7.452 |
| 24 | -37.285 | 12.638 | -68.141 | 1.133 | 21.859 | -7.077 |
| 24 | -37.285 | 12.691 | -68.194 | 1.140 | 21.806 | -6.698 |
| 24 | -37.285 | 12.744 | -68.247 | 1.146 | 21.753 | -6.315 |
| 24 | -37.285 | 12.797 | -68.300 | 1.152 | 21.700 | -5.927 |
| 24 | -37.285 | 12.849 | -68.352 | 1.158 | 21.648 | -5.534 |
| 24 | -37.285 | 12.902 | -68.405 | 1.164 | 21.595 | -5.137 |
| 24 | -37.285 | 12.954 | -68.457 | 1.170 | 21.543 | -4.735 |
| 24 | -37.285 | 13.005 | -68.508 | 1.177 | 21.492 | -4.329 |
| 24 | -37.285 | 13.057 | -68.559 | 1.183 | 21.441 | -3.918 |
| 24 | -37.285 | 13.107 | -68.610 | 1.189 | 21.390 | -3.502 |
| 24 | -37.285 | 13.158 | -68.661 | 1.195 | 21.339 | -3.081 |
| 24 | -37.285 | 13.208 | -68.711 | 1.201 | 21.289 | -2.656 |
| 24 | -37.285 | 13.257 | -68.760 | 1.208 | 21.240 | -2.225 |
| 24 | -37.285 | 13.306 | -68.809 | 1.214 | 21.191 | -1.790 |
| 24 | -37.285 | 13.355 | -68.858 | 1.220 | 21.142 | -1.350 |
| 24 | -37.285 | 13.403 | -68.906 | 1.226 | 21.094 | -.905 |
| 24 | -37.285 | 13.450 | -68.953 | 1.232 | 21.047 | -.455 |
| 24 | -37.285 | 13.403 | -68.906 | 1.226 | 21.094 | -.905 |
| 24 | -37.285 | 13.408 | -68.911 | 1.227 | 21.089 | -.860 |
| 24 | -37.285 | 13.412 | -68.915 | 1.227 | 21.085 | -.815 |
| 24 | -37.285 | 13.417 | -68.920 | 1.228 | 21.080 | -.770 |
| 24 | -37.285 | 13.422 | -68.925 | 1.229 | 21.075 | -.726 |
| 24 | -37.285 | 13.427 | -68.930 | 1.229 | 21.070 | -.681 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 24 | -37.285 | 13.431 | -68.934 | 1.230 | 21.066 | -.636 |
| 24 | -37.285 | 13.436 | -68.939 | 1.231 | 21.061 | -.590 |
| 24 | -37.285 | 13.441 | -68.944 | 1.231 | 21.056 | -.545 |
| 24 | -37.285 | 13.446 | -68.949 | 1.232 | 21.051 | -.500 |
| 24 | -37.285 | 13.450 | -68.953 | 1.232 | 21.047 | -.455 |
| 24 | -37.285 | 13.455 | -68.958 | 1.233 | 21.042 | -.410 |
| 24 | -37.285 | 13.460 | -68.963 | 1.234 | 21.037 | -.364 |
| 24 | -37.285 | 13.464 | -68.967 | 1.234 | 21.033 | -.319 |
| 24 | -37.285 | 13.469 | -68.972 | 1.235 | 21.028 | -.274 |
| 24 | -37.285 | 13.474 | -68.977 | 1.236 | 21.023 | -.228 |
| 24 | -37.285 | 13.478 | -68.981 | 1.236 | 21.019 | -.183 |
| 24 | -37.285 | 13.483 | -68.986 | 1.237 | 21.014 | -.137 |
| 24 | -37.285 | 13.488 | -68.991 | 1.237 | 21.009 | -.091 |
| 24 | -37.285 | 13.492 | -68.995 | 1.238 | 21.005 | -.046 |
| 24 | -37.285 | 13.492 | -68.995 | 1.238 | 21.005 | -.046 |
| 24 | -37.285 | 13.493 | -68.996 | 1.238 | 21.004 | -.041 |
| 24 | -37.285 | 13.493 | -68.996 | 1.238 | 21.004 | -.037 |
| 24 | -37.285 | 13.494 | -68.997 | 1.238 | 21.003 | -.032 |
| 24 | -37.285 | 13.494 | -68.997 | 1.238 | 21.003 | -.027 |
| 24 | -37.285 | 13.495 | -68.998 | 1.238 | 21.002 | -.023 |
| 24 | -37.285 | 13.495 | -68.998 | 1.238 | 21.002 | -.018 |
| 24 | -37.285 | 13.496 | -68.999 | 1.238 | 21.001 | -.014 |
| 24 | -37.285 | 13.496 | -68.999 | 1.238 | 21.001 | -.009 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.005 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.005 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.004 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.004 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.003 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.003 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.002 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.002 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.001 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.001 |
| 24 | -37.285 | 13.497 | -69.000 | 1.239 | 21.000 | -.000 |
| 25 | -40.591 | 4.848 | -60.351 | 0.000 | 29.649 | -40.591 |
| 25 | -40.591 | 5.035 | -60.538 | .064 | 29.462 | -39.986 |
| 25 | -40.591 | 5.235 | -60.738 | .129 | 29.262 | -39.332 |
| 25 | -40.591 | 5.451 | -60.954 | .193 | 29.046 | -38.621 |
| 25 | -40.591 | 5.685 | -61.187 | .257 | 28.813 | -37.846 |
| 25 | -40.591 | 5.937 | -61.440 | .322 | 28.560 | -37.000 |
| 25 | -40.591 | 6.211 | -61.714 | .386 | 28.286 | -36.072 |
| 25 | -40.591 | 6.508 | -62.011 | .450 | 27.989 | -35.051 |
| 25 | -40.591 | 6.832 | -62.335 | .515 | 27.665 | -33.921 |
| 25 | -40.591 | 7.186 | -62.689 | .579 | 27.311 | -32.666 |
| 25 | -40.591 | 7.572 | -63.075 | .643 | 26.925 | -31.266 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 25 | -40.591 | 7.995 | -63.498 | .708 | 26.502 | -29.695 |
| 25 | -40.591 | 8.458 | -63.961 | .772 | 26.039 | -27.923 |
| 25 | -40.591 | 8.964 | -64.467 | .836 | 25.533 | -25.913 |
| 25 | -40.591 | 9.516 | -65.019 | .901 | 24.981 | -23.620 |
| 25 | -40.591 | 10.115 | -65.618 | .965 | 24.382 | -20.988 |
| 25 | -40.591 | 10.760 | -66.263 | 1.029 | 23.737 | -17.952 |
| 25 | -40.591 | 11.442 | -66.945 | 1.094 | 23.055 | -14.430 |
| 25 | -40.591 | 12.147 | -67.650 | 1.158 | 22.350 | -10.331 |
| 25 | -40.591 | 12.847 | -68.350 | 1.222 | 21.650 | -5.554 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.000 |
| 25 | -40.591 | 12.847 | -68.350 | 1.222 | 21.650 | -5.554 |
| 25 | -40.591 | 12.915 | -68.418 | 1.229 | 21.582 | -5.035 |
| 25 | -40.591 | 12.983 | -68.486 | 1.235 | 21.514 | -4.508 |
| 25 | -40.591 | 13.050 | -68.553 | 1.242 | 21.447 | -3.973 |
| 25 | -40.591 | 13.116 | -68.619 | 1.248 | 21.381 | -3.430 |
| 25 | -40.591 | 13.182 | -68.685 | 1.254 | 21.315 | -2.879 |
| 25 | -40.591 | 13.247 | -68.750 | 1.261 | 21.250 | -2.320 |
| 25 | -40.591 | 13.311 | -68.814 | 1.267 | 21.186 | -1.753 |
| 25 | -40.591 | 13.374 | -68.877 | 1.274 | 21.123 | -1.177 |
| 25 | -40.591 | 13.436 | -68.939 | 1.280 | 21.061 | -.593 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.000 |
| 25 | -40.591 | 13.436 | -68.939 | 1.280 | 21.061 | -.593 |
| 25 | -40.591 | 13.442 | -68.945 | 1.281 | 21.055 | -.534 |
| 25 | -40.591 | 13.448 | -68.951 | 1.282 | 21.049 | -.475 |
| 25 | -40.591 | 13.454 | -68.957 | 1.282 | 21.043 | -.416 |
| 25 | -40.591 | 13.460 | -68.963 | 1.283 | 21.037 | -.357 |
| 25 | -40.591 | 13.467 | -68.970 | 1.283 | 21.030 | -.297 |
| 25 | -40.591 | 13.473 | -68.976 | 1.284 | 21.024 | -.238 |
| 25 | -40.591 | 13.479 | -68.982 | 1.285 | 21.018 | -.179 |
| 25 | -40.591 | 13.485 | -68.988 | 1.285 | 21.012 | -.119 |
| 25 | -40.591 | 13.491 | -68.994 | 1.286 | 21.006 | -.060 |
| 25 | -40.591 | 13.491 | -68.994 | 1.286 | 21.006 | -.060 |
| 25 | -40.591 | 13.492 | -68.995 | 1.286 | 21.005 | -.054 |
| 25 | -40.591 | 13.492 | -68.995 | 1.286 | 21.005 | -.048 |
| 25 | -40.591 | 13.493 | -68.996 | 1.286 | 21.004 | -.042 |
| 25 | -40.591 | 13.493 | -68.996 | 1.286 | 21.004 | -.036 |
| 25 | -40.591 | 13.494 | -68.997 | 1.286 | 21.003 | -.030 |
| 25 | -40.591 | 13.495 | -68.998 | 1.286 | 21.002 | -.024 |
| 25 | -40.591 | 13.495 | -68.998 | 1.286 | 21.002 | -.018 |
| 25 | -40.591 | 13.496 | -68.999 | 1.287 | 21.001 | -.012 |
| 25 | -40.591 | 13.496 | -68.999 | 1.287 | 21.001 | -.006 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.000 |
| 25 | -40.591 | 13.496 | -68.999 | 1.287 | 21.001 | -.006 |
| 25 | -40.591 | 13.496 | -68.999 | 1.287 | 21.001 | -.005 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.005 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.004 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.004 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.003 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.002 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.002 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.001 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.001 |
| 25 | -40.591 | 13.497 | -69.000 | 1.287 | 21.000 | -.000 |
| 26 | -44.118 | 3.740 | -59.243 | 0.000 | 30.757 | -44.118 |
| 26 | -44.118 | 3.899 | -59.402 | .067 | 30.598 | -43.620 |
| 26 | -44.118 | 4.071 | -59.574 | .133 | 30.426 | -43.076 |
| 26 | -44.118 | 4.259 | -59.762 | .200 | 30.238 | -42.482 |
| 26 | -44.118 | 4.463 | -59.966 | .267 | 30.034 | -41.829 |
| 26 | -44.118 | 4.688 | -60.191 | .334 | 29.809 | -41.109 |
| 26 | -44.118 | 4.935 | -60.438 | .400 | 29.562 | -40.310 |
| 26 | -44.118 | 5.208 | -60.711 | .467 | 29.289 | -39.420 |
| 26 | -44.118 | 5.511 | -61.014 | .534 | 28.986 | -38.423 |
| 26 | -44.118 | 5.848 | -61.351 | .601 | 28.649 | -37.298 |
| 26 | -44.118 | 6.226 | -61.729 | .667 | 28.271 | -36.021 |
| 26 | -44.118 | 6.650 | -62.153 | .734 | 27.847 | -34.559 |
| 26 | -44.118 | 7.128 | -62.631 | .801 | 27.369 | -32.873 |
| 26 | -44.118 | 7.669 | -63.172 | .868 | 26.828 | -30.909 |
| 26 | -44.118 | 8.283 | -63.786 | .934 | 26.214 | -28.600 |
| 26 | -44.118 | 8.978 | -64.481 | 1.001 | 25.519 | -25.856 |
| 26 | -44.118 | 9.763 | -65.266 | 1.068 | 24.734 | -22.556 |
| 26 | -44.118 | 10.638 | -66.141 | 1.134 | 23.859 | -18.544 |
| 26 | -44.118 | 11.590 | -67.092 | 1.201 | 22.908 | -13.618 |
| 26 | -44.118 | 12.574 | -68.077 | 1.268 | 21.923 | -7.527 |
| 26 | -44.118 | 11.590 | -67.092 | 1.201 | 22.908 | -13.618 |
| 26 | -44.118 | 11.688 | -67.191 | 1.208 | 22.809 | -13.066 |
| 26 | -44.118 | 11.786 | -67.289 | 1.215 | 22.711 | -12.501 |
| 26 | -44.118 | 11.884 | -67.387 | 1.221 | 22.613 | -11.925 |
| 26 | -44.118 | 11.983 | -67.486 | 1.228 | 22.514 | -11.336 |
| 26 | -44.118 | 12.082 | -67.585 | 1.235 | 22.415 | -10.735 |
| 26 | -44.118 | 12.181 | -67.684 | 1.241 | 22.316 | -10.120 |
| 26 | -44.118 | 12.279 | -67.782 | 1.248 | 22.218 | -9.492 |
| 26 | -44.118 | 12.378 | -67.881 | 1.255 | 22.119 | -8.851 |
| 26 | -44.118 | 12.476 | -67.979 | 1.261 | 22.021 | -8.196 |
| 26 | -44.118 | 12.574 | -68.077 | 1.268 | 21.923 | -7.527 |
| 26 | -44.118 | 12.671 | -68.174 | 1.275 | 21.826 | -6.843 |
| 26 | -44.118 | 12.767 | -68.270 | 1.281 | 21.730 | -6.145 |
| 26 | -44.118 | 12.863 | -68.366 | 1.288 | 21.634 | -5.431 |
| 26 | -44.118 | 12.958 | -68.461 | 1.295 | 21.539 | -4.703 |
| 26 | -44.118 | 13.051 | -68.554 | 1.301 | 21.446 | -3.959 |
| 26 | -44.118 | 13.144 | -68.647 | 1.308 | 21.353 | -3.199 |
| 26 | -44.118 | 13.235 | -68.738 | 1.315 | 21.262 | -2.424 |

## TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 26 | -44.118 | 13.324 | -68.827 | 1.321 | 21.173 | -1.632 |
| 26 | -44.118 | 13.411 | -68.914 | 1.328 | 21.086 | -.824 |
| 26 | -44.118 | 13.324 | -68.827 | 1.321 | 21.173 | -1.632 |
| 26 | -44.118 | 13.333 | -68.836 | 1.322 | 21.164 | -1.552 |
| 26 | -44.118 | 13.342 | -68.845 | 1.323 | 21.155 | -1.472 |
| 26 | -44.118 | 13.350 | -68.853 | 1.323 | 21.147 | -1.392 |
| 26 | -44.118 | 13.359 | -68.862 | 1.324 | 21.138 | -1.311 |
| 26 | -44.118 | 13.368 | -68.871 | 1.325 | 21.129 | -1.230 |
| 26 | -44.118 | 13.377 | -68.880 | 1.325 | 21.120 | -1.149 |
| 26 | -44.118 | 13.385 | -68.888 | 1.326 | 21.112 | -1.068 |
| 26 | -44.118 | 13.394 | -68.897 | 1.327 | 21.103 | -.987 |
| 26 | -44.118 | 13.403 | -68.906 | 1.327 | 21.094 | -.906 |
| 26 | -44.118 | 13.411 | -68.914 | 1.328 | 21.086 | -.824 |
| 26 | -44.118 | 13.420 | -68.923 | 1.329 | 21.077 | -.743 |
| 26 | -44.118 | 13.429 | -68.932 | 1.329 | 21.068 | -.661 |
| 26 | -44.118 | 13.437 | -68.940 | 1.330 | 21.060 | -.579 |
| 26 | -44.118 | 13.446 | -68.949 | 1.331 | 21.051 | -.497 |
| 26 | -44.118 | 13.455 | -68.957 | 1.331 | 21.043 | -.414 |
| 26 | -44.118 | 13.463 | -68.966 | 1.332 | 21.034 | -.332 |
| 26 | -44.118 | 13.472 | -68.975 | 1.333 | 21.025 | -.249 |
| 26 | -44.118 | 13.480 | -68.983 | 1.333 | 21.017 | -.166 |
| 26 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.083 |
| 26 | -44.118 | 13.480 | -68.983 | 1.333 | 21.017 | -.166 |
| 26 | -44.118 | 13.481 | -68.984 | 1.333 | 21.016 | -.158 |
| 26 | -44.118 | 13.482 | -68.985 | 1.333 | 21.015 | -.150 |
| 26 | -44.118 | 13.483 | -68.986 | 1.334 | 21.014 | -.141 |
| 26 | -44.118 | 13.483 | -68.986 | 1.334 | 21.014 | -.133 |
| 26 | -44.118 | 13.484 | -68.987 | 1.334 | 21.013 | -.125 |
| 26 | -44.118 | 13.485 | -68.988 | 1.334 | 21.012 | -.116 |
| 26 | -44.118 | 13.486 | -68.989 | 1.334 | 21.011 | -.108 |
| 26 | -44.118 | 13.487 | -68.990 | 1.334 | 21.010 | -.100 |
| 26 | -44.118 | 13.488 | -68.991 | 1.334 | 21.009 | -.091 |
| 26 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.083 |
| 26 | -44.118 | 13.489 | -68.992 | 1.334 | 21.008 | -.075 |
| 26 | -44.118 | 13.490 | -68.993 | 1.334 | 21.007 | -.067 |
| 26 | -44.118 | 13.491 | -68.994 | 1.334 | 21.006 | -.058 |
| 26 | -44.118 | 13.492 | -68.995 | 1.334 | 21.005 | -.050 |
| 26 | -44.118 | 13.493 | -68.996 | 1.334 | 21.004 | -.042 |
| 26 | -44.118 | 13.494 | -68.997 | 1.334 | 21.003 | -.033 |
| 26 | -44.118 | 13.495 | -68.997 | 1.334 | 21.003 | -.025 |
| 26 | -44.118 | 13.495 | -68.998 | 1.335 | 21.002 | -.017 |
| 26 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.008 |
| 26 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.008 |
| 26 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.007 |
| 26 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.007 |
| 26 | -44.118 | 13.496 | -68.999 | 1.335 | 21.001 | -.006 |

# TABLE A-VI (CONT)

| RAY | BETA | THETA(RAY) | NORMAL | S | ALPHA2 | RHO 2 |
|---|---|---|---|---|---|---|
| 26 | -44.118 | 13.497 | -68.999 | 1.335 | 21.001 | -.005 |
| 26 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.004 |
| 26 | -44.113 | 13.497 | -69.000 | 1.335 | 21.000 | -.003 |
| 26 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.002 |
| 26 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.002 |
| 26 | -44.118 | 13.497 | -69.000 | 1.335 | 21.000 | -.001 |

## TABLE A-VII

## COORDINATE (X,Y) VALUES, S, ALPHA2 AND RHO2 FOR THE FINAL DESIGN OF SURFACE NUMBER 1 IN FIGURE 20

| Begin RAY | Next RAY | Xsurf (next) | Ysurf (next) | S (next) | Alpha2 (next) | RHO 2 (next) |
|---|---|---|---|---|---|---|
| 1 | 1 | .362 | .000 | .133 | 21.000 | -.000 |
| 1 | 2 | .465 | .040 | .133 | 21.066 | -.642 |
| 1 | 3 | .568 | .079 | .133 | 21.139 | -1.324 |
| 1 | 4 | .671 | .119 | .133 | 21.220 | -2.050 |
| 1 | 5 | .774 | .159 | .133 | 21.309 | -2.824 |
| 1 | 6 | .877 | .199 | .132 | 21.408 | -3.650 |
| 1 | 7 | .980 | .239 | .131 | 21.518 | -4.534 |
| 1 | 8 | 1.082 | .280 | .130 | 21.640 | -5.480 |
| 1 | 9 | 1.185 | .321 | .129 | 21.778 | -6.496 |
| 1 | 10 | 1.287 | .361 | .127 | 21.932 | -7.589 |
| 1 | 11 | 1.389 | .403 | .126 | 22.106 | -8.765 |
| 1 | 12 | 1.491 | .444 | .123 | 22.303 | -10.035 |
| 1 | 13 | 1.593 | .486 | .121 | 22.526 | -11.407 |
| 1 | 14 | 1.694 | .528 | .118 | 22.779 | -12.892 |
| 1 | 15 | 1.796 | .570 | .114 | 23.068 | -14.503 |
| 1 | 16 | 1.896 | .613 | .110 | 23.399 | -16.251 |
| 1 | 17 | 1.997 | .657 | .105 | 23.778 | -18.150 |
| 1 | 18 | 2.097 | .701 | .100 | 24.213 | -20.215 |
| 1 | 19 | 2.197 | .746 | .094 | 24.712 | -22.460 |
| 1 | 20 | 2.296 | .791 | .086 | 25.286 | -24.899 |
| 1 | 21 | 2.394 | .838 | .078 | 25.943 | -27.545 |
| 1 | 22 | 2.492 | .885 | .068 | 26.693 | -30.408 |
| 1 | 23 | 2.589 | .934 | .057 | 27.543 | -33.492 |
| 1 | 24 | 2.685 | .984 | .045 | 28.500 | -36.797 |
| 1 | 25 | 2.780 | 1.035 | .030 | 29.563 | -40.313 |
| 1 | 26 | 2.873 | 1.089 | .014 | 30.725 | -44.019 |

## TABLE A-VIII

## COORDINATE VALUES (X,Y) FOR THE RAY DIAGRAM IN FIGURE 22

| RAY | X0 | Y0 | X1 | Y1 | X2 | Y2 | Xc | Yc | X4 | Y4 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | .110 | .287 | .110 | .362 | .000 | 4.000 | 0.0 | 50.00 | -.00 |
| 2 | 0 | .150 | .390 | .150 | .465 | .040 | 4.000 | 0.0 | 50.00 | -.52 |
| 3 | 0 | .189 | .493 | .189 | .568 | .079 | 4.000 | 0.0 | 50.00 | -1.06 |
| 4 | 0 | .229 | .596 | .229 | .671 | .119 | 4.000 | 0.0 | 50.00 | -1.65 |
| 5 | 0 | .268 | .699 | .268 | .774 | .159 | 4.000 | 0.0 | 50.00 | -2.27 |
| 6 | 0 | .308 | .802 | .308 | .877 | .199 | 4.000 | 0.0 | 50.00 | -2.93 |
| 7 | 0 | .348 | .906 | .348 | .980 | .239 | 4.000 | 0.0 | 50.00 | -3.65 |
| 8 | 0 | .387 | 1.009 | .387 | 1.082 | .280 | 4.000 | 0.0 | 50.00 | -4.41 |
| 9 | 0 | .427 | 1.112 | .427 | 1.185 | .321 | 4.000 | 0.0 | 50.00 | -5.24 |
| 10 | 0 | .466 | 1.215 | .466 | 1.287 | .361 | 4.000 | 0.0 | 50.00 | -6.13 |
| 11 | 0 | .506 | 1.318 | .506 | 1.389 | .403 | 4.000 | 0.0 | 50.00 | -7.09 |
| 12 | 0 | .546 | 1.421 | .546 | 1.491 | .444 | 4.000 | 0.0 | 50.00 | -8.14 |
| 13 | 0 | .585 | 1.524 | .585 | 1.593 | .486 | 4.000 | 0.0 | 50.00 | -9.28 |
| 14 | 0 | .625 | 1.628 | .625 | 1.694 | .528 | 4.000 | 0.0 | 50.00 | -10.53 |
| 15 | 0 | .664 | 1.731 | .664 | 1.796 | .570 | 4.000 | 0.0 | 50.00 | -11.90 |
| 16 | 0 | .704 | 1.834 | .704 | 1.896 | .613 | 4.000 | 0.0 | 50.00 | -13.41 |
| 17 | 0 | .744 | 1.937 | .744 | 1.997 | .657 | 4.000 | 0.0 | 50.00 | -15.08 |
| 18 | 0 | .783 | 2.040 | .783 | 2.097 | .701 | 4.000 | 0.0 | 50.00 | -16.94 |
| 19 | 0 | .823 | 2.143 | .823 | 2.197 | .746 | 4.000 | 0.0 | 50.00 | -19.02 |
| 20 | 0 | .862 | 2.247 | .862 | 2.296 | .791 | 4.000 | 0.0 | 50.00 | -21.35 |
| 21 | 0 | .902 | 2.350 | .902 | 2.394 | .838 | 4.000 | 0.0 | 50.00 | -23.99 |
| 22 | 0 | .942 | 2.453 | .942 | 2.492 | .885 | 4.000 | 0.0 | 50.00 | -27.00 |
| 23 | 0 | .981 | 2.556 | .981 | 2.589 | .934 | 4.000 | 0.0 | 50.00 | -30.44 |
| 24 | 0 | 1.021 | 2.659 | 1.021 | 2.685 | .984 | 4.000 | 0.0 | 50.00 | -34.41 |
| 25 | 0 | 1.060 | 2.762 | 1.060 | 2.780 | 1.035 | 4.000 | 0.0 | 50.00 | -39.03 |
| 26 | 0 | 1.100 | 2.866 | 1.100 | 2.873 | 1.089 | 4.000 | 0.0 | 50.00 | -44.45 |

# CHART PROGRAM LISTING

```
10 PRINT "**********************************************************************"
20 PRINT "*                                                                    *"
30 PRINT "*                               CHART                                *"
40 PRINT "*                                                                    *"
50 PRINT "**********************************************************************"
60    DISP "THIS IS CHART !!!"
70    WAIT 2500
80 Main:OPTION BASE 0
90       !
100      COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
110      COM F,Z3,Z4,Norm,Thetai,Tdelta
120      COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
130      !
140      COM Stuff$,Redraw$,Digitize$,Ray_trace$
150      COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
160      COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
170      !
180      COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
190      COM INTEGER Family,Surf_no
200      COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
210      !
220      COM Y_bullet,Smax_family,Percent_image
230      COM X1(251),Y1(251),Xc(251),X2(251),Y2(251),X2max(251),Y2max(251)
240      COM Y3(251),Y4(251),Beta(251),S2(251),Xsurfmax(251),Ysurfmax(251)
250      !
260      COM Qa(251),Smax(251),Alpha2(251),Rho2(251),Hit(2001)
270      !
280    Main$="Y"            ! SET CONDITION
290    DISP ""
300    PRINTER IS 16
310    NORMAL
320    FIXED 2
330    PRINT PAGE
340    OVERLAP
350    Y$="O"
360    INPUT "SERIAL OR OVERLAP FOR I/O PROCESSING (DEFAULT=OVERLAP) ? S/O",Y$
370      IF (Y$="S") OR (Y$="s") THEN SERIAL
380    CALL Dialogue
390    DEG
400      PRINT PAGE,"   When entering values for Smax in CHART please enter reaso
nable"
410      PRINT "values. This is required because the horizontal axis for CHART is
 divided"
420      PRINT "into 10 intervals. The program takes your entry for Smax"
430      PRINT "and divides by 10 to determine the tick spacing. The vertical "
440      PRINT "tick marks are under program control, enter anything you wish."
450      PRINT LIN(2),"PRESS CONT"
460      PAUSE
470      PRINT PAGE
480 Gymnastics:   Y_hard$="N"
490               INPUT "DO TOU WANT A HARD COPY OF THE COMPUTED DATA? Y/N",Y_ha
rd$
500                 GOSUB Yhard
510                 CALL Header
520                 GOSUB Printer
530                 CALL Header_end
540               PRINT LIN(2)
550             Main$="N"    ! RESET CONDITION
560 Header:       DISP " WORKING ..."
570 Initialize_run: Flag=0          ! RESET CONDITION
580                 Y0_min=Ya*(Talpha-Trhoi)/Talpha
590                 Y0_max=Yb*(Talpha-Trhoi)/Talpha
600                 Xnray=Nray             ! AVOIDS MIXED MODE ARITHMETIC
610                 Dely=(Y0_max-Y0_min)/Xnray
620                 Y0_min=Y0_min-Dely
630    .              IF Ya=0 THEN Y0_min=Y0_min+Dely
640                 Nray=Nray+Add_ray
```

```
650                 Ns=0          ! COUNTER FOR THE (X,Y) ARRAY
660                 Nc=0
670                 X1(Ns)=0
680                 Y1(Ns)=0
690                 Beta(0)=0
700 Y0_loop:   FOR Y0_loop=1 TO Nray     ! COMPUTE THE INTERCEPT OF THE INCIDENT
710                                    ! AND THE FIRST SURFACE
720            Xy0_loop=Y0_loop      ! AVOIDS MIXED MODE ARITHMETIC
730            Flag=0                ! RESET FLAG
740            X=0
750            Rho=Rho_initial
760            Trho=TAN(Rho)
770            Y=Y0_min+Dely*Xy0_loop
780            Ns=Ns+1
790            Y1(Ns)=Y
800 One:    CALL X1pos(X,Y)
810       X1(Ns)=X
820 Next_y0_loop: NEXT Y0_loop
830               INPUT "ARE YOU USING THE CRT (C) OR THE 9872A PLOTTER (P) ? C/
P",Plot$
840                 IF (Plot$="C") OR (Plot$="c") THEN Chart
850               INPUT "PLEASE ENTER THE SELECT CODE OF THE GRAPHICS DEVICE (DE
FAULT = 7 ).",Pselect
860               INPUT "PLEASE ENTER THE HPIB ADDRESS OF THE GRAPHICS DEVICE (D
EFAULT = 5 ).",Hpib
870 Chart:   CALL Chart
880        GOSUB Yhard_end
890        PRINT LIN(2),"PRESS CONT"
900        PAUSE
910        PRINT
920        PRINT PAGE,"   If the scale you have chosen did not allow the compute
r "
930        PRINT "to number all of the curves you may do so manually by "
940        PRINT "answering yes (Y) to the next prompt."
950        PRINT LIN(1),"   If your answere is yes, the graphics display of the
"
960        PRINT "plot will come on and a cursor will be in the lower left"
970        PRINT "corner. Position the cursor by using the DISPLAY"
980        PRINT "controls. The letter or number will be drawn with its  "
990        PRINT "left side over the center of the cursor. You MUST BE
 CAREFUL  "
1000       PRINT "because mistakes are difficult, if not impossible to erase"
1010       PRINT "(consult the graphics rom manual (page 91) to erase a letter).
"
1020       PRINT LIN(2)," When you have completed all of your lettering,"
1030       PRINT "PRESS CONT."
1040       PRINT LIN(2),"PRESS CONT"
1050       PAUSE
1060       PRINT PAGE
1070       Y$="N"
1080       INPUT "DO YOU WANT TO NUMBER ANY OF THE CURVES ? Y/N",Y$
1090         IF (Y$="Y") OR (Y$="y") THEN CSIZE 2.5
1100         IF (Y$="Y") OR (Y$="y") THEN LETTER
1110         IF (Y$="Y") OR (Y$="y") THEN CSIZE 15/4.54
1120         IF (Y$="Y") OR (Y$="y") THEN EXIT GRAPHICS
1130       Dump_crt$="N"
1140       INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
1150         IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
1160       Flag=0        ! RESET CONDITION
1170       Y$="N"
1180       INPUT "DO YOU WANT ANOTHER PLOT DRAWN TO A DIFFERENT SCALE ? Y/N",Y$
1190         IF (Y$="Y") OR (Y$="y") THEN Chart
1200 Rerun: PRINTER IS 16
1210        PRINT PAGE
1220        Y$="N"
1230        INPUT "ARE YOU GOING TO MAKE ANY MORE RUNS? Y/N",Y$
1240           IF (Y$="N") OR (Y$="n") THEN Family
```

```
1250 Rerun2: Change$="N"
1260         INPUT "ARE YOU GOING TO CHANGE ANY PARAMETERS (n2,n3,Alpha,Nray, OR
 RHO-INITIAL)? Y/N",Change$
1270           IF (Change$="N") OR (Change$="n") THEN Gymnastics
1280         GOSUB Printer
1290         INPUT "WHAT IS THE NEW VALUE OF RHO-INITIAL ?",Rho_initial
1300         Rho_initial=-1*ABS(Rho_initial)
1310         Trhoi=TAN(Rho_initial)
1320         INPUT "WHAT IS THE NEW VALUE OF ALPHA?",Alpha
1330         Talpha=TAN(Alpha)
1340 N2:     INPUT "WHAT IS THE NEW VALUE OF n2?",N2
1350         INPUT "WHAT IS THE NEW VALUE OF n3?",N3
1360           IF N3>N2 THEN BEEP
1370           IF N3>N2 THEN DISP "n3 MUST BE < n2, PLEASE RE-ENTER THE VALUES
"
1380           IF N3>N2 THEN WAIT 2500
1390           IF N3>N2 THEN N2
1400           IF N3<N2 THEN Theta_critical=ASN(N3/N2)
1410 Ya:     INPUT "WHAT IS THE NEW VALUE OF Ya (INCHES) ?",Ya
1420         INPUT "WHAT IS THE NEW VALUE OF Yb (INCHES) ?",Yb
1430           IF Ya>=Yb THEN BEEP
1440           IF Ya>=Yb THEN DISP "Ya MUST BE < Yb; PLEASE RE-ENTER THE VALUE
S"
1450           IF Ya>=Yb THEN WAIT 2500
1460           IF Ya>=Yb THEN Ya
1470         Aperture=Yb-Ya
1480 Y_bullet:INPUT "WHAT IS THE NEW VALUE OF Y_bullet ( MUST BE  > Yb ) ?",Y_bu
llet
1490           IF Y_bullet<Yb THEN BEEP
1500           IF Y_bullet<Yb THEN DISP "Y_bullet MUST BE > Yb : PLEASE RE-ENT
ER Y_bullet."
1510           IF Y_bullet<Yb THEN WAIT 2500
1520           IF Y_bullet<Yb THEN Y_bullet
1530         INPUT "WHAT IS THE NEW VALUE OF THE No. OF RAYS ?",Nray
1540         INPUT "WHAT IS THE NEW VALUE OF FOCUS (INCHES) ?",F
1550      GOTO Gymnastics
1560 Family: Family$="N"
1570         PRINTER IS 16
1580         INPUT "DO YOU WANT TO DRAW A FAMILY OF SURFACES FROM THE DESIGN CHA
RT ? Y/N",Family$
1590           IF (Family$="N") OR (Family$="n") THEN Finished
1600 Fam1:   PRINT PAGE,"  Xmax is the maximum length along the GLM axis which y
ou"
1610         PRINT "want to be displayed."
1620         PRINT "  A rule of theumb is to add one inch to the focal length."
1630         PRINT "For example: if the Focal length is 4 inches, key in 5 and"
1640         PRINT "PRESS CONT."
1650         PRINT LIN(2),"PRESS CONT"
1660         PAUSE
1670         Y_hard$="N"
1680         INPUT "DO YOU WANT A HARD COPY OF THE COMPUTED DATA ? Y/N",Y_hard$
1690           IF (Y_hard$="Y") OR (Y_hard$="y") THEN GOSUB Yhard
1700           CALL Family
1710         GOSUB Yhard_end
1720         Dump_crt$="N"
1730         INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
1740           IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
1750         Y$="N"
1760         INPUT "DO YOU WANT TO DRAW THE PLOT USING A DIFFERENT SCALE ? Y/N",
Y$
1770           IF (Y$="Y") OR (Y$="y") THEN Fam1
1780         Main1$="Y"          ! SET CONDITION
1790         Y_hard$="N"
1800         INPUT "DO YOU WANT A HARD COPY OF QA,Smax,Alpha2 and Rho2 ? Y/N",Y_
hard$
1810 .         GOSUB Yhard
1820           CALL Header
```

```
1830         FOR I=1 TO Nray
1840             IF Line_count>Line_max THEN CALL Header
1850           PRINT USING 1870;I,Qa(I),Smax(I),Alpha2(I),Rho2(I)
1860           Line_count=Line_count+1
1870           IMAGE 14X,DDD,4(5X,DDD.DDD)
1880         NEXT I
1890           CALL Header_end
1900           GOSUB Yhard_end
1910         PRINT LIN(10),"PRESS CONT"
1920         Main1$="N"        ! RESET CONDITION
1930         PAUSE
1940 Stuff: PRINTER IS 16
1950        PRINT PAGE,"   Please study the display and choose the surface"
1960        PRINT "which you want to use to generate a final surface."
1970        PRINT "   When you have decided which surface to use, PRESS CONT"
1980        PRINT LIN(1),"   Key in the number of the surface you want to use"
1990        PRINT "in response to the prompt."
2000        PRINT LIN(1),"   The computer will then determine the aperture of "
2010        PRINT "the surface and trace 250 rays through the aperture."
2020        PRINT LIN(1),"   When you are ready to proceed PRESS CONT."
2030        PAUSE
2040        GRAPHICS    ! IF A NOTE TO OPERATOR IS DESIRED, THEN ENTER SETGU,
2050        WAIT 2500   ! LORG 2, MOVE 2,2 ,CSIZE 2.5 ,LABEL "PRESS CONT WHEN
2060        BEEP        ! READY" ,SETGU CSIZE 15/4.54 AFTER THE SECOND BEEP
2070        WAIT 250
2080        BEEP
2090        PAUSE
2100        EXIT GRAPHICS
2110        INPUT "WHICH SURFACE (1,2,3,etc.) ?",Family
2120        Family=INT(Family)
2130          IF Family<=0 THEN Family=1
2140          IF Family>Nray_family THEN Family=Nray_family
2150        DISP "WORKING ..."
2160 Re_stuff:  CALL Stuff
2170          GOSUB Yhard_end
2180        Dump_crt$="N"
2190        INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
2200          IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
2210        Flag=0        ! RESET CONDITION
2220        Redraw$="N"
2230        INPUT "DO YOU WANT ANOTHER PLOT DRAWN TO A DIFFERENT SCALE ? Y/N",Re
draw$
2240          IF (Redraw$="Y") OR (Redraw$="y") THEN Re_stuff
2250 Stuff_trace: Y$="N"
2260            INPUT "DO YOU WANT TO TRACE RAYS THROUGH THE COMPUTED SURFACE ?
Y/N",Y$
2270              IF (Y$="N") OR (Y$="n") THEN Rerun3
2280            DISP "WORKIN' ON THE RAY TRACE DATA..."
2290            FOR Y0_loop=1 TO Nray_stuff
2300              Surf_no=3
2310              Rho2=Rho2(Y0_loop)
2320              X=X2(Y0_loop)
2330              Y=Y2(Y0_loop)
2340                CALL Binary_search(X,Y)
2350              Y3(Y0_loop)=Y
2360              X=Z3
2370              Surf_no=4
2380                CALL Binary_search(X,Y)
2390              Y4(Y0_loop)=Y
2400            NEXT Y0_loop
2410            Nray_trace=Nray_stuff
2420            Ray_trace$="Y"
2430            BEEP
2440            Y$="N"
2450            INPUT "DO YOU WANT A HARD COPY OF THE RAY TRACE DATA ? Y/N",Y_ha
rd$
2460              GOSUB Yhard
```

```
2470                  CALL Header
2480 Stuff_trace1:FOR I=1 TO Nray_stuff
2490                    IF Line_count>Line_max THEN CALL Header
2500                  PRINT USING 2510;I,0,Y1(I),X1(I),Y1(I),X2(I),Y2(I),Xc(I),0,Z4,
Y4(I)
2510                  IMAGE DDD,2X,D,3(2X,DD.DDD,3X,DD.DDD),2X,D.D,3X,DD.DD,3X,4D.2D
2520                  Line_count=Line_count+1
2530                NEXT I
2540                  CALL Header_end
2550                  GOSUB Yhard_end
2560                PRINT LIN(2),"PRESS CONT"
2570                BEEP
2580                PAUSE
2590 Re_graph:        CALL Graph
2600            Redraw$="N"
2610            INPUT "DO YOU WANT ANOTHER PLOT DRAWN TO A DIFFERENT SCALE ? Y/N",Re
draw$
2620              IF (Redraw$="Y") OR (Redraw$="y") THEN Re_graph
2630 Density: PRINTER IS 16
2640              PRINT PAGE,"  The point of maximum ray density is determined by yo
u visually."
2650              PRINT "by placing the cross-hairs (they will appear automatically)
"
2660              PRINT "over the position of maximum ray density."
2670              PRINT "   This is accomplished by using the DISPLAY controls (up
,down,<-,->)."
2680              PRINT "When you the get cursor in the area of interest, use the S
HIFT button"
2690              PRINT "with the DISPLAY controls (both the shift button and disp
lay control"
2700              PRINT "should be depressed simultaneously) for fine positioing."
2710              PRINT LIN(2),"   The position of the vertical hair is critical,"
2720              PRINT "because its location is used for the position of the "
2730              PRINT "image plane (Z3) on the GLM axis."
2740              PRINT LIN(2),"CAUTION : Do not let the image plane intercept any
 rays"
2750              PRINT "in the interior of the lens. If this is done, those rays"
2760              PRINT "will be included in the histogram."
2770              PRINT LIN(2),"PRESS CONT"
2780              PAUSE
2790              PRINT PAGE,"   The histogram is an illustration of the density"
2800              PRINT "of the rays that intercept the image plane versus radial  "
2810              PRINT "distance from the GLM axis."
2820              PRINT LIN(2),"   When the position has been located, PRESS CONT."
2830              PRINT LIN(2),"PRESS CONT"
2840              PAUSE
2850              Digitize$="Y"
2860                CALL Graph
2870                IF Plot$="P" THEN Dense   ! P FOR HP-9872 PLOTTER
2880              Dump_crt$="N"
2890              INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
2900                IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
2910              PRINT PAGE
2920              DISP "WORKIN' ON THE HISTOGRAM..."
2930 Dense:         CALL Density         ! PRODUSE A HISTOGRAM OF THE RAY DENSITY
2940                IF Plot$="P" THEN Denser
2950              Digitize$="N"    ! RESET CONDITION
2960              INPUT "DO YOU WANT A HARD COPY OF HISTOGRAM ? Y/N",Dump_crt$
2970                IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
2980 Denser:      Y$="N"
2990              INPUT "DO YOU WANT TO REDRAW THE HISTOGRAM FOR A DIFFERENT IMAGE P
LANE ? Y/N",Y$
3000                IF (Y$="Y") OR (Y$="y") THEN Flag=0
3010                IF (Y$="Y") OR (Y$="y") THEN Re_graph
3020              Ray_trace$="N"
3030 Rerun3:  Y$="N"
3040             INPUT "DO YOU WANT TO TRY ANOTHER SURFACE (THIS MEANS STARTING OVER
```

```
) ? Y/N",Y$
3050           IF (Y$="Y") OR (Y$="y") THEN Main
3060         GOTO Finished
3070 Printer:PRINTER IS 16
3080            IF (Change$="Y") OR (Change$="y") THEN Y_hard$="N"
3090            IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN Y_hard$="Y"
3100            IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINTER IS 0
3110            IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINT CHR$(27)&"&100T"
3120            IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN Crt
3130          FIXED 0
3140          FIXED 2
3150            IF Date$="" THEN 3170
3160          PRINT "DATE : ";Date$
3170          PRINT LIN(1),"Alpha = ";Alpha;"degrees";TAB(48);"Rho-initial = ";R
ho_initial
3180          PRINT "TAN(Alpha) = ";Talpha;TAB(48);"TAN(RHO-INITIAL) = ";Trhoi
3190          PRINT LIN(1),"RHO1 = ";Rho1;"degrees";TAB(48);"Theta(critical) = "
;Theta_critical;"degrees"
3200          PRINT LIN(1),"GLM Radius = ";Y_bullet;"inches"
3210          PRINT "Aperture = ";Aperture;TAB(24);"Ya = ";Ya;"inch";TAB(48);"Yb
 = ";Yb;"inches"
3220          FIXED 5
3230          PRINT LIN(1),"n1 = ";N1;TAB(24);"n2 = ";N2;TAB(48);"n3";N3
3240          FIXED 0
3250          PRINT LIN(1),"Number of Rays = ";Nray
3260          FIXED 2
3270          PRINT LIN(1),"Design Focal Point = ";F;" inches"
3280          PRINT
3290 Crt:       IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN DUMP GRAPHICS
3300            IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINT CHR$(27)&"&136T"
3310            IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN PRINTER IS 16
3320          Change$="N"           ! RESET CONDITION
3330          Dump_crt$="N"
3340          Y_hard$="N"
3350        RETURN
3360 Yhard:  IF (Y_hard$="Y") OR (Y_hard$="y") THEN Flag$="1"
3370         IF (Y_hard$="N") OR (Y_hard$="n") THEN Flag$="0"
3380         IF Flag$="1" THEN PRINTER IS 0
3390         IF Flag$="1" THEN PRINT CHR$(27)&"&100T"
3400       RETURN
3410 Yhard_end: IF Flag$="1" THEN PRINT CHR$(27)&"&136T"
3420            IF Flag$="1" THEN PRINTER IS 16
3430          RETURN
3440 Finished: DISP "FINISHED"
3450       END
3460 SUB Snell
3470   OPTION BASE 0
3480   !
3490   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
3500   COM F,Z3,Z4,Norm,Thetai,Tdelta
3510   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
3520   !
3530   COM Stuff$,Redraw$,Digitize$,Ray_trace$
3540   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
3550   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
3560   !
3570   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
3580   COM INTEGER Family,Surf_no
3590   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
3600   !
3610  DEG
3620 One: Thetai=90-(Alpha+ABS(Rho))    ! RHO MUST BE DECLARED ELESWHERE
3630      Alpha2=Alpha
3640      Norm=Alpha2-90
3650      Sin_iprime=N1/N2*SIN(Thetai)
3660      Thetar=ASN(Sin_iprime)
3670      Rho1=Alpha2+Thetar-90     ! USE ALPHA2 INSTEAD OF NORM BECAUSE THE
```

```
3680                                      ! SIGN OF THE ANGLE IS DESIRED
3690      Trho1=TAN(Rho1)
3700 SUBEND
3710 SUB Plot
3720    OPTION BASE 0
3730    !
3740    COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
3750    COM F,Z3,Z4,Norm,Thetai,Tdelta
3760    COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
3770    !
3780    COM Stuff$,Redraw$,Digitize$,Ray_trace$
3790    COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
3800    COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
3810    !
3820    COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
3830    COM INTEGER Family,Surf_no
3840    COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
3850    !
3860    COM Y_bullet,Smax_family,Percent_image
3870    COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
3880    COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
3890    !
3900    COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
3910    !
3920    ! THE ORIGINAL OF THIS ROUTINE WAS OBTAINED FROM THE HP UTILITIES LIBRARY
3930    !     9845B UTILITY LIBRARY
3940    !     PROGRAM CARTRIDGE 2
3950    !     09845-10205
3960    !     PROGRAM REGPLT
3970    ! SUB LAXES WAS ALSO BORROWED FROM THE LIBRARY
3980    !
3990    !    THIS ROUTINE IS USED TO SET UP THE LIMIT,LOCATE,SCALE AND CLIP STATE
MENT
4000    ! STATEMENTS FOR THE FOLLOWING ROUTINES:
4010    !          1) CHART
4020    !          2) FAMILY
4030    !          3) STUFF
4040    !          4) DENSITY
4050    !          5) RAY_TRACE
4060    !
4070    Flaga=Flagc=0
4080 Set_up:     IF (Plot$="P") OR (Plot$="p") THEN P9872a
4090 Crt:   PLOTTER IS 13,"GRAPHICS"
4100        GOTO 4120
4110 P9872a:   PLOTTER IS Pselect,Hpib,"9872A"
4120           GCLEAR
4130           LINE TYPE 1
4140           LDIR 0
4150           LORG 5
4160           LIMIT 0,184,0,140          ! ALL UNITS IN MILLIMETERS
4170     DATA -2,-1,1,2
4180     READ Um,Dm,Md,Mu
4190     DATA .39794,.69897,.87506
4200     READ Log2,Log5,Log7
4210     !
4220       IF (Digitize$="Y") OR (Digitize$="y") THEN Digit_scale
4230       IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray_trace
4240       IF (Chart$="Y") OR (Chart$="y") THEN Chart
4250       IF (Family$="Y") OR (Family$="y") THEN Family
4260       IF (Stuff$="Y") OR (Stuff$="y") THEN Stuff
4270     !
4280 Chart:  Xmin=0
4290       INPUT "WHAT IS THE MINIMUM VALUE OF S (INCHES) (DEFAULT = 0 INCH) ?"
,Xmin
4300       Xmax=2
4310       INPUT "WHAT IS THE MAXIMUM VALUE OF S (INCHES) (DEFAULT = 2.0 INCH)
?",Xmax
```

```
4320          IF Flagc=1 THEN 4380
4330        Ymin=Alpha-1
4340        Ymax=90
4350        INPUT "WHAT IS THE MINIMUM VALUE OF ALPHA2 (DEGREES) (DEFAULT = ALPH
A-1  DEG) ?",Ymin
4360        INPUT "WHAT IS THE MAXIMUM VALUE OF ALPHA2 (DEGREES) (DEFAULT = 90 D
EG) ?",Ymax
4370          IF Flaga=1 THEN 4430
4380          IF Xmin>Xmax THEN BEEP
4390          IF Xmin>Xmax THEN DISP "Smin IS > Smax : PLEASE RE-ENTER VALUES IN
 CORRECT ORDER"
4400          IF Xmin>Xmax THEN WAIT 2500
4410          IF Xmin>Xmax THEN Flagc=1
4420          IF Xmin>Xmax THEN Chart
4430          IF Ymin>Ymax THEN BEEP
4440          IF Ymin>Ymax THEN DISP "ALPHA2min IS > ALPHA2max : PLEASE RE-ENTER
 VALUES IN CORRECT ORDER"
4450          IF Ymin>Ymax THEN WAIT 2500
4460          IF Ymin>Ymax THEN Flaga=1
4470          IF Ymin>Ymax THEN 4330
4480        Xorg=Xmin
4490        Yorg=Ymin
4500        Flaga=Flagc=0
4510      GOTO Chart_scale
4520 Family: Xmin=Ymin=Xorg=Yorg=0
4530         Ymax=Y_bullet
4540         Xmax=5
4550         INPUT "WHAT IS THE MAXIMUM VALUE OF X (INCHES) (DEFAULT = 5 INCHES)
 ?",Xmax
4560           IF Xmax<=Xmin THEN BEEP
4570           IF Xmax<=Xmin THEN DISP "Xmax MUST BE > Xmin. PLEASE RE-ENTER T
HE VALUES IN THE CORRECT ORDER."
4580           IF Xmax<=Xmin THEN WAIT 2500
4590           IF Xmax<=Xmin THEN Family
4600 Family_scale: LOCATE 15,130,35,100
4610                 GOSUB Same
4620               SCALE Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),Ymin-.5*ABS(Ytic),Y
_bullet+.5*ABS(Ytic)
4630                CLIP Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),Ymin-.5*ABS(Ytic),Y
_bullet+.5*ABS(Ytic)
4640                 GOSUB Same_axes
4650            SUBEXIT
4660 Stuff: Xmin=Ymin=Xorg=Yorg=0
4670        Xmax=5
4680        Ymax=Y_bullet
4690        INPUT "WHAT IS THE MAXIMUM VALUE OF X (INCHES) (DEFAULT = 5 INCHES)
?",Xmax
4700          IF Xmax<=Xmin THEN BEEP
4710          IF Xmax<=Xmin THEN DISP "Xmax MUST BE > Xmin. PLEASE RE-ENTER TH
E VALUES IN THE CORRECT ORDER."
4720          IF Xmax<=Xmin THEN WAIT 2500
4730          IF Xmax<=Xmin THEN Stuff
4740 Stuff_scale: LOCATE 15,130,35,100
4750                 GOSUB Same
4760              SCALE Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),Ymin-.5*ABS(Ytic),Y_
bullet+.5*ABS(Ytic)
4770               CLIP Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),Ymin-.5*ABS(Ytic),Y_
bullet+.5*ABS(Ytic)
4780                GOSUB Same_axes
4790            SUBEXIT
4800 Ray_trace: Xmin=Xorg=Yorg=0
4810            Xmax=5
4820            INPUT "WHAT IS THE MAXIMUM VALUE OF X (INCHES) (DEFAULT = 5 INCH
ES) ?",Xmax
4830              IF Xmax<=Xmin THEN BEEP
4840              IF Xmax<=Xmin THEN DISP "Xmax MUST BE > Xmin. PLEASE RE-ENTE
R THE VALUES IN THE CORRECT ORDER."
```

```
4850              IF Xmax<=Xmin THEN WAIT 2500
4860              IF Xmax<=Xmin THEN Ray_trace
4870            Ymax=Y_bullet
4880            Ymin=-Ymax
4890 Ray_trace_scale: LOCATE 15,130,25,100
4900                    GOSUB Same
4910                  SCALE Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),-Y_bullet-.25*AB
S(Ytic),Y_bullet+.25*ABS(Ytic)
4920                   CLIP Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),-Y_bullet-.25*AB
S(Ytic),Y_bullet+.25*ABS(Ytic)
4930                    GOSUB Same_axes
4940                  CLIP Xmin,100,Ymin,Ymax
4950                SUBEXIT
4960 Chart_scale: LOCATE 18,133,28,100       ! ALL UNITS IN GDU'S
4970                GOSUB Same               ! DRAW THE DESIGN CHART
4980              Xtic=(Xmax-Xmin)/10
4990              SCALE Xmin,Xmax+.25*ABS(Xtic),Ymin,Ymax+.25*ABS(Ytic)
5000              CLIP Xmin,Xmax,Ymin,Ymax
5010                GOSUB Same_axes
5020              FRAME
5030            SUBEXIT
5040 Digit_scale: LOCATE 0,130,30,100        ! ALL UNITS IN GDU'S
5050              Xmax=Y_bullet                 ! DRAW THE HISTOGRAM
5060              Xmin=-Xmax
5070              Xorg=Yorg=0
5080              Ymax=1
5090              Ymin=0
5100                GOSUB Same
5110              SCALE -Y_bullet-.15*ABS(Ytic),Y_bullet+.15*ABS(Ytic),-.15*ABS(
Ytic),1+.15*ABS(Ytic)
5120               CLIP -Y_bullet-.15*ABS(Ytic),Y_bullet+.15*ABS(Ytic),-.15*ABS(
Ytic),1+.15*ABS(Ytic)
5130                GOSUB Same_axes
5140            SUBEXIT
5150 Same_axes:   CALL Laxes(Xtic,Ytic,Xorg,Yorg,1,1,2,Xmin-Xfudge,Xmax,Ymin-Yfu
dge,Ymax)
5160            CLIP Xmin,Xmax,Ymin,Ymax
5170           RETURN
5180 Same: Lx=LGT(Xmax-Xmin)
5190       Ly=LGT(Ymax-Ymin)
5200       Xfudge=.20*(Xmax-Xmin)
5210       Yfudge=.20*(Ymax-Ymin)
5220 Tic_marks: Testxtic=FRACT(Lx)+(Lx<0)
5230            Xtic=10^(INT(Lx)-1)*(1+1.5*((Testxtic>Log2) AND (Testxtic<Log5))
+4*((Testxtic>=Log5) AND (Testxtic<=Log7))+6.5*(Testxtic>Log7))
5240            Testytic=FRACT(Ly)+(Ly<0)
5250            Ytic=10^(INT(Ly)-1)*(1+1.5*((Testytic>Log2) AND (Testytic<Log5))
+4*((Testytic>=Log5) AND (Testytic<=Log7))+6.5*(Testytic>Log7))
5260          RETURN
5270     SUBEXIT
5280 SUB Laxes(Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize,Xmin,Xmax,Ymin,Ymax)
5290   OPTION BASE 0
5300   !
5310   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
5320   COM F,Z3,Z4,Norm,Thetai,Tdelta
5330   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
5340   !
5350   COM Stuff$,Redraw$,Digitize$,Ray_trace$
5360   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
5370   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
5380   !
5390   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
5400   COM INTEGER Family,Surf_no
5410   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
5420   !
5430   COM Y_bullet,Smax_family,Percent_image
5440   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
```

```
5450   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
5460   !
5470   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
5480   !
5490   !
5500   !    THIS ROUTINE IS USED TO DRAW AND LABEL THE AXES FOR THE FOLLOWING
5510   ! ROUTINES:
5520   !         1) CHART
5530   !         2) FAMILY
5540   !         3) STUFF
5550   !         4) DENSITY
5560   !         5) RAY_TRACE
5570   !
5580    Flag=1
5590    DEG
5600    LINE TYPE 1
5610    LDIR 0
5620    LORG 5
5630      IF (Xmin>=Xmax) OR (Ymin>=Ymax) THEN SUBEXIT
5640    GRAPHICS
5650    Xfudge=.02*(Xmax-Xmin)
5660    Yfudge=.02*(Ymax-Ymin)
5670    Xmaj=1
5680    Ymaj=1
5690    Minticize=2
5700        IF (Digitize$="Y") OR (Digitize$="y") THEN Digit_axis
5710        IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray_trace_axis
5720        IF (Chart$="Y") OR (Chart$="y") THEN Chart_axis
5730        IF (Family$="Y") OR (Family$="y") THEN Family_axis
5740        IF (Stuff$="Y") OR (Stuff$="y") THEN Stuff_axis
5750 Chart_axis: LINE TYPE 3
5760            GRID Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,2*Minticsize
5770            LINE TYPE 1
5780            GOTO Labelx
5790 Family_axis: AXES Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize
5800             GOTO Labelx
5810 Stuff_axis: AXES Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize
5820            GOTO Labelx
5830 Ray_trace_axis: AXES Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize
5840                GOTO Labelx_ray
5850 Digit_axis: AXES .1,.1,0,0,1,1,2
5860            GOTO Labelx_d
5870 Labelx: FIXED 3
5880        LDIR 90
5890        LORG 8
5900 Parx:   FOR A=Xorg TO Xmax STEP ABS(Xtic)
5910            MOVE A,Yorg-Yfudge
5920          LABEL USING 5950;A
5930          IMAGE DD.D
5940          IMAGE DD.DD
5950          IMAGE #,K
5960        NEXT A
5970 Labely: LDIR 0
5980        LORG 8
5990 Pary:   FOR A=Yorg TO Ymax STEP ABS(Ytic)       ! FOR CHART AND FAMILY
6000            MOVE Xorg-Xfudge,A
6010          LABEL USING 5950;A
6020        NEXT A
6030        GOTO Label_
6040 Labelx_ray: LDIR 90
6050            LORG 8
6060 Parx_ray:  FOR A=Xorg TO Xmax STEP ABS(Xtic)
6070                MOVE A,Yorg-Yfudge
6080              LABEL USING 5950;A
6090            NEXT A
6100 Labely_ray: LDIR 0
6110            LORG 8
```

```
6120 Pary_ray:  FOR A=-Y_bullet TO Y_bullet STEP ABS(Ytic)      ! FOR RAY_TRACE
6130                MOVE Xorg-Xfudge,A
6140                  IF A=0 THEN LABEL USING 5950;A
6150                  IF A=0 THEN Next_a
6160                LABEL USING 6170;A
6170                IMAGE DD.D
6180 Next_a:      NEXT A
6190              GOTO Label_
6200 Labelx_d: LDIR 90
6210           LORG 8
6220           FOR A=-Y_bullet TO Y_bullet STEP .1
6230               MOVE A,Yorg-Yfudge
6240               IF A=0 THEN LABEL USING 5950;A
6250               IF A=0 THEN GOTO Next_aa
6260             LABEL USING 6170;A
6270 Next_aa:  NEXT A
6280 Labely_d: LDIR 0
6290           LORG 8
6300           FOR A=0 TO Y_bullet STEP .1
6310               MOVE Xorg-Xfudge,A
6320               IF A=0 THEN LABEL USING 5950;A
6330               IF A=0 THEN GOTO Nexta
6340             LABEL USING 6170;A
6350 Nexta:    NEXT A
6360 Label_: SETGU
6370         LDIR 0
6380         LORG 5
6390         CSIZE 2.5
6400           IF (Digitize$="Y") OR (Digitize$="y") THEN Digit_label
6410           IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray_trace_label
6420           IF (Chart$="Y") OR (Chart$="y") THEN Chart_label
6430           IF (Family$="Y") OR (Family$="y") THEN Family_label
6440           IF (Stuff$="Y") OR (Stuff$="y") THEN Stuff_label
6450 Chart_label: Centerx=74.00
6460              Centery=64
6470              !
6480                MOVE .5*Centerx,9
6490              LABEL USING 6500;F
6500              IMAGE "Design Focal Point : ",DD.DD," inches"
6510                MOVE 1.5*Centerx,9
6520              LABEL USING 6530;Nray_chart
6530              IMAGE "Number of Rays : ",K
6540                MOVE .5*Centerx,5
6550              LABEL USING 6560;Alpha
6560              IMAGE "Alpha : "DD.DD" deg"
6570                MOVE 1.5*Centerx,5
6580              LABEL USING 6590;N2
6590              IMAGE "n2 : ",D.DDDDD
6600              !
6610              CSIZE 3
6620                MOVE Centerx,15
6630              LABEL "S : Distance Along Refracted Ray in the Lens ( Inches )
"

6640              LDIR 90
6650                MOVE .8,Centery
6660              LABEL "Alpha 2 : Angle of the Tangent Line at"
6670                MOVE 4.8,Centery
6680              LABEL "the Second Surface ( Degrees )"
6690              !
6700              GOTO End_label
6710 Family_label: Centerx=Centerx_family=72.5
6720               Centery=Centery_family=66.5
6730               !
6740                 IF (Stuff$="Y") OR (Stuff$="y") THEN Stuff_label
6750                 MOVE .5*Centerx,9
6760               LABEL USING 6500;F
6770                 MOVE 1.5*Centerx,9
```

```
6780                   LABEL USING 6530;Nray_family
6790                     MOVE .5*Centerx,5
6800                   LABEL USING 6560;Alpha
6810                     MOVE 1.5*Centerx,5
6820                   LABEL USING 6590;N2
6830                   !
6840                     MOVE Centerx,23
6850                   LABEL "Distance Along GLM Axis (inches)"
6860                   CSIZE 3
6870                     MOVE Centerx,15
6880                   LABEL "FAMILY OF SURFACES"
6890                   LDIR 90
6900                     MOVE 1.5,Centery
6910                   LABEL "Radial Distance from the Axis (inches)"
6920                   !
6930                   GOTO End_label
6940 Stuff_label: Centerx=72.5
6950              Centery=66.5
6960              !
6970                MOVE .5*Centerx,9
6980              LABEL USING 6500;F
6990                MOVE 1.5*Centerx,9
7000              LABEL USING 6530;Nray_stuff
7010                MOVE .5*Centerx,5
7020              LABEL USING 6560;Alpha
7030                MOVE 1.5*Centerx,5
7040              LABEL USING 6590;N2
7050                MOVE Centerx,5
7060              LABEL USING 7070;Aperture
7070              IMAGE "Aperture : "DD.DDD" inch"
7080              !
7090                MOVE Centerx,23
7100              LABEL "Distance Along GLM Axis (inches)"
7110              CSIZE 3
7120                MOVE Centerx,15
7130              LABEL USING 7140;Family
7140              IMAGE "SURFACE NUMBER "K
7150              !
7160              LDIR 90
7170                MOVE 1.5,Centery
7180              LABEL "Radial Distance from the Axis (inches)"
7190              !
7200              GOTO End_label
7210 Ray_trace_label: Centerx=72.5                ! CHANGE IN GRAPH AS WELL
7220                  Centery=62.5
7230                  !
7240                    MOVE 1.5*Centerx,9
7250                  LABEL USING 6530;Nray_trace
7260                    MOVE .5*Centerx,10
7270                  LABEL USING 7280;Rho_initial
7280                  IMAGE "Incident Ray Angle : ",3D.DD," deg"
7290                    MOVE .5*Centerx,6
7300                  LABEL USING 7070;Aperture
7310                    MOVE 1.5*Centerx,6
7320                  LABEL USING 6590;N2
7330                    MOVE .5*Centerx,2
7340                  LABEL USING 6560;Alpha
7350                    MOVE Centerx,23
7360                  LABEL "Distance Along the GLM Axis ( inches )"
7370                  !
7380                  CSIZE 3
7390                  MOVE Centerx,15
7400                  LABEL USING 7410;Family
7410                  IMAGE "SURFACE NUMBER ",K
7420                  !
7430                  LDIR 90
7440                  MOVE 1.3,Centery
```

```
7450                  LABEL "Radial Distance From the Axis ( inches )"
7460                  !
7470                  GOTO End_label
7480 Digit_label: Centerx=61.56
7490              Centery=65
7500              !
7510                MOVE 1.5*Centerx,13
7520              LABEL USING 6530;Nray_trace
7530                MOVE Centerx/2,9
7540              LABEL USING 7550;Hit_total
7550              IMAGE "Number of Hits on the Image Plane : ",K
7560                MOVE 1.5*Centerx,9
7570              LABEL USING 7580;Percent_image
7580              IMAGE "% of Rays to Image Plane : ",3D.DD
7590                MOVE 1.5*Centerx,5
7600              LABEL USING 7610;Z3
7610              IMAGE "Image Plane : ",DD.DD," inches"
7620                MOVE .5*Centerx,5
7630              LABEL USING 7640;N_increment
7640              IMAGE "Number of Increments [0,1.1] : ",K
7650              !
7660              CSIZE 3
7670                MOVE Centerx,20
7680              LABEL "Normalized Number of Hits  vs  Distance from the GLM Ax
is"
7690                MOVE Centerx/2,13
7700              LABEL USING 7710;Family
7710              IMAGE "SURFACE NUMBER ",K
7720              !
7730 End_label: CSIZE 15/4.54
7740            LDIR 0
7750            LORG 5
7760            SETUU
7770          SUBEND
7780 SUB X1pos(X,Y)
7790   OPTION BASE 0
7800   !
7810   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
7820   !
7830   INTEGER Z
7840  DEG
7850  DEF FNY1(Y)=Y*Talpha/(Talpha-Trhoi)  ! FINDS THE Y-VALUE OF THE INTERCEPT
7860                                      ! OF THE RAY AND THE FIRST SURFACE
7870  DEF FNX1(Y)=Y/Talpha    ! FINDS THE X-VALUE OF THE INTERCEPT OF THE RAY
7880                          ! AND THE FIRST SURFACE
7890      Y=FNY1(Y)
7900      X=FNX1(Y)
7910      SUBEND
7920                                    SUB Char
7930 One:  FOR Z=0 TO 79
7940           IF Z=79 THEN 7960
7950         PRINT CHR$(228);
7960           IF Z=79 THEN PRINT CHR$(228)
7970       NEXT Z
7980     SUBEXIT
7990  SUBEND
8000 SUB Dialogue
8010   OPTION BASE 0
8020   !
8030   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
8040   COM F,Z3,Z4,Norm,Thetai,Tdelta
8050   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
8060   !
8070   COM Stuff$,Redraw$,Digitize$,Ray_trace$
8080   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
8090   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
8100   !
```

```
8110   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
8120   COM INTEGER Family,Surf_no
8130   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_rav,Y0,Ya,Yb,Aperture
8140   !
8150   COM Y_bullet,Smax_family,Percent_image
8160   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
8170   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
8180   !
8190   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
8200   !
8210   !
8220   !   THIS ROUTINE INITIALIZES MOST OF THE VARIABLES USED IN THE PROGRAM
8230   !
8240  DEG
8250 Initialize_top:Alpha=21
8260             Talpha=TAN(Alpha)
8270             Tol=.00001
8280             Z3=Z4=50
8290             N1=1
8300             N2=4
8310             N3=1
8320             F=4
8330             Y_bullet=1.1
8340             Ya=0                 ! WHEN YA # 0 THEN THEN THE BORDERS OF THE
8350             Yb=Y_bullet          ! APERTURE ARE ILLUSTRATED BY DRAWING A
8360             Aperture=Yb-Ya       ! RAY AT Y=Ya AND Y=Yb. THIS IS DONE BY
8370             Add_ray=0            ! ADDING A RAY TO ALL NRAY_x DECLARATIONS
8380             Nray=10
8390             Rho_initial=Rho=0
8400             Trhoi=Trho=TAN(Rho_initial)
8410             Line_count=0
8420             Line_max=45
8430             N_increment=100
8440             Pselect=7
8450             Hpib=5
8460             Plot$="C"
8470             Date$=""
8480             Family$="N"
8490             Stuff$="N"
8500             Digitize$="N"
8510             Redraw$="N"
8520             Ray_trace$="N"
8530   PRINT PAGE,"Note to the operator:  "
8540   PRINT LIN(2),"   When a request for information is placed on the screen"
8550   PRINT "two options exist:"
8560   PRINT LIN(1),"      1) key in the required information and PRESS CONT."
8570   PRINT "      2) PRESS CONT if the data/parameter or other response "
8580   PRINT "         has not or will not change from the previous entry. If in"
8590   PRINT "         doubt, key in the correct response or data and PRESS CONT.
"
8600   PRINT LIN(2),"PRESS CONT"
8610   PAUSE
8620   PRINT PAGE,"The sign convention is :"
8630   PRINT LIN(1),"      1) the origin is placed at the vertex of the first su
rface"
8640          PRINT "          and the GLM axis."
8650   PRINT "      2) distances: positive to the right of the origin."
8660   PRINT "                    positive above the origin (Radial distance)."
8670   PRINT "                    positive out of the meridian plane (screen)."
8680   PRINT "                    (ie. a right hand system.)"
8690   PRINT "      3) angles: POSITIVE IF counter-clockwise"
8700   PRINT "                 rotation from the GLM axis to the ray"
8710   PRINT LIN(2),"PRESS CONT"
8720   PAUSE
8730 Alpha: PRINT PAGE,"    Alpha is the angle of inclination (ie. the cone"
8740        PRINT "half-angle) of the FIRST surface with respect to the GLM-axis
."
```

```
8750         PRINT LIN(3),"    The current value of ALPHA is";Alpha;"degrees"
8760         INPUT "WHAT IS NEW VALUE OF ALPHA (DEGREES)?",Alpha
8770         Talpha=TAN(Alpha)
8780 Index: PRINT PAGE,"     The lens system is assumed to be operating in air."
8790     PRINT "Therefore the first index of refraction is defined as n1= 1.00000
."
8800     PRINT LIN(2),"    It is assumed that n1 < n2"
8810     PRINT "                   and n2 > n3."
8820 N2: INPUT "WHAT IS n2 (DEFAULT VALUE = 4) ?",N2
8830 N3: INPUT "WHAT IS n3 (DEFAULT VALUE = 1) ?",N3
8840       IF N3<N2 THEN Theta_critical=ASN(N3/N2)
8850       IF N3>N2 THEN BEEP
8860       IF N3>N2 THEN DISP "n2 MUST BE > n3. PLEASE RE-ENTER n2 AND n3."
8870       IF N3>N2 THEN WAIT 4000
8880       IF N3>N2 THEN N2
8890 Rho_initial: PRINT PAGE,"   The rays incident on the first surface are "
8900              PRINT "assumed to be parallel to the GLM-axis. "
8910              PRINT LIN(5),"PRESS CONT"
8920              PAUSE
8930              Rho=Rho_initial
8940              Trho=TAN(Rho)
8950                CALL Snell        ! COMPUTE RHO1
8960 Y_bullet: PRINT PAGE,"   The GLM is assumed to be symmetric around the long
itudinal"
8970           PRINT "axis. The maximum radius, called Y_bullet, is ";Y_bullet;"
inches."
8980           PRINT LIN(2),"   If your design requires a different radius, plea
se"
8990           PRINT "enter the new value now."
9000           INPUT "Y_bullet (INCHES) ?",Y_bullet
9010 Aperture: PRINT PAGE,"   The aperture is the difference in radial distance
from the GLM axis,"
9020           PRINT "projected onto the first surface, into which light is allo
wed to enter"
9030           PRINT "the GLM optical system. The parameter values are:"
9040           PRINT LIN(2),"      Ya = the lower aperture limit (default value =
 ";Ya;"inch)"
9050           PRINT "      Yb = the upper aperture limit (default value = ";Yb;"
inch)"
9060           PRINT "           and is always less than or equal to Y_bullet."
9070           INPUT "WHAT IS Ya (INCHES) ?",Ya
9080           INPUT "WHAT IS Yb (INCHES) ?",Yb
9090             IF Ya>=Yb THEN BEEP
9100             IF Ya>=Yb THEN DISP "Ya MUST BE < Yb. Please enter correct valu
es."
9110             IF Ya>=Yb THEN WAIT 3000
9120             IF Ya>=Yb THEN Aperture
9130             IF Yb>Y_bullet THEN BEEP
9140             IF Yb>Y_bullet THEN DISP "Y_bullet MUST BE > Yb: PLEASE RE-
ENTER Y_bullet,Ya AND Yb."
9150             IF Yb>Y_bullet THEN WAIT 3000
9160             IF Yb>Y_bullet THEN Y_bullet
9170           Aperture=Yb-Ya
9180             IF Ya<>0 THEN Add_ray=1
9190 Nray: PRINT PAGE,"   The program will trace";Nray;" rays through the system
 usless "
9200       PRINT "you specify another value."
9210       PRINT LIN(2),"CAUTION: Entering too many lines will clutter the des
ign chart."
9220             PRINT "          Try";Nray;" rays, then decide if you want more
 or less rays."
9230         INPUT " HOW MANY RAYS DO YOU WANT THE PROGRAM TO TRACE ?",Nray
9240           IF Nray<2 THEN BEEP
9250           IF Nray<2 THEN DISP "YOU MUST USE AT LEAST TWO (2) RAYS. PLEASE R
E-ENTER THE CORRECT VALUE."
9260           IF Nray<2 THEN WAIT 2500
9270           IF Nray<2 THEN Nray
```

```
9280 Focus: PRINT PAGE,"   The focal point is the location on the GLM-axis at wh
ich "
9290        PRINT "you want all of the rays to pass through."
9300        PRINT LIN(2),"   The minimum value of the focal point is ";Y_bullet/
Talpha;" inches."
9310        PRINT "This value has been chosen to prevent the angle of the tangen
t line"
9320        PRINT "of the second surface from exceeding 90 degrees at y = ";Y_bu
llet
9330        PRINT "(ie. at y = Y_bullet inches )."
9340 PRINT LIN(2),"   The default value is ";F;" inches."
9350        INPUT "WHAT IS THE VALUE OF THE FOCAL POINT ?",F
9360 Date: PRINT PAGE
9370       INPUT "WHAT IS TODAY'S DATE ?",Date$
9380 Start: PRINTER IS 16
9390 Gymnastics: SUBEND
9400 SUB Chart
9410   OPTION BASE 0
9420   !
9430   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
9440   COM F,Z3,Z4,Norm,Thetai,Tdelta
9450   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
9460   !
9470   COM Stuff$,Redraw$,Digitize$,Ray_trace$
9480   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
9490   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
9500   !
9510   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
9520   COM INTEGER Family,Surf_no
9530   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
9540   !
9550   COM Y_bullet,Smax_family,Percent_image
9560   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
9570   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
9580   !
9590   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
9600   !
9610   !   THIS ROUTINE COMPUTES AND DRAWS A FAMILY OF CURVES REPRESENTING THE
9620   ! RELATIONSHIP BETWEEN THE DISTANCE ALONG THE RAY REFRACTED AT THE FIRST
9630   ! SURFACE OF THE LENS AND THE SLOPE (DERIVED FROM SNELL'S LAW) REQUIRED
9640   ! TO SUCCESSFULLY REFRACT THAT RAY THROUGH THE  DESIRED FOCAL POINT.
9650   !
9660   !   THE NOTABLE VARIABLES ARE:
9670   !       1) BETA : THE ANGLE OF A LINE DRAWN FROM THE FOCAL POINT TO THE
9680   !                 INTERCEPT OF THE RAY AND THE FIRST SURFACE
9690   !       2) RHO1 : THE ANGLE OF THE REFRACTED FROM THE FIRST SURFACE WITH
9700   !                 RESPECT TO THE GLM-AXIS (REF)
9710   !       3) RHO2 : THE REQUIRED ANGLE OF THE REFRACTED RAY FROM THE SECOND
9720   !                 SURFACE WRT REF
9730   !       4) HIGH : RHO1 > BETA
9740   !       5) LOW : RHO1 < BETA
9750   !       6) ALPHA2 : THE SLOPE OF THE SECOND SURFACE WRT REF REQUIRED TO
9760   !                   SATISFY SNELL'S LAW
9770   !       7) S : THE DISTANCE ALONG THE RAY REFRACTED FROM THE FIRST
9780   !              SURFACE, MEASURED FROM THE FIRST SURFACE
9790   !       8) Q : INTERCEPT POINT OF THE INCIDENT RAY AND THE FIRST SURFACE
9800   !       9) A : X-INTERCEPT OF THE RAY REFRACTED AT Q
9810   !      10) QA : DISTANCE FROM Q TO A
9820   !
9830    DEG
9840   Nray_chart=Nray
9850     IF (Stuff$="Y") OR (Stuff$="y") THEN Nray_chart=Nray_stuff
9860   Chart$="Y"
9870     CALL Header
9880   Flag=0
9890 Draw_axes:   CALL Plot
```

```
9900 Beta: FOR I=1 TO Nray_chart     ! COMPUTE BETA FOR EACH RAY
9910         Tbeta=Y1(I)/(X1(I)-F)
9920         Beta(I)=ATN(Tbeta)
9930         A=X1(I)-Y1(I)/Trho1         ! COMPUTE THE DISTANCE ALONG THE REFRACTED
9940         Qa2=(X1(I)-A)^2+Y1(I)^2    ! RAY TO THE X-INTERCEPT
9950         Qa(I)=SQR(Qa2)              ! THIS IS THE MAX ALLOWED VALUE OF S
9960       NEXT I
9970 Chart: FOR Nbeta=1 TO Nray_chart   ! COMPUTE ALPHA2 AND S FOR EACH RAY
9980          S0=0
9990          Smax=Qa(Nbeta)
10000         Nstep=Smax/20
10010         J=0
10020 S:   FOR S=S0 TO Smax STEP Nstep
10030        J=J+1
10040          IF ABS(Beta(Nbeta))>ABS(Rho1) THEN High
10050          IF S<0 THEN Next_beta
10060 Low:     CALL Low_region(S,Alpha2,Nbeta)
10070          IF Tdelta<0 THEN Negative
10080          IF Rho2>0 THEN Negative
10090          IF Alpha2>=90 THEN Negative
10100          IF Line_count>Line_max THEN CALL Header
10110        PRINT USING Image_data;Nbeta,Beta(Nbeta),Thetai,Norm,Rho2,S,Alpha2
10120        Line_count=Line_count+1
10130          GOSUB Graph
10140        GOTO Next_s
10150 High:    CALL High_region(S,Alpha2,Nbeta)
10160          IF Tdelta<0 THEN Negative
10170          IF Rho2>0 THEN Negative
10180          IF Alpha2>=90 THEN Negative
10190          IF Line_count>Line_max THEN CALL Header
10200        PRINT USING Image_data;Nbeta,Beta(Nbeta),Thetai,Norm,Rho2,S,Alpha2
10210        Line_count=Line_count+1
10220          GOSUB Graph
10230        GOTO Next_s
10240 Zero_cross_over:A=F*SIN(Alpha)/SIN(Alpha+ABS(Beta(Nbeta)))    !THIS
10250                 B=S*COS(ABS(Beta(Nbeta))-ABS(Rho1))   ! CONDITION EXISTS IN
10260                 D=A-B                           ! HIGH WHEN ALPHA2 > 90 DEGREES
10270                 E=S*SIN(ABS(Beta(Nbeta))-ABS(Rho1))
10280                 Tdelta=E/D
10290                 Arho2=ABS(Beta(Nbeta))+ATN(Tdelta)
10300                   IF Arho2<0 THEN Negative
10310                 Rho2=-1*Arho2
10320 Norm_z_cross:A=COS(Rho2)-N2/N3*COS(Rho1)
10330              B=N2/N3*SIN(ABS(Rho1))-SIN(ABS(Rho2))
10340              Tnorm=B/A
10350              Norm=ATN(Tnorm)
10360              Thetai=Norm+ABS(Rho1)
10370              Thetar=Norm+ABS(Rho2)
10380              Alpha2=Norm+90
10390              IF Line_count>Line_max THEN CALL Header
10400        PRINT USING Image_data;Nbeta,Beta(Nbeta),Thetai,Norm,Rho2,S,Alpha2
10410              Line_count=Line_count+1
10420                GOSUB Graph
10430 Next_s:Smax(Nbeta)=S
10440         Alpha2(Nbeta)=Alpha2
10450         NEXT S
10460         S=S-Nstep
10470           IF S<Qa(Nbeta) THEN Negative
10480 Next_beta:  IF Line_count>Line_max THEN CALL Header
10490            PRINT
10500            Line_count=Line_count+1
10510            Flag=2                        ! LABEL THE PLOT WITH NBETA
10520              GOSUB Graph
10530            Flag=1   ! RESET
10540          NEXT Nbeta
10550        Flag=3                            ! TELL GRAPH THIS IS THE LAST BETA
10560          CALL Graph
```

```
10570       Chart$="N"
10580         CALL Header_end
10590       SUBEXIT
10600 Negative:  IF Nstep<Tol THEN Next_beta
10610          S=S-Nstep   ! RHO2 < 0 FOR LOW OR ALPHA2 > 0 FOR HIGH OR LOW
10620          S0=S
10630          Nstep=Nstep/10
10640          Rho2=0
10650          GOTO S
10660 Graph: GRAPHICS
10670        SETUU
10680        LINE TYPE 1
10690        LDIR 0
10700        LORG 2
10710        CSIZE 2.5
10720          IF J=1 THEN MOVE S,Alpha2
10730          IF J=2 THEN DRAW S,Alpha2     ! DO NOT CHANGE
10740          IF (J>=3) AND (Flag<>2) THEN DRAW S,Alpha2 ! DO NOT CHANGE
10750          IF Flag=2 THEN LABEL USING 10770;Nbeta
10760        CSIZE 15/4.54
10770        IMAGE K
10780     RETURN
10790 Image_data:IMAGE DDD,4X,3(4D.3D,5X),4D.3D,5X,2D.3D,5X,4D.3D
10800 ! IMAGE STATEMENT IS FOR Nbeta,Beta(Nbeta),Thetai,Norm,Rho2,S,Alpha2
10810  SUBEND
10820 SUB Low_region(S,Alpha2,INTEGER N)
10830  OPTION BASE 0
10840  !
10850  COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
10860  COM F,Z3,Z4,Norm,Thetai,Tdelta
10870  COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
10880  !
10890  COM Stuff$,Redraw$,Digitize$,Ray_trace$
10900  COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
10910  COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
10920  !
10930  COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
10940  COM INTEGER Family,Surf_no
10950  COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
10960  !
10970  COM Y_bullet,Smax_family,Percent_image
10980  COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
10990  COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
11000  !
11010  COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
11020  !
11030  DEG
11040 Tdelta:A=SIN(Alpha+ABS(Beta(N)))   ! COMPUTE RHO2 = f(S)
11050        B=F*SIN(Alpha)/A
11060        C=S*COS(ABS(Rho1)-ABS(Beta(N)))
11070        D=B-C
11080        E=S*SIN(ABS(Rho1)-ABS(Beta(N)))
11090        Tdelta=E/D
11100        Delta=ATN(Tdelta)
11110        Arho2=ABS(Beta(N))-Delta
11120        Rho2=-1*Arho2
11130 Normal:A=COS(Rho2)-N2/N3*COS(Rho1)   ! COMPUTE ALPHA2 = f(RHO2)
11140        B=SIN(ABS(Rho2))-N2/N3*SIN(ABS(Rho1))
11150        Tanorm=B/A
11160        Anorm=ATN(Tanorm)
11170        Thetai=Anorm-ABS(Rho1)
11180        Thetar=Anorm-ABS(Rho2)
11190        Norm=-1*Anorm
11200        Alpha2=90+Norm
11210 SUBEND
11220 SUB High_region(S,Alpha2,INTEGER N)
11230   OPTION BASE 0
```

```
11240    !
11250    COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
11260    COM F,Z3,Z4,Norm,Thetai,Tdelta
11270    COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
11280    !
11290    COM Stuff$,Redraw$,Digitize$,Ray_trace$
11300    COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
11310    COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
11320    !
11330    COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
11340    COM INTEGER Family,Surf_no
11350    COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
11360    !
11370    COM Y_bullet,Smax_family,Percent_image
11380    COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
11390    COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
11400    !
11410    COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
11420    !
11430   DEG
11440 High:A=SIN(Alpha+ABS(Beta(N)))   ! COMPUTE RHO2 = f(S)
11450      B=F*SIN(Alpha)/A
11460      C=S*COS(ABS(Beta(N))-ABS(Rho1))
11470      D=B-C
11480      E=S*SIN(ABS(Beta(N))-ABS(Rho1))
11490      Tdelta=E/D
11500      Arho2=ABS(Beta(N))+ATN(Tdelta)
11510      Rho2=-1*Arho2
11520 Normal:A=N2/N3*SIN(ABS(Rho1))-SIN(ABS(Rho2))  ! COMPUTE ALPHA2 = f(RHO2)
11530        B=N2/N3*COS(Rho1)-COS(Rho2)
11540        Tanorm=A/B
11550        Anorm=ATN(Tanorm)
11560        Thetai=ABS(Rho1)-Anorm
11570        Thetar=ABS(Rho2)-Anorm
11580        Norm=-1*Anorm
11590        Alpha2=90+Norm
11600 SUBEND
11610 SUB Family
11620    OPTION BASE 0
11630    !
11640    COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
11650    COM F,Z3,Z4,Norm,Thetai,Tdelta
11660    COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
11670    !
11680    COM Stuff$,Redraw$,Digitize$,Ray_trace$
11690    COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
11700    COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
11710    !
11720    COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
11730    COM INTEGER Family,Surf_no
11740    COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
11750    !
11760    COM Y_bullet,Smax_family,Percent_image
11770    COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
11780    COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
11790    !
11800    COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
11810   !
11820   !    THIS ROUTINE COMPUTES AND DRAWS A FAMILY OF SURFACES USING THE
11830   ! THE DESIGN CHART DRAWN IN SUBROUTINE CHART
11840   !    THE ROUTINE TAKES THE END POINTS OF EACH CURVE OF ALPHA2 VS S
11850   ! AND DRAWS A SURFACE USING ALPHA2 TO DRAW A LINE TO INTERCEPT THE NEXT
11860   ! RAY. THESE SURFACES ARE THEN THE MAXIMUM SURFACES FOR EACH PARAMETER
11870   ! BETA.
11880   !
11890   !    WHEN STUFF$="Y" THE ROUTINE GOES THROUGH THE I LOOP ONCE (ie FOR
11900   ! ONE SURFACE) AND THE J LOOP NRAY_STUFF TIMES.
```

```
11910  !
11920  DEG
11930  Family$="Y"
11940    CALL Header
11950  Ymax=Y_bullet
11960  LINE TYPE 1
11970  LDIR 0
11980  LORG 5
11990  CSIZE 15/4.54
12000  Flag=0              ! RESET CONDITION
12010  Nray_family=Nray
12020    IF (Stuff$="Y") OR (Stuff$="y") THEN Nray_family=Nray_stuff
12030 Draw_axes:  CALL Plot                        ! DRAWS AND LABELS THE AXES
12040             Step_family=INT(Nray_family/10)
12050               IF Step_family<=0 THEN Step_family=1
12060 First_surf:LINE TYPE 8           ! DRAW THE FIRST SURFACE
12070               MOVE 0,0
12080             X=Ymax/Talpha
12090             DRAW X,Ymax
12100             DRAW 100,Ymax
12110               IF (Stuff$="Y") OR (Stuff$="y") THEN Delta_b
12120             EXIT GRAPHICS
12130             BEEP
12140             Y$="N"
12150             INPUT "DO YOU WANT TO DRAW IN THE INCIDENT RAYS ? Y/N",Y$
12160               IF (Y$="N") OR (Y$="n") THEN Delta_b
12170             GRAPHICS
12180             LINE TYPE 3
12190             FOR I=Step_family TO Nray_family STEP Step_family
12200               MOVE 0,Y1(I)
12210               DRAW X1(I),Y1(I)
12220               A=X1(I)-Y1(I)/Trho1
12230               DRAW A,0
12240             NEXT I
12250 Delta_b:GRAPHICS
12260            IF (Stuff$="Y") OR (Stuff$="y") THEN Step_family=1
12270          B1=Y1(Step_family)-X1(Step_family)*Trho1
12280          B2=Y1(Step_family+1)-X1(Step_family+1)*Trho1
12290          Deltab=B2-B1
12300 Starting_points:FOR I=Step_family TO Nray_family STEP Step_family
12310         J=I                                      ! PRINTING PURPOSES
12320         Flag_over=0                              ! RESET CONDITION
12330         Flag_neg=0                               ! RESET CONDITION
12340         Sfirst=Slast=Smax(I)
12350           IF (Stuff$="Y") OR (Stuff$="y") THEN Sfirst=Slast=Smax(Family)
12360         S2(I)=Slast                              ! FOR STUFF ONLY
12370         Delta_xsurf=S2(I)*COS(Rho1)
12380         Delta_ysurf=S2(I)*SIN(Rho1)
12390         Xsurf21=X1(I)+Delta_xsurf
12400         Ysurf21=Y1(I)+Delta_ysurf
12410           IF (Stuff$="Y") OR (Stuff$="y") THEN Xsurf21=X1(Family)+Delta_xsu
rf
12420           IF (Stuff$="Y") OR (Stuff$="y") THEN Ysurf21=Y1(Family)+Delta_ysu
rf
12430           IF Ysurf21<0 THEN GOSUB Neg_ysurf      ! THE STARTING POINT
12440         X2(I)=Xsurf21                            ! IS BENEATH THE X-AXIS
12450         Y2(I)=Ysurf21
12460         Xsurfmax(I)=Xsurf21 !THESE ARE THE MAXIMUM (X,Y) COORDINATES OF THE
12470         Ysurfmax(I)=Ysurf21 ! RAY IN THE LENS
12480           IF Flag_neg=1 THEN GOSUB New_s
12490         Trho21=Y2(I)/(X2(I)-F)
12500         Rho2(I)=ATN(Trho21)
12510         Alpha2i=Alpha2(I)
12520           IF (Stuff$="Y") OR (Stuff$="y") THEN Alpha2i=Alpha2(Family)
12530           IF Alpha2i>=90 THEN Next_i
12540 .       Talpha2i=TAN(Alpha2i)
12550         Bsurf21=Ysurf21-Xsurf21*Talpha2i
```

```
12560        Xsurflast=Xsurf21
12570        Ysurflast=Ysurf21
12580        Bsurflast=Bsurf21
12590          IF Line_count>Line_max THEN CALL Header
12600        PRINT USING 13470;I,J,Rho2(J),Xsurflast,Ysurflast,Slast,Alpha2i
12610        Line_count=Line_count+1
12620        LINE TYPE 1
12630          CSIZE 2.5
12640          LORG 2
12650          MOVE Xsurf21,Ysurf21                   ! THE STARTING POINT
12660 Surfaces:FOR J=I+1 TO Nray_family              ! DRAW THE PARTICULAR SURFACE
12670            Bnext=B1+(J-1)*Deltab
12680            Numxsurf=Bnext-Bsurflast
12690            Xsurfnext=Numxsurf/(Talpha2i-Trho1)
12700            Ysurfnext=Xsurfnext*Trho1+Bnext
12710            X2(J)=Xsurfnext     ! ONLY USED FOR STUFF. DON'T USE AN IF
12720            Y2(J)=Ysurfnext     ! STATEMENT IN ORDER TO SAVE TIME
12730            X1=0
12740              CALL X1pos(X1,Ysurfnext)
12750              IF Xsurfnext<=X1 THEN GOSUB Y1
12760                DRAW Xsurfnext,Ysurfnext
12770            Xs2=(X1(J)-Xsurfnext)^2              ! COMPUTE NEXT VALUE OF S BY
12780            Ys2=(Y1(J)-Ysurfnext)^2              ! EXTENDING THE RAY FROM THE
12790            Snext=SQR(Xs2+Ys2)                   ! PRESENT POINT TO THE NEXT
12800            S2(J)=Snext                          ! ALPHA2 VS S LINE
12810          IF ABS(Beta(J))>ABS(Rho1) THEN CALL High_region(Snext,Alpha2i,J)
12820          IF ABS(Beta(J))<=ABS(Rho1) THEN CALL Low_region(Snext,Alpha2i,J)
12830        Rho2(J)=Rho2
12840        Alpha2(J)=Alpha2i
12850          IF Line_count>Line_max THEN CALL Header
12860        PRINT USING 13470;I,J,Rho2(J),X2(J),Y2(J),S2(J),Alpha2(J)
12870        Line_count=Line_count+1
12880          IF Alpha2i>=90 THEN Next_i
12890        Talpha2i=TAN(Alpha2(J))
12900        Bsurfnext=Ysurfnext-Xsurfnext*Talpha2i
12910        Bsurflast=Bsurfnext
12920        Xsurflast=Xsurfnext
12930        Ysurflast=Ysurfnext
12940          IF (Stuff$="Y") OR (Stuff$="y") THEN 12970
12950        X2max(I)=Xsurflast     ! USED TO DETERMINE APERTURE FOR STUFF
12960        Y2max(I)=Ysurflast
12970          IF Flag_over=1 THEN Next_i
12980 Next_j:  NEXT J
12990 Next_i:  IF Line_count>Line_max THEN CALL Header
13000        PRINT
13010        Line_count=Line_count+1
13020          IF (Stuff$="Y") OR (Stuff$="y") THEN 13030
13030          IF Flag_over=0 THEN GOSUB Label_surf
13040          IF (Stuff$="Y") OR (Stuff$="y") THEN 13080
13050      NEXT I
13060    LORG 5            ! RESET
13070    CSIZE 15/4.54
13080      IF (Stuff$="Y") OR (Stuff$="y") THEN GOSUB Yb
13090    Flag=3
13100      CALL Graph
13110           Family$="N"
13120             CALL Header_end
13130         SUBEXIT
13140 Y1: Flag_over=1
13150     Xsurfnext=Bsurflast/(Talpha-Talpha2i)
13160     Ysurfnext=Xsurfnext*Talpha
13170       MOVE Xsurfnext,Ysurfnext           ! THE ENDPOINT
13180     CSIZE 3
13190     LORG 1
13200       IF (Stuff$="N") OR (Stuff$="n") THEN LABEL USING 13410;I
13210       IF (Stuff$="Y") OR (Stuff$="y") THEN LABEL USING 13410;Family
13220       MOVE Xsurfnext,Ysurfnext     ! PLACE THE PEN BACK ON TOP OF THE POINT
```

```
13230   RETURN
13240 New_s: Sy2=Y2(I)-Y1(I)
13250        Sx2=X2(I)-X1(I)
13260        S2(I)=SQR(Sx2+Sy2)
13270      RETURN
13280 Neg_ysurf: Flag_neg=1              ! IF, FOR SOME NUMERICAL REASON YB1 < 0
13290            A=X1(I)-Y1(I)/Trhol
13300              IF (Stuff$="Y") OR (Stuff$="y") THEN A=X1(Family)-Y1(Family)/
Trhol
13310            Xsurf21=A                ! START AT THE X-INTERCEPT
13320            Ysurf21=0
13330          RETURN
13340 Label_surf:   MOVE Xsurflast,Ysurflast  ! THE ENDPOINT
13350             CSIZE 3
13360             LORG 1
13370               IF (Stuff$="N") OR (Stuff$="n") THEN LABEL USING 13410;I
13380               IF (Stuff$="Y") OR (Stuff$="y") THEN LABEL USING 13410;Famil
y
13390               MOVE Xsurflast,Ysurflast ! PLACE THE PEN ON TOP OF THE POINT
13400            RETURN
13410  IMAGE K
13420 Yb: Bsm=Y2max(Family)-X2max(Family)*Trhol  ! COMPUTE THE UPPER LIMIT
13430     X1=Bsm/(Talpha-Trhol)  ! OF THE APERTURE. THIS IS THE FINAL VALUE OF
13440     Yb=Y1=X1*Talpha        ! THE APERTURE
13450     Aperture=Yb-Ya
13460   RETURN
13470  IMAGE 2X,DDD,5X,DDD,5(5X,DDD.DDD)
13480 ! THE IMAGE STATEMENT IS FOR: I,J,Rho2(J),X2(J),Y2(J),S2(J),Alpha2(J)
13490 SUBEND
13500 SUB Ray_trace
13510   OPTION BASE 0
13520   !
13530   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rhol,Trhol,Rho2,Trho2
13540   COM F,Z3,Z4,Norm,Thetai,Tdelta
13550   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
13560   !
13570   COM Stuff$,Redraw$,Digitize$,Ray_trace$
13580   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
13590   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
13600   !
13610   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
13620   COM INTEGER Family,Surf_no
13630   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
13640   !
13650   COM Y_bullet,Smax_family,Percent_image
13660   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
13670   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
13680   !
13690   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
13700   !
13710   !
13720   !   THIS ROUTINE CONTROLS THE RAY DIAGRAM FOR AN ARBITRARY NUMBER
13730   ! OF RAYS WHICH IS ENTERED BY THE OPERATOR IN RESPONSE TO A PROMPT
13740   !
13750   !   THE ROUTINE CONTROLS THE FITTING OF A POLYNOMIAL TO A SET OF POINTS
13760   ! WHEN AN ARBITRARY RAY DOES NOT COINCIDE WITH A RAY COMPUTED IN STUFF
13770   !
13780  DEG
13790  Ray_trace$="Y"
13800  Select=16
13810  EXIT GRAPHICS
13820  Y$="N"
13830  INPUT "DO YOU WANT A HARD COPY OF ALL OF THE GENERATED COEFICIENTS ? Y/N"
,Y$
13840    IF (Y$="Y") OR (Y$="y") THEN Select=0
13850  Graphics=1
13860  X=1
```

```
13870  N=4
13880    CALL Driver(N,Select,X,Graphics)
13890  Ray_trace$="N"
13900  SUBEND
13910 SUB Driver(N,Select,Input,Graphics)
13920   !
13930   OPTION BASE 1
13940   !
13950   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
13960   COM F,Z3,Z4,Norm,Thetai,Tdelta
13970   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
13980   !
13990   COM Stuff$,Redraw$,Digitize$,Ray_trace$
14000   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
14010   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
14020   !
14030   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
14040   COM INTEGER Family,Surf_no
14050   !
14060   DIM Xx(N),Yy(N),A$[2],B$[2],Coeffs(0:10)
14070   PRINTER IS Select
14080   Q=4
14090 P: Degree=3
14100      CALL Polynomial(Xx(*),Yy(*),N,Degree,Coeffs(*),Regss,Resss,Totalss,Re
gms,Resms,F,Dfreg,Dfres,Dftot,Abort)
14110    PRINT "Coefficients:"
14120    FOR I=0 TO Degree
14130      PRINT USING 14140;I,Coeffs(I)
14140      IMAGE "A("DDDD")=",K
14150    NEXT I
14160    PRINT LIN(1)
14170    GOSUB Aovtable
14180      IF Graphics AND (Select=16) THEN WAIT 2000
14190      IF Graphics THEN CALL Plot_cubic(Q,A,B,Coeffs(*),Xx(*),Yy(*),N,Degree
e,Select)
14200  SUBEXIT
14210 Print: PRINTER IS Select
14220        PRINT LIN(2),SPA(12);"DATA"
14230        FOR I=1 TO N
14240          PRINT USING 14250;I,Xx(I),Yy(I)
14250          IMAGE "Point #"DDDD":",5X,"X2="DDD.DDDD,5X,"Y2="DDD.DDDD
14260        NEXT I
14270        PRINT LIN(2)
14280        PRINTER IS 16
14290      RETURN
14300 Aovtable: PRINT USING 14310
14310           IMAGE "Source     "5X" Df"11X" SS"13X" MS"12X"F",/
14320           PRINT USING 14330;Dfreg,Regss,Regms,F
14330           IMAGE "Regression"5X,MDD,4X,M7D.3D,4X,M7D.3D,5X,M4D.3D
14340           PRINT USING 14350;Dfres,Resss,Resms
14350           IMAGE "Residual  "5X,MDD,4X,M7D.3D,4X,M7D.3D
14360           PRINT USING 14370;Dftot,Totalss
14370           IMAGE "Total     "5X,MDD,4X,M7D.3D,/,/,/,/,/
14380         RETURN
14390     SUBEND
14400 SUB Polynomial(X(*),Y(*),N,Degree,Coeffs(*),Regss,Resss,Totalss,Regms,Resm
s,F,Dfreg,Dfres,Dftot,Abort)
14410   OPTION BASE 0
14420   DIM Matrix(Degree,Degree),Inv(Degree,Degree),B(Degree)
14430   REDIM Coeffs(Degree)
14440     IF Degree>N-2 THEN SUBEXIT ! Check for higher degree than possible
14450   Dfreg=Degree
14460   Dfres=N-1-Degree
14470   Dftot=Dfreg+Dfres
14480   FOR K=0 TO Degree          !  Set up system of equations
14490 .   FOR J=K TO Degree
14500       Matrix(K,J)=0
```

```
14510         FOR I=1 TO N
14520           Matrix(K,J)=Matrix(K,J)+FNG(K)*FNG(J)
14530         NEXT I
14540       Matrix(J,K)=Matrix(K,J)
14550     NEXT J
14560     B(K)=0
14570       FOR I=1 TO N
14580         B(K)=B(K)+Y(I)*FNG(K)
14590       NEXT I
14600   NEXT K
14610   MAT Inv=INV(Matrix)       !  Solve the system of equations
14620   MAT Coeffs=Inv*B
14630   FOR I=1 TO N
14640     X1=X1+X(I)
14650     X2=X2+X(I)*X(I)
14660     Y1=Y1+Y(I)
14670     Y2=Y2+Y(I)*Y(I)
14680     Z=Z+X(I)*Y(I)
14690   NEXT I
14700   Y1=Y1/N
14710   X1=X1/N
14720   Totalss=Y2-N*Y1*Y1           ! Total Sum of Squares
14730   GOSUB Regss                  ! Regression Sum of Squares
14740   Resss=Totalss-Regss          ! Residual Sum of Squares
14750   Regms=Regss/Dfreg
14760   Resms=Resss/Dfres
14770   F=Regms/Resms
14780 SUBEXIT
14790 Regss:  Regss=0
14800         FOR I=1 TO N
14810           J=0
14820             FOR L=0 TO Degree
14830               J=J+X(I)^L*Coeffs(L)
14840             NEXT L
14850             Regss=Regss+(J-Y1)^2
14860         NEXT I
14870       RETURN
14880     SUBEXIT
14890 DEF FNG(M)=X(I)^M
14900 SUBEND
14910 SUB Plot_cubic(Q,A,B,Coeffs(*),X(*),Y(*),N,Degree,Select)
14920     IF Whichdevice=1 THEN GRAPHICS
14930   DATA -2,-1,1,2
14940   READ Um,Dm,Md,Mu
14950   DATA .39794,.69897,.87506
14960   READ Log2,Log5,Log7
14970   Xmin=FNMin(X(*),N)
14980   Xmax=FNMax(X(*),N)
14990   Lx=LGT(Xmax-Xmin)
15000   Ymin=FNMin(Y(*),N)
15010   Ymax=FNMax(Y(*),N)
15020   Ly=LGT(Ymax-Ymin)
15030   Xfudge=.20*(Xmax-Xmin)
15040   Yfudge=.20*(Ymax-Ymin)
15050 Setup:   IF Plot$="P" THEN P9872
15060 Crt: PLOTTER IS "GRAPHICS"
15070      GOTO 15090
15080 P9872: PLOTTER IS Pselect,Hpib,"9872A"
15090        GCLEAR
15100        GRAPHICS
15110        LOCATE 23,123,0,100
15120        FRAME
15130        LINE TYPE 1
15140        SCALE Xmin-Xfudge,Xmax,Ymin-Yfudge,Ymax
15150        Testxtic=FRACT(Lx)+(Lx<0)
15160        Testytic=FRACT(Ly)+(Ly<0)
```

```
15170        Xtic=10^(INT(Lx)-1)*(1+1.5*((Testxtic>Log2) AND (Testxtic<Log5))+4*
((Testxtic>=Log5) AND (Testxtic<=Log7))+6.5*(Testxtic>Log7))
15180        Ytic=10^(INT(Ly)-1)*(1+1.5*((Testytic>Log2) AND (Testytic<Log5))+4*
((Testytic>=Log5) AND (Testytic<=Log7))+6.5*(Testytic>Log7))
15190          CALL Laxes_cubic(Xtic,Ytic,Xmin,Ymin,1,1,2,Xmin-Xfudge,Xmax,Ymin-
Yfudge,Ymax)
15200        LORG 5
15210        FOR I=1 TO N
15220          MOVE X(I),Y(I)
15230          LABEL USING 15240;"*"
15240          IMAGE A
15250        NEXT I
15260        LORG 1
15270        PENUP
15280        EXIT GRAPHICS
15290        SUBEXIT
15300 Polynomial: SETUU   ! CHECK THE EARLIER PROGRAM FOR THESE PARAMETER VALUES
15310             CLIP 0,123,0,100
15320             LINE TYPE 1
15330             LINE TYPE 6
15340             FOR I=Xmin TO Xmax+Xfudge STEP (Xmax+Xfudge-Xmin)/30
15350               J=0
15360                 FOR L=0 TO Degree
15370                   J=J+I^L*Coeffs(L)
15380                 NEXT L
15390               PLOT I,J,Md            ! OR DRAW ????
15400             NEXT I
15410             PENUP
15420             EXIT GRAPHICS
15430           SUBEXIT
15440 DEF FNMax(X(*),N)
15450     X=X(1)
15460     FOR I=2 TO N
15470       X=MAX(X,X(I))
15480     NEXT I
15490   RETURN X
15500 DEF FNMin(X(*),N)
15510     X=X(1)
15520     FOR I=2 TO N
15530       X=MIN(X,X(I))
15540     NEXT I
15550   RETURN X
15560 SUB Header
15570   OPTION BASE 0
15580   !
15590   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
15600   COM F,Z3,Z4,Norm,Thetai,Tdelta
15610   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
15620   !
15630   COM Stuff$,Redraw$,Digitize$,Ray_trace$
15640   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
15650   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
15660   !
15670   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
15680   COM INTEGER Family,Surf_no
15690   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
15700   !
15710   COM Y_bullet,Smax_family,Percent_image
15720   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
15730   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
15740   !
15750   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
15760   !
15770   INTEGER Z,Ztab1,Ztab2,Ztab3,Char_min,Char_max
15780   !
15790   !   THIS ROUTINE DRAWS THE HEADER FOR PRINTED OUTPUT FOR THE FOLOWING
15800   ! ROUTINES:
```

```
15810   !         1) CHART
15820   !         2) FAMILY
15830   !         3) STUFF
15840   !         4) RAY_TRACE
15850   !
15860   !
15870   !
15880   !    THIS ROUTINE DRAWS AND PRINTS THE HEADERS FOR TABULAR DATA.
15890   !
15900   DEG
15910   Ztab1=35     ! THE ZTAB'S CAN BE USED TO POSITION TABLE HEADINGS
15920   Ztab2=27     ! FOR THE PRINTED OUTPUT
15930   Ztab3=13
15940     IF (Main$="Y") OR (Main$="y") THEN Main
15950     IF (Main1$="Y") OR (Main1$="y") THEN Main1
15960     IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray_trace
15970     IF (Chart$="Y") OR (Chart$="y") THEN Chart
15980     IF (Family$="Y") OR (Family$="y") THEN Family
15990     IF (Stuff$="Y") OR (Stuff$="y") THEN Family
16000 Main: PRINT PAGE;LIN(2)
16010         GOSUB Char_long
16020       PRINT LIN(1);TAB(Ztab2);"INITIAL PARAMETER VALUES";LIN(1)
16030         GOSUB Char_long
16040       PRINT LIN(2)
16050     SUBEXIT
16060 Main1:   IF Line_count>Line_max THEN GOSUB Header_end_long
16070        PRINT PAGE;LIN(2)
16080        Line_count=0
16090          GOSUB Char_long
16100        PRINT LIN(1);TAB(Ztab3),"RAY        QA          Smax        Alpha2
 RHO 2",LIN(1)
16110                DISP TAB(Ztab3),"RAY        QA          Smax        Alpha2
 RHO 2"
16120          GOSUB Char_long
16130        PRINT LIN(2)
16140      SUBEXIT
16150 Chart:   IF Line_count>Line_max THEN GOSUB Header_end_long
16160        PRINT PAGE;LIN(2)
16170        Line_count=0
16180          GOSUB Char_long
16190        PRINT LIN(1),"RAY       BETA       THETAI(RAY)    NORMAL       RHO 2
       S         ALPHA 2";LIN(1)
16200                DISP "RAY       BETA       THETAI(RAY)    NORMAL       RHO 2
       S         ALPHA 2"
16210          GOSUB Char_long
16220        PRINT LIN(2)
16230      SUBEXIT
16240 Family:   IF Line_count>Line_max THEN GOSUB Header_end_long
16250         PRINT PAGE;LIN(2)
16260         Line_count=0
16270           GOSUB Char_long
16280         PRINT LIN(1);"   Begin   Next     Xsurf        Ysurf          S
  Alpha2      RHO 2"
16290                PRINT "   Ray     Ray      (next)       (next)       (next)
  (next)      (next)";LIN(1)
16300                 DISP "   Begin   Next     Xsurf        Ysurf          S
  Alpha2      RHO 2"
16310           GOSUB Char_long
16320         PRINT LIN(2)
16330       SUBEXIT
16340 Ray_trace:   IF Line_count>Line_max THEN GOSUB Header_end_long
16350            PRINT PAGE;LIN(2)
16360            Line_count=0
16370              GOSUB Char_long
16380            PRINT LIN(1),"RAY  X0    Y0       X1      Y1       X2      Y2
    Xe    Yc      X4         Y4    ";LIN(1)
16390                    DISP "RAY  X0    Y0       X1      Y1       X2      Y2
```

```
     Xc   Yc      X4          Y4   "
16400              GOSUB Char_long
16410            PRINT LIN(2)
16420          SUBEXIT
16430 Char_long: FOR Z=0 TO 79
16440                IF Z=79 THEN PRINT CHR$(228)
16450                IF Z=79 THEN 16470
16460              PRINT CHR$(228);
16470            NEXT Z
16480          RETURN
16490 Char_short: Char_min=12    ! THIS ROUTINE CAN BE USED FOR SMALL TABLES
16500             Char_max=64
16510              FOR Z=Char_min TO Char_max
16520                  IF Z=Char_min THEN PRINT TAB(Char_min);
16530                  IF Z=Char_max THEN PRINT CHR$(228)
16540                  IF Z=Char_max THEN 16560
16550                PRINT CHR$(228);
16560              NEXT Z
16570            RETURN
16580 Header_end_long: PRINT LIN(2)
16590                  GOSUB Char_long
16600                RETURN
16610 Header_end_shor: PRINT LIN(2)
16620                  GOSUB Char_short
16630                RETURN
16640   SUBEND
16650 SUB Header_end
16660   !
16670   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
16680   COM F,Z3,Z4,Norm,Thetai,Tdelta
16690   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
16700   !
16710   INTEGER Z,Char_min,Char_max
16720   ! THIS ROUTINE DRAWS THE FINAL BOTTOM LINE OF A TABLE. SUB HEADER ABOVE
16730   ! DRAWS THE BOTTOM LINES ON PAGES OF TABLES WHILE OUTPUT IS STILL BEING
16740   ! CALCULATED
16750   !
16760   PRINT LIN(2)
16770   FOR Z=0 TO 79
16780       IF Z=79 THEN PRINT CHR$(228)
16790       IF Z=79 THEN 16810
16800     PRINT CHR$(228);
16810   NEXT Z
16820 SUBEXIT
16830 Short: Char_min=20     ! THIS ROUTINE CAN BE USED TO FINISH A SMALL TABLE
16840        Char_max=60     ! TO USE, INSERT AN IF STATEMENT AFTER THE PRINT LIN
(2) ABOVE
16850        FOR Z=Char_min TO Char_max
16860          IF Z=Char_min THEN PRINT TAB(Char_min);
16870          IF Z=Char_max THEN PRINT CHR$(228)
16880          IF Z=Char_max THEN 16900
16890          PRINT CHR$(228);
16900        NEXT Z
16910     SUBEXIT
16920 SUB Stuff
16930   OPTION BASE 0
16940   !
16950   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
16960   COM F,Z3,Z4,Norm,Thetai,Tdelta
16970   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
16980   !
16990   COM Stuff$,Redraw$,Digitize$,Ray_trace$
17000   COM INTEGER I,J,Y0_loop.Nray,Nray_chart,Nray_family,Nray_stuff
17010   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
17020   !
17030   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
17040   COM INTEGER Family,Surf_no
```

```
17050   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
17060   !
17070   COM Y_bullet,Smax_family,Percent_image
17080   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
17090   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
17100   !
17110   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
17120   !
17130   !
17140   !   THIS ROUTINE COMPUTES 250 POINTS OF THE SURFACE CHOSEN IN FAMILY
17150   ! THESE POINTS ARE THEN USED IN OTHER ROUTINES TO TRACE AN ARBITRARY
17160   ! NUMBER OF RAYS THROUGH THE DESIGN SURFACE
17170   !
17180   !
17190   DEG
17200     IF Y2max(Family)=Y1(Family) THEN Yb=Y1(Family)
17210     IF Y2max(Family)=Y1(Family) THEN Initialize
17220   !
17230 Yb: Bsm=Y2max(Family)-X2max(Family)*Trho1  ! COMPUTE THE UPPER LIMIT
17240     X1=Bsm/(Talpha-Trho1)  ! OF THE APERTURE. THIS IS AN ESTIMATE FOR
17250     Yb=Y1=X1*Talpha  ! THE APERTURE MAY CHANGE IN THE FINAL DESIGN
17260     Aperture=Yb-Ya  ! THE ESTIMATE IS REQUIRED TO CALCULATE DELY BELOW
17270 Initialize:   IF (Redraw$="Y") OR (Redraw$="y") THEN Redraw! INDICATES
17280               Ya=Y1(Family)
17290               Aperture=Yb-Ya
17300               Y0_min=Ya*(Talpha-Trhoi)/Talpha  ! REDRAW TO ANOTHER SCALE
17310               Y0_max=Yb*(Talpha-Trhoi)/Talpha
17320               Nray_stuff=250
17330               INPUT "HOW MANY RAYS DO YOU WANT TO DRAW THE SURFACE WITH ( DE
FAULT = 250 MAX) ?",Nray_stuff
17340                 IF Ya=0 THEN Add_ray=0
17350                 IF Ya<>0 THEN Add_ray=1
17360               Xnray=Nray_stuff            ! AVOIDS MIXED MODE ARITHMETIC
17370               Dely=(Y0_max-Y0_min)/Xnray
17380               Y0_min=Y0_min-Dely
17390                 IF Ya=0 THEN Y0_min=Y0_min+Dely
17400               Nray_stuff=Nray_stuff+Add_ray
17410               Flag=0
17420               Ns=0
17430               X2(Ns)=0
17440               Y2(Ns)=0
17450               Beta(Ns)=0
17460 Start: Stuff$="Y"
17470        FOR I=1 TO Nray_stuff    ! COMPUTE THE COORDINATES OF THE INTERCEPT
17480          Flag=0                 ! OF THE INCIDENT RAY AND THE FIRST SURFACE
17490          X=0
17500          Rho=Rho_initial
17510          Trho=TAN(Rho)
17520          Y=Y0_min+Dely*I
17530          Ns=Ns+1
17540          Y1(Ns)=Y
17550            CALL X1pos(X,Y)
17560          X1(Ns)=X
17570 Next_i: NEXT I
17580         BEEP
17590         Y$="N"
17600         INPUT "DO YOU WANT A HARD COPY OF THE COMPUTED DATA ? Y/N",Y$
17610           IF (Y$="Y") OR (Y$="y") THEN GOSUB Yhard
17620           CALL Chart
17630         BEEP
17640         Y$="N"
17650         INPUT "DO YOU WANT A HARD COPY OF THE CHART ? Y/N",Y$
17660           IF (Y$="Y") OR (Y$="y") THEN GOSUB Dump_it
17670         Y$="N"
17680         INPUT "DO YOU WANT A HARD COPY OF THE COMPUTED DATA ? Y/N",Y$
17690           IF (Y$="Y") OR (Y$="y") THEN GOSUB Yhard
17700           CALL Family
```

```
17710         Stuff$="N"
17720       SUBEXIT
17730 Redraw: Stuff$="Y"
17740           CALL Plot
17750           LINE TYPE 8
17760             MOVE 0,0
17770           DRAW Y_bullet/Talpha,Y_bullet
17780           DRAW 100,Y_bullet
17790             MOVE X2(1),Y2(1)
17800           LINE TYPE 1
17810         FOR I=2 TO Nray_stuff
17820           DRAW X2(I),Y2(I)
17830         NEXT I
17840           MOVE X2(I-1),Y2(I-1)
17850         LORG 2
17860         LABEL USING 17870;Family
17870         IMAGE K
17880       Redraw$="N"
17890       Flag=3
17900        CALL Graph
17910      SUBEXIT
17920 Dump_it: PRINTER IS 0
17930          PRINT CHR$(27)&"&100T"
17940          DUMP GRAPHICS
17950          PRINT CHR$(27)&"&136T"
17960          PRINTER IS 16
17970        RETURN
17980 Yhard: Flag$="1"
17990        PRINTER IS 0
18000      RETURN
18010     SUBEXIT
18020 SUB Binary_search(X,Y)
18030   OPTION BASE 0
18040   !
18050   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
18060   COM F,Z3,Z4,Norm,Thetai,Tdelta
18070   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
18080   !
18090   COM Stuff$,Redraw$,Digitize$,Ray_trace$
18100   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
18110   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
18120   !
18130   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
18140   COM INTEGER Family,Surf_no
18150   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
18160   !
18170   COM Y_bullet,Smax_family,Percent_image
18180   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
18190   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
18200   !
18210   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
18220   !
18230   !
18240   !   THIS ROUTINE FINDS THE INTERCEPT OF THE RAY WITH THE FOLLOWING:
18250   !
18260   !        1) FIRST SURFACE
18270   !        2) SECOND SURFACE
18280   !        3) FIRST IMAGE PLANE
18290   !        4) SECOND IMAGE PLANE
18300   !
18310   !
18320 DEG
18330 DEF FNA(X,Y)=X-Y/Trho2
18340 DEF FNAi(X,Y)=X-Y/Talpha
18350 DEF FNXn(A)=A-Y_bullet/Trho1    ! FINDS THE INTERCEPT OF THE RAY AND Y=-Y_B
ULLET
18360   Trho2=TAN(Rho2)      ! RHO2 MUST BE DECLARED ELSEWHERE
```

```
18370     IF Surf_no=4 THEN Four
18380   Xc(Y0_loop)=FNA(X,Y)       ! FIND THE X-INTERCEPT OF RHO2
18390 Three: Dy=(Z3-X)*Trho2
18400        Y=Y+Dy
18410        X=Z3
18420      SUBEXIT
18430 Four: Dy=(Z4-X)*Trho2
18440       Y=Y+Dy
18450       X=Z4
18460     SUBEXIT
18470 SUB Graph
18480   OPTION BASE 0
18490   !
18500   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
18510   COM F,Z3,Z4,Norm,Thetai,Tdelta
18520   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
18530   !
18540   COM Stuff$,Redraw$,Digitize$,Ray_trace$
18550   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family.Nray_stuff
18560   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
18570   !
18580   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
18590   COM INTEGER Family,Surf_no
18600   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
18610   !
18620   COM Y_bullet,Smax_family,Percent_image
18630   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
18640   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
18650   !
18660   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
18670   !
18680   !    THIS ROUTINE DRAWS THE GLM FOR STUFF AND RAY_TRACE SUBROUTINES
18690   ! AND DIGITIZES THE IMAGE PLANE POSITION (Z3)
18700   !
18710   !
18720 DEG
18730   IF Flag=3 THEN Wait1
18740   IF (Digitize$="Y") OR (Digitize$="y") THEN Digit
18750 Draw_axes:    CALL Plot
18760 First_surf: FIXED 2
18770             LINE TYPE 8
18780               MOVE 0,0
18790             X=Y_bullet/Talpha
18800             DRAW X,Y_bullet
18810             DRAW 100,Y_bullet
18820               MOVE 0,0
18830             DRAW X,-Y_bullet
18840             DRAW 100,-Y_bullet
18850 X2pos:  MOVE X2(1),Y2(1)
18860        FOR I=2 TO Nray_stuff
18870          DRAW X2(I),Y2(I)
18880        NEXT I
18890 X2neg:   MOVE X2(1),-Y2(1)
18900        FOR I=2 TO Nray_stuff
18910          DRAW X2(I),-Y2(I)
18920        NEXT I
18930        EXIT GRAPHICS
18940        Y$="N"
18950        INPUT "DO YOU WANT A HARD COPY OF THE PLOT WITHOUT RAYS ? Y/N",Y$
18960          IF (Y$="Y") OR (Y$="y") THEN GOSUB Dump_it
18970        GRAPHICS
18980          IF (Redraw$="Y") OR (Redraw$="y") THEN Second_image
18990 First_image: LINE TYPE 5
19000                MOVE Z3,Y_bullet
19010              DRAW Z3,-Y_bullet
19020 Second_image: LINE TYPE 6
```

```
19030                 MOVE Z4,Y_bullet
19040               DRAW Z4,-Y_bullet
19050 REM DRAW THE RAYS
19060   IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Nray_graph=Nray_trace
19070   IF (Stuff$="Y") OR (Stuff$="y") THEN Nray_graph=Nray_stuff
19080 First_: LINE TYPE 1
19090         CLIP 0,100,-Y_bullet,Y_bullet
19100         FOR I=1 TO Nray_graph
19110             MOVE 0,Y1(I)
19120           DRAW X1(I),Y1(I)
19130         NEXT I
19140 Second_: FOR I=1 TO Nray_graph
19150              MOVE X1(I),Y1(I)
19160            DRAW X2(I),Y2(I)
19170          NEXT I
19130 First_image_p: FOR I=1 TO Nray_graph
19190                    MOVE X2(I),Y2(I)
19200                  DRAW Z3,Y3(I)
19210                NEXT I
19220 Decision: WAIT 2000
19230           EXIT GRAPHICS
19240           BEEP
19250           Y$="N"
19260           INPUT "DO YOU WANT TO PLOT TO THE SECOND IMAGE PLANE ? Y/N",Y$
19270           GRAPHICS
19280             IF (Y$="N") OR (Y$="n") THEN Wait1
19290 Second_image_p: FOR I=1 TO Nray_graph
19300                     MOVE Z3,Y3(I)
19310                   DRAW Z4,Y4(I)
19320                 NEXT I
19330 Wait1: BEEP  ! THIS ROUTINE PRESENTS THE PLOT TO THE OPERATOR
19340        WAIT 250 ! IF A NOTE TO THE OPERATOR IS DESIRED, ENTER SETGU,LORG 2
19350        BEEP     ! MOVE 2,2,CSIZE 2.5,LABEL "PRESS CONT",CSIZE 15/4.54,
19360        PAUSE    ! SETUU AFTER THE SECOND BEEP
19370        EXIT GRAPHICS
19380          IF (Flag=0) OR (Flag=3) THEN EXIT GRAPHICS
19390      SUBEXIT
19400 Digit: GRAPHICS                ! THIS ROUTINE CHANGES THE ANALOG POSITION
19410        POINTER F,0             ! OF THE IMAGE PLANE ON THE GRAPHICS DEVICE
19420        DIGITIZE Z3,Y           ! TO DIGITAL DATA FOR PROCESSING
19430        LINE TYPE 5
19440          MOVE Z3,Y_bullet
19450        DRAW Z3,-Y_bullet
19460        SETGU
19470        LDIR 0
19480        LINE TYPE 1
19490        LORG 5
19500        CSIZE 2.5
19510        Centerx=72.5       ! CHANGE IN LAXES AS WELL
19520          MOVE 1.5*Centerx,2
19530        LABEL USING 19540;Z3
19540        IMAGE "Image Plane : ",DD.DD," inch"
19550        SETUU
19560        CSIZE 15/4.54
19570        WAIT 2500
19580        EXIT GRAPHICS
19590      SUBEXIT
19600 Dump_it: PRINTER IS 0
19610          PRINT CHR$(27)&"&100T"
19620          DUMP GRAPHICS
19630          PRINT CHR$(27)&"&136T"
19640          PRINTER IS 16
19650        RETURN
19660      SUBEND
19670 SUB X1neg(X,Y)
19680   OPTION BASE 0
19690   !
```

```
19700   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,F,Z3,Z4
19710   !
19720 DEG
19730 Alphan=-1*Alpha
19740 Talphan=TAN(Alphan)
19750 DEF FNY1(Y)=Y*Talphan/(Talphan-Trho)   ! FINDS THE Y-VALUE OF THE INTERCEPT
19760                                         ! OF THE RAY AND THE FIRST SURFACE
19770 DEF FNX1(Y)=Y/Talphan   ! FINDS THE X-VALUE OF THE INTERCEPT OF THE RAY
19780                         ! AND THE FIRST SURFACE
19790     Y=FNY1(Y)
19800 Woops:X=FNX1(Y)
19810 SUBEND
19820 SUB Density
19830   OPTION BASE 0
19840   !
19850   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Rho1,Trho1,Rho2,Trho2
19860   COM F,Z3,Z4,Norm,Thetai,Tdelta
19870   COM Date$,Plot$,Family$,Chart$,Main$,Main1$,Flag$,Y_hard$
19880   !
19890   COM Stuff$,Redraw$,Digitize$,Ray_trace$
19900   COM INTEGER I,J,Y0_loop,Nray,Nray_chart,Nray_family,Nray_stuff
19910   COM INTEGER Nray_trace,Nray_density,Add_ray,Nc,Ns,Nbeta,Flag
19920   !
19930   COM INTEGER Line_count,Line_max,N_increment,Hit_total,Pselect,Hpib
19940   COM INTEGER Family,Surf_no
19950   COM REAL Theta_critical,Tol,N1,N2,N3,Delta_ray,Y0,Ya,Yb,Aperture
19960   !
19970   COM Y_bullet,Smax_family,Percent_image
19980   COM X1(*),Y1(*),Xc(*),X2(*),Y2(*),X2max(*),Y2max(*)
19990   COM Y3(*),Y4(*),Beta(*),S2(*),Xsurfmax(*),Ysurfmax(*)
20000   !
20010   COM Qa(*),Smax(*),Alpha2(*),Rho2(*),Hit(*)
20020   !
20030   INTEGER Inc,K,L,M,Lines
20040   !
20050   DEG
20060   ! THIS ROUTINE COMPUTES THE RAY DENSITY IN ONE DIMENSION.
20070   ! ASSUMPTIONS:
20080   !     1) THE POINT OF MAXIMUM RAY DENSITY HAS BEEN FOUND VIA A
20090   !        DIGITIZE STATEMENT.
20100   !     2) THE VALUES OF Y3 HAVE NOT BEEN CHANGED TO REFLECT THE
20110   !        NEW VALUE OF Z3
20120   !
20130   PRINTER IS 16
20140   Nray_density=Nray_stuff
20150 Re_draw: FOR I=1 TO Nray_density        ! RECOMPUTE THE VALUE OF Y3
20160           A=Xc(I)
20170           X=X2(I)
20180           Y=Y2(I)
20190           Rho2=Rho2(I)
20200           Trho2=TAN(Rho2)
20210 Same:     Deltay=(Z3-X)*Trho2
20220           Y3=Y+Deltay
20230           Y3(I)=Y3
20240           Deltay=(Z4-Z3)*Trho2
20250           Y4(I)=Y3(I)+Deltay
20260 Next_i:  NEXT I
20270 Hit_zero:   MAT Hit=ZER(2001)                        ! ZERO THE ARRAY
20280          Inc=0                    ! FIND THE No. OF HITS IN THE INTERVAL
20290          Ylast=-Y_bullet          ! [-Y_BULLET, -Y_BULLET+DELTA/2]
20300          Delta=Y_bullet/N_increment
20310          Ynext=Ylast+Delta/2
20320          Hit(Inc)=0
20330          FOR I=1 TO Nray_density
20340             IF (Y3(I)>=Ylast) AND (Y3(I)<Ynext) THEN Hit(Inc)=Hit(Inc)+1
20350          NEXT I
20360 Hit_middle:                       ! FIND THE No. OF HITS IN THE INTERVAL
```

```
20370                                    ! [-Y_bullet+DELTA/2, Y_bullet-DELTA/2)
20380             FOR L=1 TO 2*N_increment-1
20390               Inc=Inc+1
20400               Hit(Inc)=0
20410               Ylast=Ynext
20420               Ynext=Ylast+Delta
20430                 FOR I=1 TO Nray_density
20440                   IF (Y3(I)>=Ylast) AND (Y3(I)<Ynext) THEN Hit(Inc)=Hit(In
c)+1
20450                 NEXT I
20460               NEXT L
20470 Hit_last: Inc=2*N_increment    ! FIND THE No. OF HITS IN THE LAST INTERVAL
20480           Hit(Inc)=0           !  [Y_bullet-DELTA/2, Y_bullet]
20490           Ylast=Y_bullet-Delta/2
20500           Ynext=Y_bullet
20510           FOR I=1 TO Nray_density
20520             IF (Y3(I)>=Ylast) AND (Y3(I)<=Ynext) THEN Hit(Inc)=Hit(Inc)+1
20530           NEXT I
20540 Normalize: Hit_total=0                  ! NORMALIZE THE No. OF HITS ON THE
20550            FOR I=0 TO 2*N_increment     ! IMAGE PLANE (Z3) TO THE TOTAL
20560              Hit_total=Hit_total+Hit(I) ! No. OF HITS
20570            NEXT I
20580            !
20590              IF Hit_total=0 THEN None
20600            FOR I=0 TO 2*N_increment
20610              Hit(I)=Hit(I)/Hit_total
20620            NEXT I
20630 None:      Pct=Hit_total/Nray_density        ! % OF RAYS THAT ARRIVE AT THE
20640            Percent_image=Pct*100             ! IMAGE PLANE
20650 Graph: CALL Plot                             ! PLOT AND LABEL THE AXES
20660               IF Hit_total=0 THEN Label_
20670 First_bar: Xlast=-Y_bullet                   ! DRAW THE HISTOGRAM
20680            Xnext=Xlast+Delta/2  ! INTERVAL  [-Y_bullet, -Y_bullet+DELTA/2)
20690            Inc=0
20700              IF Hit(Inc)=0 THEN Middle_bars
20710            CLIP Xlast,Xnext,0,Hit(Inc)
20720            FRAME
20730            GOSUB Delt
20740 Middle_bars: FOR I=1 TO 2*N_increment-1     ! INTERVAL
20750                Inc=Inc+1           ! [-Y_bullet+DELTA/2, Y_bullet-DELTA/2)
20760                Xlast=Xnext
20770                Xnext=Xlast+Delta
20780                  IF Hit(Inc)=0 THEN Next_i_m
20790                CLIP Xlast,Xnext,0,Hit(Inc)
20800                FRAME
20810                GOSUB Delt
20820 Next_i_m:    NEXT I
20830 Last_bar: Inc=2*N_increment        ! INTERVAL [Y_bullet-DELTA/2, Y_bullet]
20840           Xlast=Y_bullet-Delta/2
20850           Xnext=Y_bullet
20860             IF Hit(Inc)=0 THEN Label_
20870           CLIP Xlast,Xnext,0,Hit(Inc)
20880           FRAME
20890           GOSUB Delt
20900 Label_: Digitize$="N"
20910         Flag=3
20920           CALL Graph
20930       SUBEXIT
20940 Delt: Lines=10
20950       Delt=(Xnext-Xlast)/Lines
20960       X=Xlast-Delt
20970       FOR J=1 TO Lines
20980         X=X+Delt
20990           MOVE X,0
21000           DRAW X,Hit(Inc)
21010       NEXT J
21020     RETURN
```

```
21030   SUBEND
21040 SUB Min(Xm(*),X,INTEGER M)
21050   OPTION BASE 0
21060   !
21070   DIM Xm(M)
21080   !
21090   !   THIS ROUTINE FINDS THE MINIMUM ELEMENT IN A ONE DIMENSIONAL ARRAY
21100   !
21110   X=Xm(0)
21120   FOR I=1 TO M
21130     X=Min(X,Xm(I))
21140   NEXT I
21150  SUBEND
21160 SUB Max(Xm(*),X,INTEGER M)
21170   OPTION BASE 0
21180   !
21190   DIM Xm(M)
21200   !
21210   !   THIS ROUTINE FINDS THE MAXIMUM ELEMENT IN A ONE DIMENSIONAL ARRAY
21220   !
21230   X=Xm(0)
21240   FOR I=1 TO M
21250     X=MAX(X,Xm(I))
21260   NEXT I
21270  SUBEND
```

# APPENDIX B

## TRACE PROGRAM DESCRIPTION AND PROGRAM LISTING

TRACE was the first program written for this thesis. TRACE was written for the purpose of automating the task of drawing the ray diagram produced when tracing rays through a conical lens with various second surfaces. The program consists of a set of subroutines called from a control program which are designed to perform a specific task(s). A detailed description of each subroutine will not be given because TRACE has extensive documentation included within the program. However, an abbreviated discussion of the geometric relationships at the first and second surfaces and the bisection routine in subroutine BINARY SEARCH follows.

TRACE occupies approximately 145 Kbytes of memory which is approximately 77% of available memory (11). A listing of TRACE is at the end of this appendix.

TRACE was designed to calculate the trajectory of up to 250 rays incident upon the first surface in the upper half-plane as illustrated in figure B-1. All of the parameters shown in figure B-1 are provided by the operator except Z4, the location of the permanent Image plane, which is set for Z4 = 50 inches. All of the parameter values provided by the operator have default values declared either in subroutine DIALOGUE, line number 13810 or the MAIN routine, line number 10.

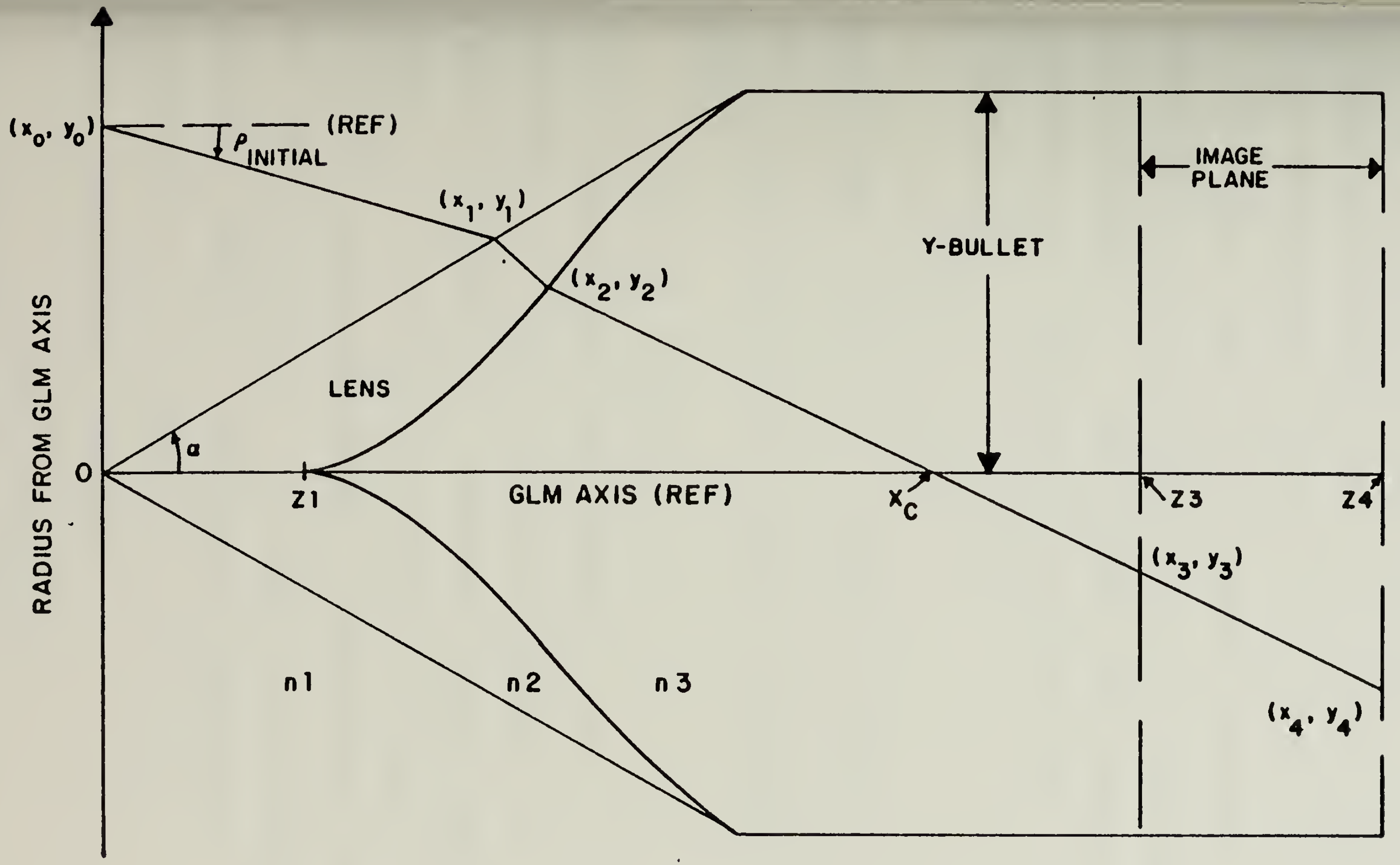


Figure B-1. Symbol Definition for the GLM.

TRACE calculates the complete trajectory of each ray before proceeding to the next ray. Each surface is assigned a number with the vertical axis, y, defined as the zeroth surface. All incident rays originate on the y-axis and are assumed to be parallel with the source at infinity. The first surface shown in figure B-2 encountered is the exterior of the conical spike which, in two dimensions, is a plane perpendicular to the meriodional plane, the paper, with a cone half-angle $\alpha$. The incident angle makes an angle of incidence $\theta_I$ with the first surface normal $\hat{n}$ at point T according to

$$\theta_I = \frac{\pi}{2} \quad (\alpha + |\rho_I|)$$

Noting that $\alpha = \alpha_2$, $\rho_R = |\mathrm{NORM}| - \theta_R$, $\mathrm{NORM} = \alpha_2 - \frac{\pi}{2}$ (B-1)

and applying Snell's Law $\sin\theta_R = (n_1/n_2)\sin\theta_I$ (B-2)

yields

$$\rho_R = \alpha_2 + \theta_R - \frac{\pi}{2}$$

the ray angle in the lens referred to as $\rho_1$ in chapter III and Appendix F. As the incident ray angle $\rho_I$ increases the angle of incidence $\theta_I$ decreases to zero when the incident causing $\theta_I$ to transition through the normal as shown in figure B-3, subroutine SNELL in TRACE detects this transition as a change in sign in $\theta_I$. Snell's Law is defined with all arguments positive. Therefore the magnitude of $\theta_I$ is used to determine $\theta_R$ and thus $\rho_R$ according to

$\sin\theta_R = (n_2/n_1)\sin\theta_I$ yielding

$$\theta_R = \alpha_2 + \theta_R - \frac{\pi}{2} \qquad \text{(B-3)}$$

which is the same result for $\theta_I > 0$.

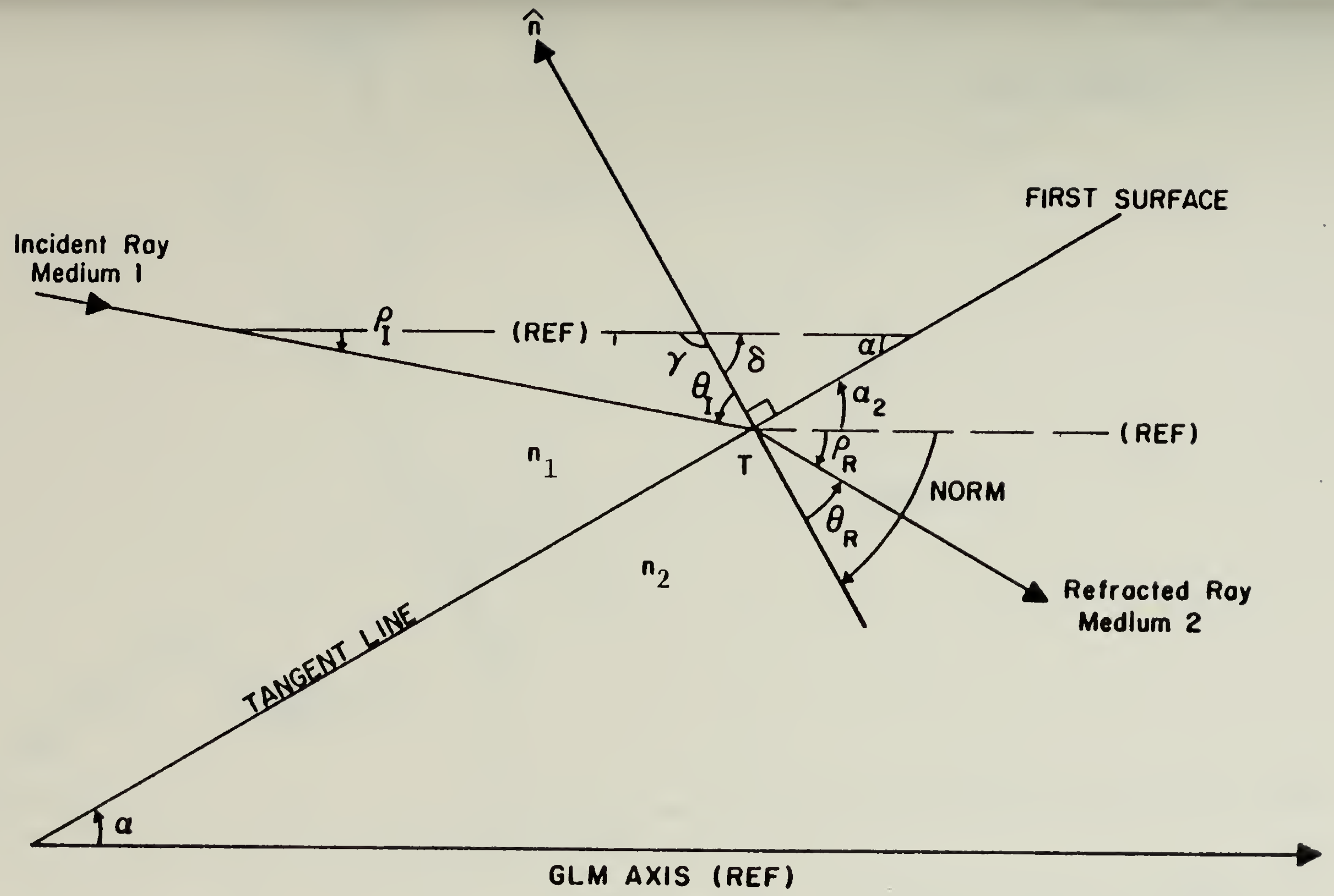


Figure B-2. Geometry and Symbol Definition for the Application of Snell's Law at the First Surface of the Conical Lens.

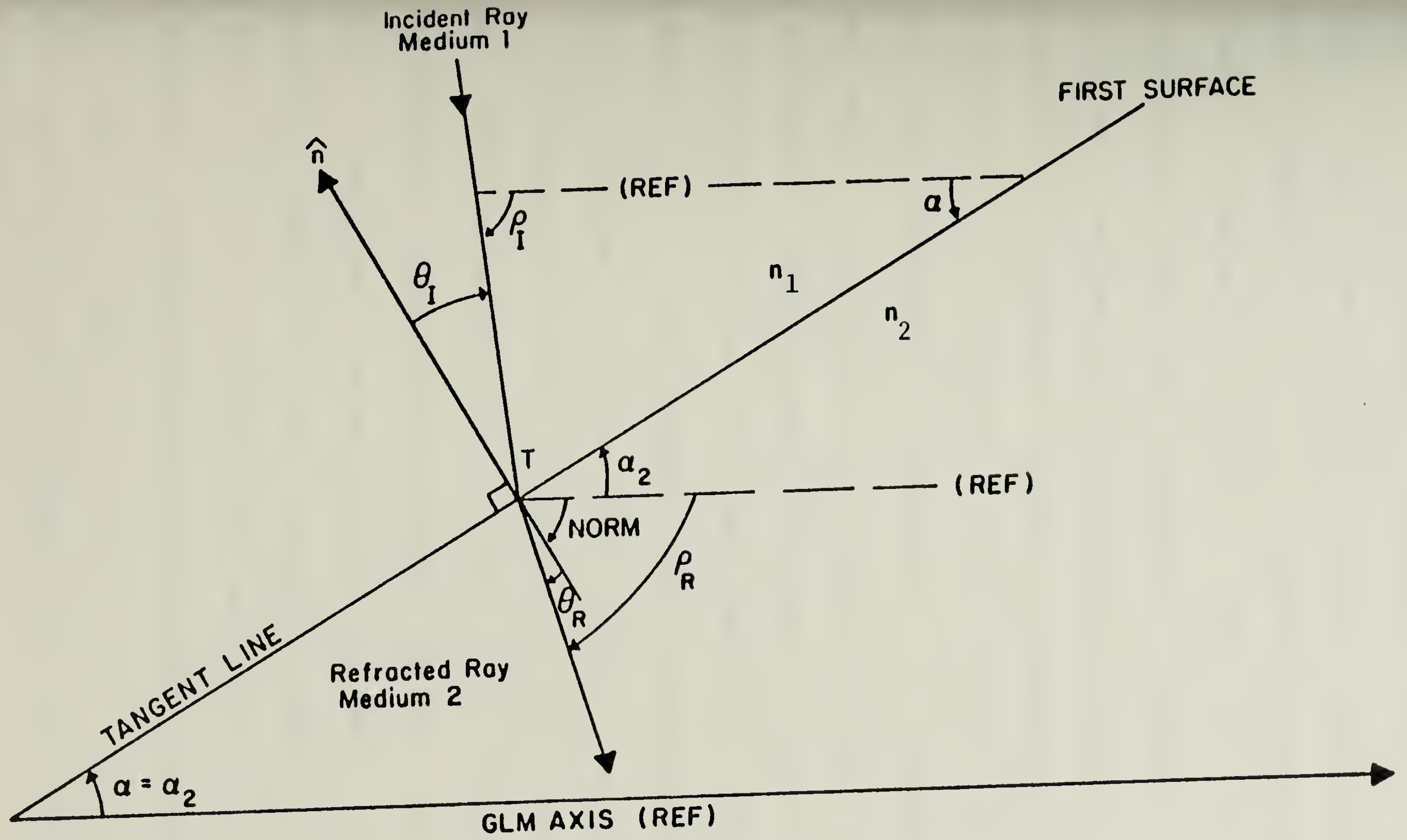


Figure B-3. Geometry and Symbol Definition for the Application of Snell's Law at the First Surface of the Conical Lens for $\theta_I < 0$.

The application of Snell's Law at the second surface of the lens is very similar to the first surface. The geometry for $\theta_I > 0$ is shown in figure B-4 and for $\theta_I < 0$ in figure B-5. Some of the rays inside the lens may intercept the GLM axis at $x < Z1$ and therefore may intercept the branch of the second surface in the lower half-plane. The geometry for this case is shown in figure B-6. A significant difference between rays intercepting the positive and negative branches of the second surface is the definition of the normal direction $\hat{n}$. In figures B-4 and B-5 the direction of $\hat{n}$ is into the lens where in figure B-6, $\hat{n}$ is defined to be out of the lens. Practically, this contradiction in definition has no effect on TRACE because the angle of the normal used in applying Snell's Law has always been the acute angle as labeled in all of the figures. The direction of $\hat{n}$ in figure B-6 was changed to reflect the change in sign of NORM.

The search for and calculation of the intercept of the ray refracted at the first surface and the second surface is accomplished in subroutine BINARY SEARCH. The primary method used is the bisection method illustrated in figure B-7. The bisection method can only be applied where the ray in the lens is known to intercept the second surface of the lens. Examples of rays which do not intercept the second surface are shown in figures C-2, C-5 and C-10. The method used for the rays which do not intercept the second surface is discussed later.

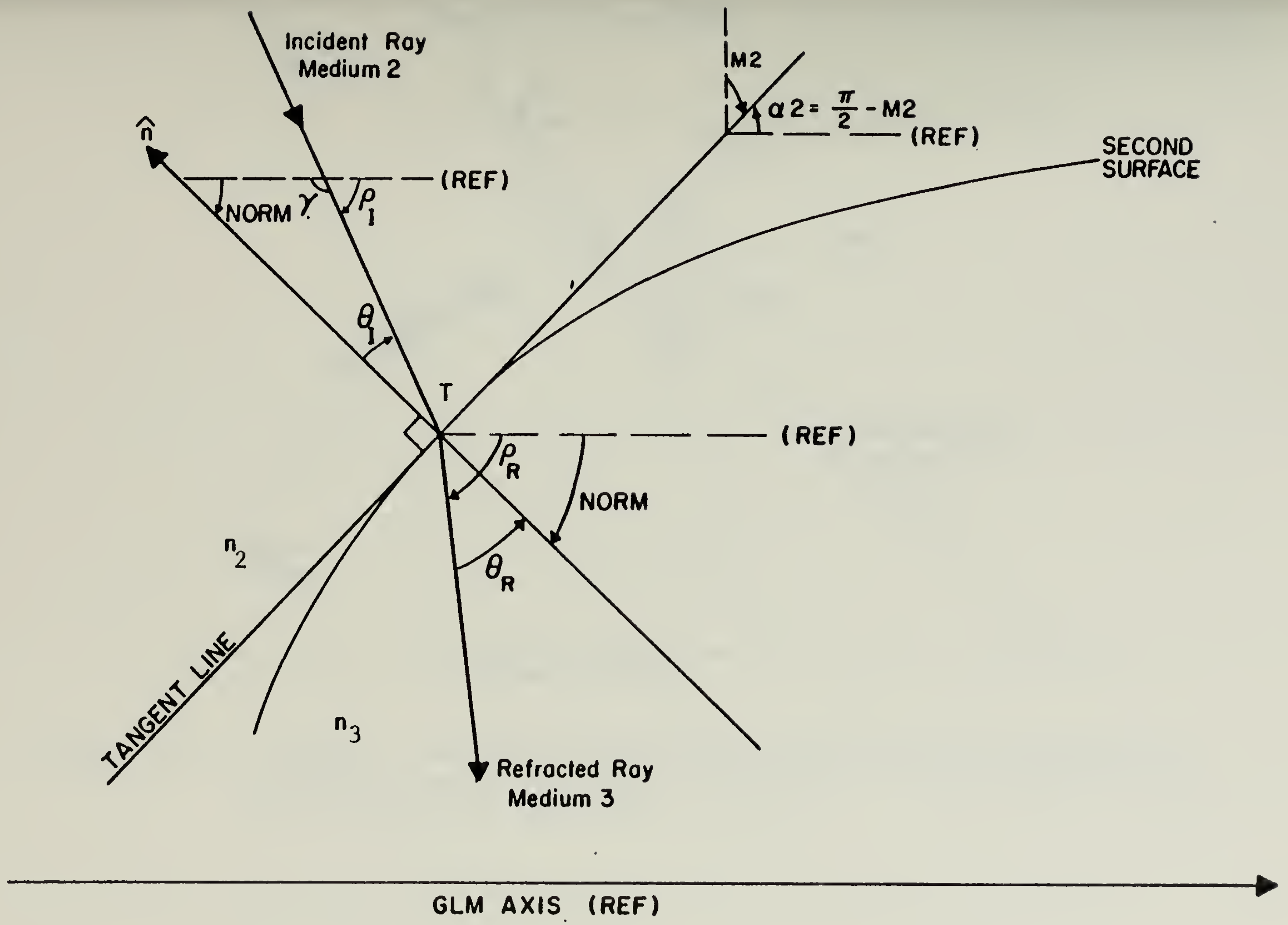


Figure B-4. Geometry and Symbol Definition for the Application of Snell's Law at the Positive Branch of the Second Surface of the Conical Lens.

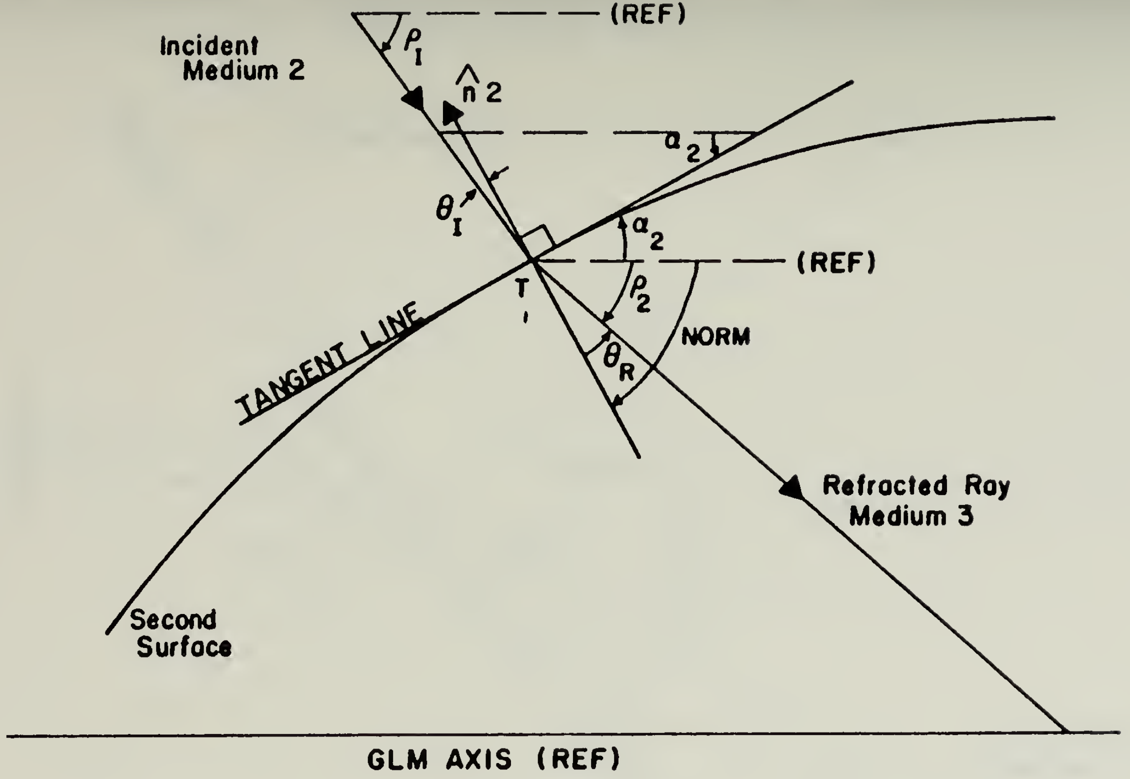


Figure B-5. Geometry and Symbol Definition for the Application of Snell's Law at the Positive Branch of the Second Surface of the Conical Lens and $\theta_I < 0$.

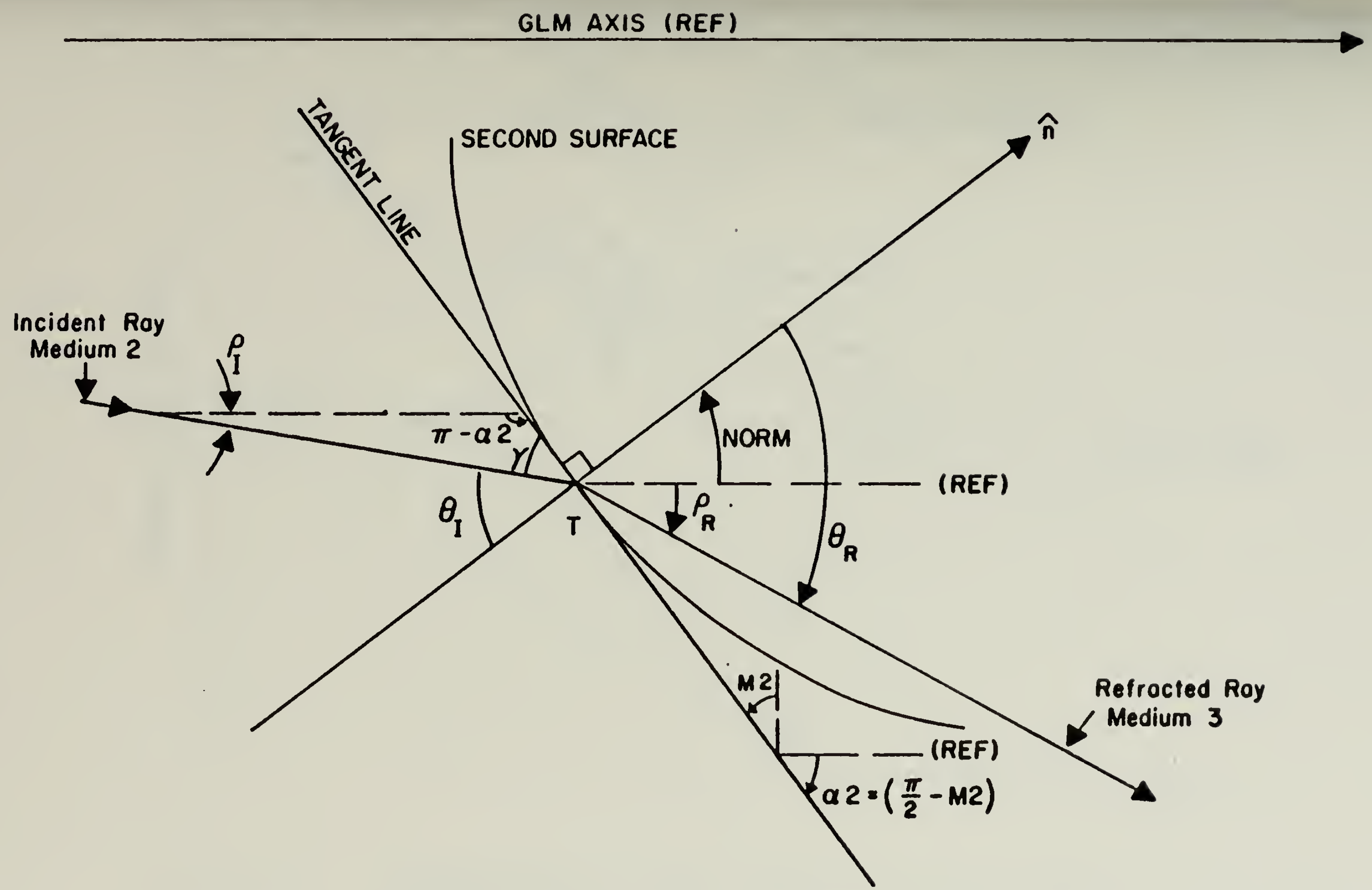


Figure B-6. Geometry and Symbol Definition for the Application of Snell's Law at Negative Branch of the Second Surface of the Conical Lens.

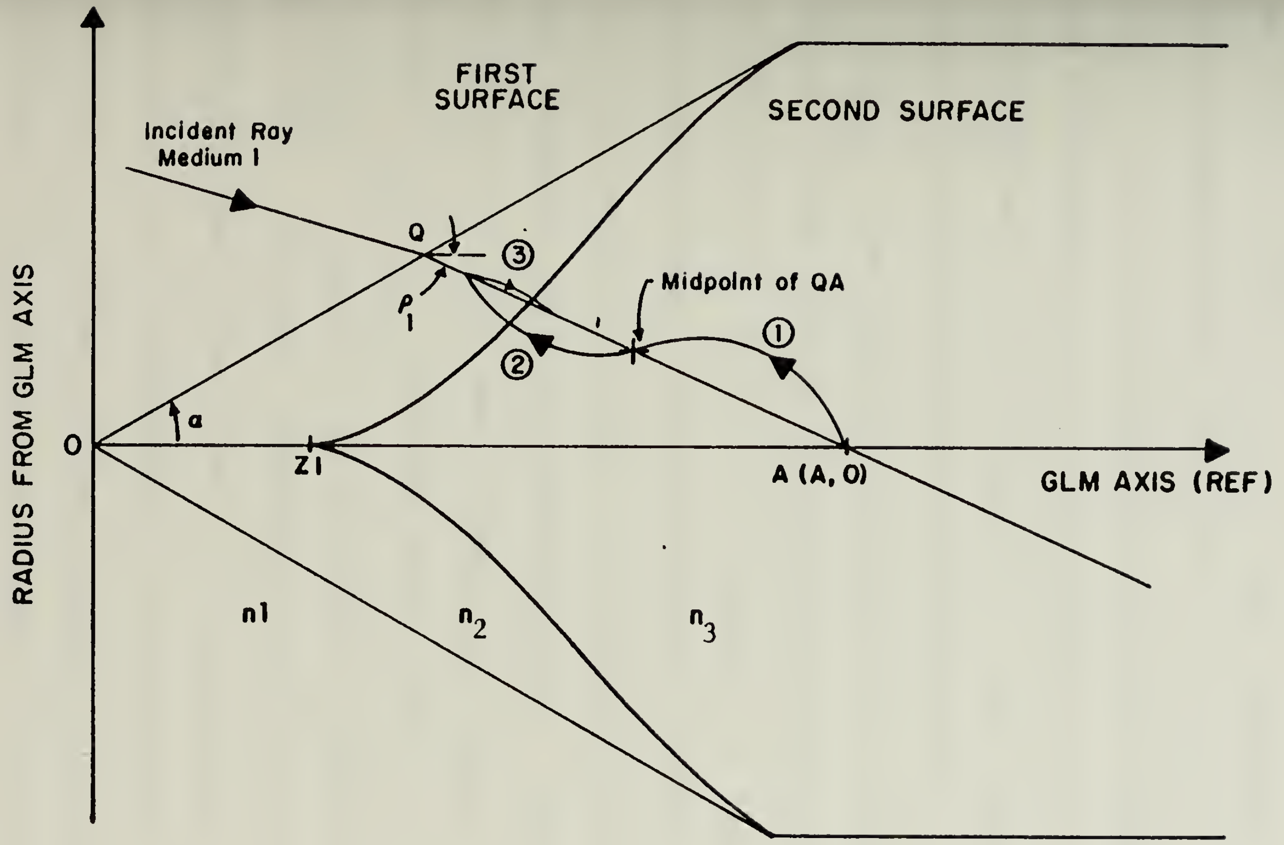


Figure B-7. Illustration of the Bisection Method Used in Subroutine BINARY SEARCH.

The bisection method converges to the intercept value quickly, especially if the ray is nearly normal to the second surface. The sequence of events is depicted in figure B-7 by the circled numbers 1 , 2 , etc. Assuming $\rho_1$ is known, the intercept of the GLM axis, A, of the ray in the lens is found. The first step in the bisection sequence, 1 is to find the midpoint of QA. The midpoint of QA, $(x_{mid}, y_{mid})$ is calculated under label Recurse _p, line 7440 of TRACE. The midpoint is then compared to the value of the second surface at $y=y_{mid}$. The midpoint in figure B-7 is behind the second surface. Step 2 finds one midpoint of the line segment from Q to the midpoint of QA. A comparison is again made with the second surface. Now, the present location is ahead of the second surface. Step 3 is executed and the location is again compared with the second surface. The process continues until the difference between the present location on the ray $(x_{mid}, y_{mid})$ and the true value of the intercept $(x_2, y_{mid})$, $|x_{mid}-x_2|$ is less than a specified tolerance, tol = $10^{-5}$. Once the intercept of the ray and the second surface has been found then go to label Done, line 8140, to define the (x, y) coordinates of the intercept. The x-coordinate is defined as the average of the present location on the ray, $x_{mid}$, and x2, the value of the second surface at $y=y_{mid}$.

The ray may pass ahead, shown by $A < Z1$, of the second surface. The test for $A < Z1$ is conducted in line 6860.

If the test is true a jump to line 7720 is performed. A test is conducted in line 7870 to determine if the ray intercepts the negative branch of the second surface. If the test is true then a jump to line 8080 is made to prepare to enter the bisection method for the negative branch, which is identical to the method for the positive branch described above. The test performed in line 7870 will not detect a ray that is just tangent to the second surface. The slow march method, lines 7840 to 8000, is employed to search and find the intercept (A, 0) and marches down the ray until the second surface intercept is found within the tolerance, or the wall of the GLM, y = -Y_ bullet is reached. If y = -Y_ bullet then the march is terminated and a jump to line 8250 via 8150 is made to exit the subroutine.

A special case exists when $\theta_I < 0$. The ray in the lens could intercept the positive branch. A test is conducted in line 7010 to determine if the ray intercepts the positive branch. If the test is true a jump is made to line 7330 for entry into the bisection method. If the test fails then the marching method is used. The application of the marching method for this case is made from line 7100 to 7320.

Another special case exists when the polynomial used to describe the second surface causes the second surface to intercept the first surface at $|y|$ < Y_ bullet. The test for this case is conducted in line 7370 for the positive branch and line 7770 for the negative branch. If the test

is true, then a jump is made to line 8280 and the subroutine is exited.

The "success" of a conical lens is whether light rays can be focused to a point. Success is determined two ways. The first is the ray diagram; compare figures 14, 22, C-2, C-10 and C-16 for the ability of each lens to focus light rays. The second method is by a histogram of the ray distribution on the image plane. The desired histogram is a delta function at the origin as shown in figures 15 and 23. Undesirable histograms are shown in figures C-3, C-7, C-13, C-14 and C-17. The histogram is calculated by dividing the image plane in the upper half-plane into an even number of intervals equal to N_increment. The division of the image plane is also performed in the lower half plane as shown in figure B-8. The intervals are arranged such that the GLM axis is straddled by the center interval, thus adding a half-interval to each extremity of the image plane. Hence, the total number of intervals is then 2*N_increment +1. The generation of the histogram is executed in subroutine DENSITY, line 17920. The algorithm checks each interval for an intercept of a ray with the image plane in lines 18440 to 18820, DENSITY then normalizes the distribution to the number of hits on the image plane, not the total number of rays in lines 18830 to 18930. The percentage of the rays striking the image plane is calculated in lines 18920 and 18930 and displayed to the operator by subroutine LAXES in lines 12870 and 12880. The

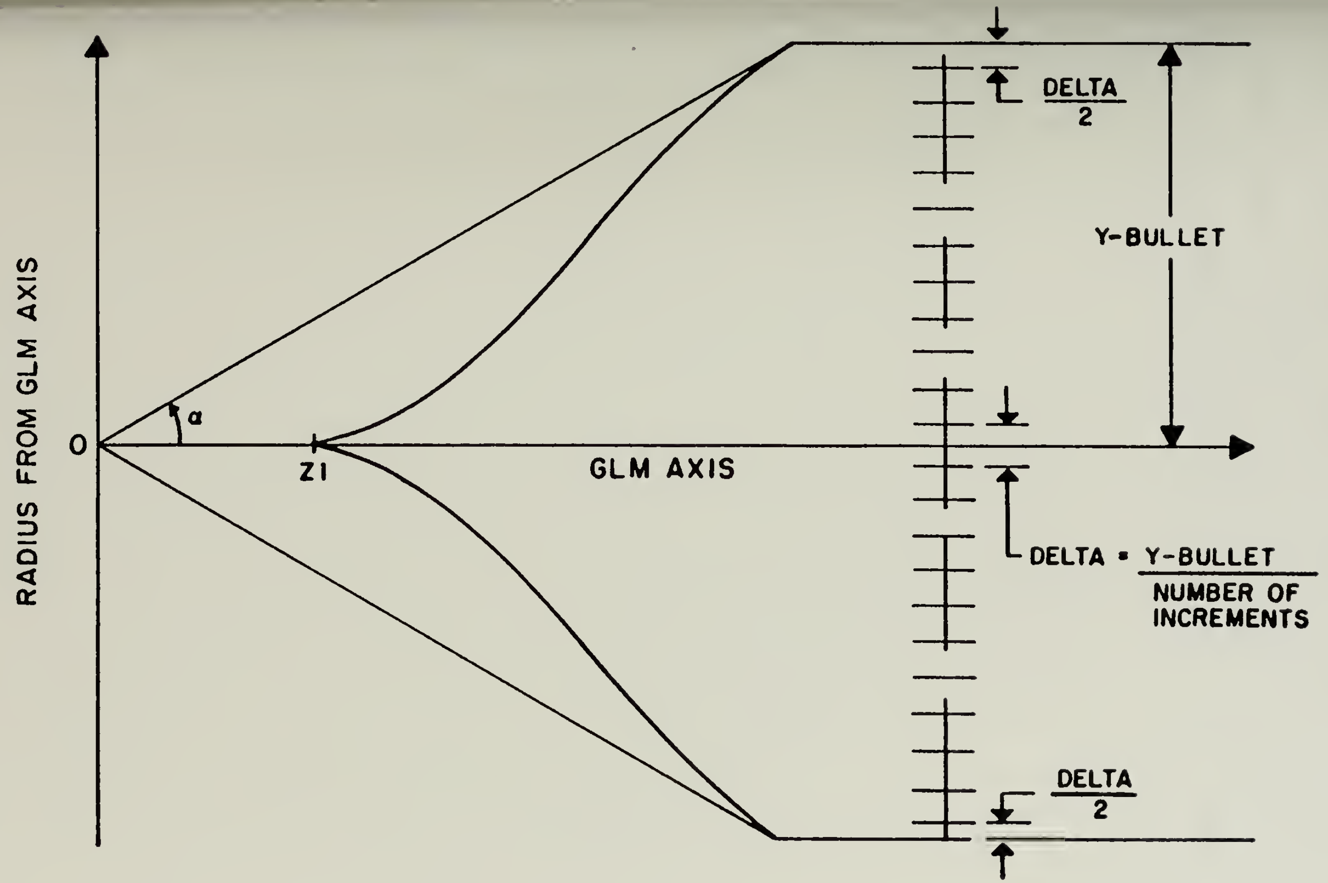


Figure B-8. Illustration of Dividing the Image Plane into a Set of N-Increment Intervals Used to Generate a Histogram of a Ray Diagram.

percentage label serves to remind the operator that the histogram represents the distribution of the rays actually striking the image plane within the boundaries of the GLM.

The subroutines in Table B-1 were copied or derived from the HP-9845B utilities library for use in TRACE.

## TABLE B-I

## SUBROUTINES DERIVED FROM THE HP-9845B UTILITIES LIBRARY

| Name | Line No. |
|---|---:|
| Main | 18 |
| Plot | 10170 |
| Laxes | 11430 |
| Dialogue | 13810 |

# TRACE PROGRAM LISTING

```
10 NORMAL
20 PRINTER IS 16
30 PRINT PAGE
40 FIXED 2
50 PRINT "*********************************************************************"
60 PRINT "*                                                                   *"
70 PRINT "*                              TRACE                                *"
80 PRINT "*                                                                   *"
90 PRINT "*********************************************************************"
100 DISP "THIS IS TRACE"
110 WAIT 1500
120 Main:OPTION BASE 0
130    !
140    COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(17),Apos(17),A(17),Z1,Z3,Z4
150    COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
160    COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
170    !
180    COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
190    COM INTEGER Linecount,Linemax
200    COM INTEGER Pselect,Hpib
210    !
220    COM REAL Nmin,Nmax,R0_max
230    COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
240    COM X(1250),Y(1250),Xc(250),N(250),R(250)
250    !
260    COM Rho(750),Phi(250),C(250),Y3(250),Hit(2001)
270    !
280    INTEGER Zz,Char_max
290    DISP ""
300    OVERLAP                ! SEE THE 9845 OPERATING AND PROGRAMMING MANUAL
310    CALL Dialogue
320    DEG
330 Comments: !    FLAGS ARE USED THROUGHOUT THE PROGRAM TO INDICATE VARIOUS
340           ! CONDITIONS WHICH INFLUENCE THE BEHAVIOR AND TREATMENT OF THE
350           ! LIGHT RAY AS IT PASSES THROUGH THE LENS SYSTEM. THE EXPLANATION
360           ! OF FLAGS USED IN THE GENERATION OF THE RAY DIAGRAMS
370           ! AND THE DECLARATION LOCATION IS GIVEN BELOW:
380           !
390           !   FLAG :
400           !      1) 0 => DECLARED IN MAIN AND BINARY SEARCH
410           !              RESET CONDITION
420           !      2) 1 => DECLARED IN BINARY SEARCH
430           !              RAY INTERCEPTS THE NEGATIVE BRANCH
440           !      3) 5 => DECLARED IN SNELL
450           !              Theta I < 0 DEGREES
460           !      4) 10 => DECLARED IN SNELL
470           !               TOTAL INTERNAL REFLECTION
480           !      5) 15 => DECLARE IN SNELL
490           !               THE RAY LEAVES THE SECOND SURFACE WITH
500           !               ABS(RHO) ≥ 90 DEGREES (THE RAY WILL NOT INTERCEP
T
510           !               THE IMAGE PLANE)
520           !      6) 20 => DECLARED IN BINARY SEARCH
530           !               THE RAY MISSES BOTH THE POSITIVE AND NEGATIVE
540           !               BRANCHES FOR ThetaI > OR < 0 DEGREES
550           !      7) 30 => DECLARED IN BINARY SEARCH
560           !               THE SECOND SURFACE BENDS AROUND AND CROSSES THE
570           !               FIRST SURFACE
580           !
590           !
600           !   Flag2 :
610           !      1) 0 => DECLARED IN MAIN AND BINARY SEARCH
620           !              RESET CONDITION
630           !      2) 1 => DECLARED IN BINARY SEARCH
640           !              THE RAY INTERCEPTS THE NEGATIVE BRANCH
650           !
660           ! NOTE: THESE ARE NOT ALL OF THE FLAGS, JUST THE ONES AFFECTING
```

```
670           !       RAYS.
680           !
690 Gymnastics:   Y_hard$="N"
700               Flag2$="1"
710               INPUT "DO TOU WANT A HARD COPY OF THE COMPUTED DATA? Y/N",Y_ha
rd$
720                 IF (Y_hard$="N") OR (Y_hard$="n") THEN Flag2$="0"
730                 GOSUB Yhard
740                 GOSUB Printer
750               BEEP
760               PAUSE
770                 IF (G$="Y") OR (G$="y") THEN GOSUB Header_grin
780                 IF (G$="Y") OR (G$="y") THEN Initialize_run
790               Y_hard$="N"
800               INPUT "DO YOU WANT A HARD COPY OF THE COEFICIENTS PRINTED ? Y/
N",Y_hard$
810                 IF (Y_hard$="N") OR (Y_hard$="n") THEN 1000
820                 GOSUB Yhard
830                 GOSUB Header_coef
840               FOR I=0 TO Np
850                 IF Linecount>Linemax THEN GOSUB Header_coef
860                 PRINT USING Image_p;I,Apos(I)
870                 Linecount=Linecount+1
880               NEXT I
890                 IF Linecount>Linemax THEN GOSUB Header_coef
900               PRINT
910               Linecount=Linecount+1
920               FOR I=0 TO Nn
930                 IF Linecount>Linemax THEN GOSUB Header_coef
940                 PRINT USING Image_n;I,Aneg(I)
950                 Linecount=Linecount+1
960               NEXT I
970 Image_p:      IMAGE 25X,"Apos("DD") = ",K
980 Image_n:      IMAGE 25X,"Aneg("DD") = ",K
990                 GOSUB Header_coef_end
1000              PRINT LIN(2),"PRESS CONT"
1010              PAUSE
1020                IF Flag2$="1" THEN Y_hard$="Y"
1030                GOSUB Yhard
1040                GOSUB Header
1050 Initialize_run:Surf_no=0
1060                Z1=Apos(0)
1070                  IF (G$="Y") OR (G$="y") THEN Z3=15
1080                Y0_min=Ya*(Talpha-Trhoi)/Talpha
1090                Y0_max=Yb*(Talpha-Trhoi)/Talpha
1100                Xnray=Nray      ! TO KEEP FROM USIN MIXED MODE ARITHMETIC
1110                Dely=(Y0_max-Y0_min)/Xnray
1120                Y0_min=Y0_min-Dely
1130                  IF Ya=0 THEN Y0_min=Y0_min+Dely
1140                Nray=Nray+Add_ray
1150                Ns=-1         ! COUNTERS
1160                Nr=-1
1170                Xc(0)=0
1180                N(0)=0
1190                Phi(0)=0
1200                C(0)=0
1210 Y0_loop:  FOR Y0_loop=1 TO Nray
1220           Flag=0                    ! RESET FLAG
1230           Flag_2=0
1240           X=0
1250           Rho=Rho_initial
1260           Trho=TAN(Rho)
1270           Xy0_loop=Y0_loop
1280           Y=Y0_min+Dely*Xy0_loop
1290           Ns=Ns+1
1300           Nr=Nr+1
1310           X(Ns)=X
```

```
1320           Y(Ns)=Y
1330           Rho(Nr)=Rho
1340 One: Surf_no=1
1350      Ns=Ns+1
1360      Nr=Nr+1
1370        CALL Binary_search(X,Y)
1380      X(Ns)=X
1390      Y(Ns)=Y
1400        CALL Snell(X,Y)
1410      Rho(Nr)=Rho
1420        IF (G$="Y") OR (G$="y") THEN Phi(Y0_loop)=Phi
1430 Two: Surf_no=2
1440      Ns=Ns+1
1450      Nr=Nr+1
1460        CALL Binary_search(X,Y)
1470      X(Ns)=X
1480      Y(Ns)=Y
1490        IF (G$="Y") OR (G$="y") THEN Blind
1500        IF Flag=20 THEN Blind
1510        IF Flag=30 THEN Flag_30
1520        CALL Snell(X,Y)
1530      Rho(Nr)=Rho
1540 Flag_15: IF Flag<>15 THEN Flag_10
1550          GOTO Blind
1560 Flag_10:   IF Flag<>10 THEN Three     ! FLAG_30 WHEN FIXED
1570          Xc(Y0_loop)=0
1580          Rho(Nr)=0
1590          GOTO Blind
1600 Flag_30:     IF Flag<>30 THEN Three
1610          Xc(Y0_loop)=0
1620          Rho(Nr)=0
1630          GOTO Blind
1640 Three: Surf_no=3    ! FIRST (MOVABLE) IMAGE PLANE
1650        Ns=Ns+1
1660          CALL Binary_search(X,Y)
1670        X(Ns)=X
1680        Y(Ns)=Y
1690 Four: Surf_no=4
1700       Ns=Ns+1
1710         CALL Binary_search(X,Y)
1720       X(Ns)=X
1730       Y(Ns)=Y
1740       GOTO Next_y0_loop
1750 Blind: Ns=Ns+1     ! THE RAY CROSSED AHEAD OF THE SECOND SURFACE
1760        X(Ns)=0     ! AND WAS NOT REFRACTED
1770        Y(Ns)=0
1780        Ns=Ns+1
1790        X(Ns)=0
1800        Y(Ns)=0
1810 Next_y0_loop: FIXED 2
1820               Nsp=Ns-1
1830                 IF (G$="Y") OR (G$="y") THEN Print_grin
1840                 IF Linecount>Linemax THEN GOSUB Header
1850               PRINT USING Image_data;X(Nsp-3);Y(Nsp-3);X(Nsp-2);Y(Nsp-2);X(
Nsp-1);Y(Nsp-1);Xc(Y0_loop);0;X(Nsp);Y(Nsp);Rho
1860               Linecount=Linecount+1
1870               GOTO Continue
1880 Print_grin:   IF Linecount>Linemax THEN GOSUB Header_grin
1890             PRINT USING Image_data_g;X(Nsp-3);Y(Nsp-3);X(Nsp-2);Y(Nsp-2);X(
Nsp-1);Y(Nsp-1);Xc(Y0_loop);0;N(Y0_loop);Rho
1900             Linecount=Linecount+1
1910 Continue: NEXT Y0_loop
1920  !
1930  '
1940 Image_data: IMAGE X(2(D.DD,X,DD.DD,3X),DD.DD,X,DD.DD,1X,6D.DD,X,D.DD,3X,3D.
DD,X,4D.DD,3X,4D.DD)
1950 ! Image_data: IMAGE D,2X,DD.DD,3X,DD.DD,2X,DD.DD,3X,DD.DD,2X,DDD.DD,3X,5D.2
```

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D,2X,D,3X,DD.DD,2X,5D.DD,3X,4D.DD  ! USE THIS FOR Y_BULLET ≥ 10 INCHES
1960  !
1970 Image_data_g: IMAGE 7X(2(D.DD,XD.DD,3X),DD.DD,X,DD.DD,3X,3D.DD,X,D.DD,3X,DD
.DD,3X,4D.DD)
1980  !
1990  !
2000             GOSUB Header_end
2010             GOSUB Yhard_end
2020           PRINT LIN(2),"FINISHED"
2030           BEEP
2040           WAIT 250
2050           BEEP
2060             IF (G$="Y") OR (G$="y") THEN Graph_grin
2070        Ray_trace$="N"
2080        INPUT "DO YOU WANT TO PLOT THE RESULTS OF THE TRACE ? Y/N",Ray_trace
$
2090          IF (Ray_trace$="N") OR (Ray_trace$="n") THEN Rerun
2100        INPUT "ON WHICH DEVICE: CRT (C) OR THE 9872 (P) ? P/C",Plot$
2110          IF (Plot$="C") OR (Plot$="c") THEN Graph
2120        INPUT "PLEASE ENTER THE SELECT CODE OF THE GRAPHICS DEVICE (DEFAULT
= 7 )",Pselect
2130        INPUT "PLEASE ENTER THE HPIB ADDRESS OF THE GRAPHICS DEVICE (DEFAULT
 = 5 )",Hpib
2140 Graph:   CALL Graph
2150        Y$="N"
2160        INPUT "DO YOU WANT ANOTHER PLOT DRAWN TO A DIFFERENT SCALE ? Y/N",Y$
2170          IF (Y$="Y") OR (Y$="y") THEN Graph
2180 Density: PRINTER IS 16
2190          PRINT PAGE,"  The point of maximum ray density is determined by yo
u visually."
2200          PRINT "by placing the cross-hairs (they will appear automatically)
"
2210          PRINT "over the position of maximum density."
2220          PRINT "   This is accomplished by using the DISPLAY controls (up
,down,<-,->)."
2230          PRINT "When you get cursor in the area of interest, use the SHIFT
 button"
2240          PRINT "with the DISPLAY controls (both shift and display control
s"
2250          PRINT "should be depressed simultaneously) for fine positioing."
2260          PRINT LIN(2),"   The position of the vertical hair is critical,"
2270          PRINT "because its location is used for the position of the "
2280          PRINT "image plane (Z3) on the GLM axis."
2290          PRINT LIN(2),"CAUTION : Do not let the image plane intercept any
 rays"
2300          PRINT "in the interior of the lens. If this is done, those rays"
2310          PRINT "will be included in the histogram."
2320          PRINT "PRESS CONT"
2330          PAUSE
2340          PRINT PAGE,"   The histogram is an illustration of the density"
2350          PRINT "of the rays that intercept the image plane versus radial "
2360          PRINT "distance from the GLM axis."
2370          PRINT LIN(2),"   When the position has been located, PRESS CONT."
2380          PAUSE
2390          Digitize$="Y"
2400            CALL Graph
2410            IF Plot$="P" THEN 2460  ! P FOR HP-9872 PLOTTER
2420          Dump_crt$="N"
2430          INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
2440            IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
2450          DISP "WORKIN' ON THE HISTOGRAM..."
2460            CALL Density        ! PRODUSE A HISTOGRAM OF THE RAY DENSITY
2470            IF Plot$="P" THEN 2520
2480          Digitize$="N"    ! RESET CONDITION
2490          Dump_crt$="N"
2500          INPUT "DO YOU WANT A HARD COPY OF HISTOGRAM ? Y/N",Dump_crt$
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2510            IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
2520          Y$="N"
2530          INPUT "DO YOU WANT TO REDRAW THE HISTOGRAM FOR A DIFFERENT IMAGE P
LANE ? Y/N",Y$
2540            IF (Y$="Y") OR (Y$="y") THEN Z3=50
2550            IF (Y$="Y") OR (Y$="y") THEN Graph
2560          Digitize$="N"          ! RESET CONDITION
2570 Rerun: PRINTER IS 16
2580        Y$="N"
2590        INPUT "ARE YOU GOING TO MAKE ANY MORE RUNS? Y/N",Y$
2600           IF (Y$="N") OR (Y$="n") THEN Finished
2610        Change$="N"
2620        INPUT "ARE YOU GOING TO CHANGE ANY PARAMETERS (n2,n3,Alpha,Nray, OR
RHO-INITIAL)? Y/N",Change$
2630          IF (Change$="N") OR (Change$="n") THEN Gymnastics
2640        GOSUB Printer
2650        PRINT "  If you do NOT want to change a particular parameter"
2660        PRINT "PRESS CONT in respose to the prompt."
2670        INPUT "WHAT IS THE NEW VALUE OF RHO-INITIAL ?",Rho_initial
2680        Rho_initial=-1*ABS(Rho_initial)
2690        Trhoi=TAN(Rho_initial)
2700        INPUT "WHAT IS THE NEW VALUE OF ALPHA?",Alpha
2710        Talpha=TAN(Alpha)
2720        INPUT "WHAT IS THE NEW VALUE OF Y_BULLET ?",Y_bullet
2730        INPUT "WHAT IS THE NEW VALUE OF Ya ?",Ya
2740        INPUT "WHAT IS THE NEW VALUE OF Yb ?",Yb
2750        Add_ray=0        ! RESET CONDITION
2760          IF Ya>=Yb THEN BEEP
2770          IF Ya>=Yb THEN DISP "Ya MUST BE < Yb. PLEASE RE-ENTER Ya AND Yb.
"
2780          IF Ya>=Yb THEN WAIT 2500
2790          IF Ya>=Yb THEN 2730
2800          IF Yb>Y_bullet THEN BEEP
2810          IF Yb>Y_bullet THEN DISP "Yb MUST BE < Y_bullet. PLEASE RE-ENT
ER."
2820          IF Yb>Y_bullet THEN WAIT 2500
2830          IF Yb>Y_bullet THEN 2720
2840        Aperture=Yb-Ya
2850          IF Ya<>0 THEN Add_ray=1
2860        INPUT "WHAT IS THE NEW VALUE OF n2?",N2
2870        INPUT "WHAT IS THE NEW VALUE OF n3?",N3
2880        INPUT "WHAT IS THE NEW VALUE OF Z3 ?",Z3
2890        INPUT "WHAT IS THE NEW VALUE OF THE No. OF RAYS?",Nray
2900        INPUT "WHAT IS THE NEW No. OF INCREMENTS (1000 MAX ) ?",N_increment
2910        Change$="Y"
2920     GOTO Gymnastics
2930 Graph_grin: J=1
2940             FOR I=1 TO Nray
2950               R(I)=SQR(X(J)^2+Y(J)^2)
2960               C(I)=N(I)*R(I)*SIN(Phi(I))
2970               J=J+5
2980             NEXT I
2990             Y$="N"
3000             INPUT "DO YOU WANT TO PRINT A TABLE OF GRIN AND THE GRIN CONSTA
NT ? Y/N",Y$
3010               IF (Y$="N") OR (Y$="n") THEN 3150
3020           Y_hard$="N"
3030           INPUT "DO YOU WANT A HARD COPY OF THE TABLE ? Y/N",Y_hard$
3040             GOSUB Yhard
3050             GOSUB Header_c
3060           FOR I=1 TO Nray
3070               IF Linecount>Linemax THEN GOSUB Header_c
3080             PRINT USING 3090;I,N(I),R(I),Phi(I),SIN(Phi(I)),C(I)
3090             IMAGE 3X,4D,5(5X,4D.4D)
3100             Linecount=Linecount+1
3110           NEXT I
3120             GOSUB Header_end
```

```
3130                GOSUB Yhard_end
3140              Y$="N"
3150              INPUT "DO YOU WANT TO PLOT THE RESULTS OF THE TRACE ? Y/N",Y$
3160                IF (Y$="N") OR (Y$="n") THEN Grinc
3170              INPUT "ON WHICH DEVICE: CRT (C) OR THE 9872 (P) ? C/P ",Plot$
3180                CALL Graph
3190              Y$="N"
3200              INPUT "DO YOU WANT ANOTHER PLOT DRAWN TO A DIFFERENT SCALE ? Y/N"
,Y$
3210                IF (Y$="Y") OR (Y$="y") THEN CALL Graph
3220              Dump_crt$="N"
3230              INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
3240                IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
3250 Grinc: Grinc$="N"
3260           INPUT "DO YOU WANT A GRAPH OF THE GRIN CONSTANT C(r) vs r ? Y/N",Gri
nc$
3270             IF (Grinc$="N") OR (Grinc$="n") THEN Rerun_grin
3280           INPUT "ON WHICH DEVICE: CRT (C) OR THE 9872 (P) ? C/P ",Plot$
3290 Grinc1: Grinc$="Y"
3300             CALL Graph
3310            Grinc$="N"        ! RESET CONDITION
3320           Dump_crt$="N"
3330              INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
3340             IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Printer
3350           Y$="N"
3360           INPUT "DO YOU WANT ANOTHER PLOT DRAWN ? Y/N",Y$
3370             IF (Y$="Y") OR (Y$="y") THEN Grinc1
3380 Rerun_grin: Y$="N"
3390                G$="Y"        ! INSURANCE
3400                INPUT "ARE YOU GOING TO MAKE ANOTHER RUN ? Y/N",Y$
3410                  IF (Y$="N") OR (Y$="n") THEN Finished
3420                Change$="N"
3430                INPUT "ARE YOU GOING TO CHANGE ANY PARAMETERS (n2,n3,Alpha,Nray
, OR RHO-INITIAL)? Y/N",Change$
3440                  IF (Change$="N") OR (Change$="n") THEN Gymnastics
3450                GOSUB Printer
3460                PRINT "  If you do NOT want to change a particular parameter"
3470                PRINT "PRESS CONT in respose to the prompt."
3480                Add_ray=0    ! RESET CONDITION
3490                INPUT "WHAT IS THE NEW VALUE OF RHO-INITIAL ?",Rho_initial
3500                Rho_initial=-1*ABS(Rho_initial)
3510                Trhoi=TAN(Rho_initial)
3520                INPUT "WHAT IS THE NEW VALUE OF ALPHA?",Alpha
3530                INPUT "WHAT IS THE NEW VALUE OF Nmin ?",Nmin
3540                  IF (Grin$="C") OR (Grin$="c") THEN 3570
3550                INPUT "WHAT IS THE NEW VALUE OF Nmax ?",Nmax
3560                INPUT "WHAT IS THE NEW VALUE OF R0_max ?",R0_max
3570                INPUT "WHAT IS THE NEW VALUE OF n3?",N3
3580                INPUT "WHAT IS THE NEW VALUE OF THE No. OF RAYS?",Nray
3590                INPUT "WHAT IS THE NEW VALUE OF Ya ?",Ya
3600                INPUT "WHAT IS THE NEW VALUE OF Yb ?",Yb
3610                  IF Ya>=Yb THEN BEEP
3620                  IF Ya>=Yb THEN DISP "Ya MUST BE < Yb. PLEASE ENTER AGAIN"
3630                  IF Ya>=Yb THEN WAIT 2500
3640                  IF Ya>=Yb THEN 3590
3650                Aperture=Yb-Ya
3660                  IF Ya<>0 THEN Add_ray=Add_ray+1
3670                Change$="Y"
3680                  GOSUB Printer
3690                Change$="N"          ! RESET CONDITION
3700         GOTO Gymnastics
3710 Printer:    IF (Change$="Y") OR (Change$="y") THEN Y_hard$="N"
3720             IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN Y_hard$="Y"
3730             IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINTER IS 0
3740             IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINT CHR$(27)&"&l00T"
3750             IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN Crt
3760             GOSUB Header_initial
```

```
3770            IF (G$="Y") OR (G$="y") THEN Grin1
3780          FIXED 0
3790            IF Linecount>Linemax THEN GOSUB Header_initial
3800            IF Date$="" THEN 3830
3810          PRINT "Curve No: ";Curve,TAB(48);"Date: ";Date$
3820          GOTO 3840
3830          PRINT "Curve No: ";Curve
3840          Linecount=Linecount+1
3850 Grin1:   FIXED 2
3860            IF Linecount>Linemax THEN GOSUB Header_initial
3870          PRINT LIN(1),"RHO(INITIAL) = ";Rho_initial;"degrees";TAB(48);"Alph
a = ";Alpha;"degrees"
3880          Linecount=Linecount+1
3890            IF Linecount>Linemax THEN GOSUB Header_initial
3900          PRINT "TAN(RHO-INITIAL) = ";Trhoi;TAB(48);"Tan(Alpha) = ";Talpha
3910          Linecount=Linecount+1
3920          FIXED 5
3930            IF (G$="N") OR (G$="n") THEN Grin2
3940            IF Linecount>Linemax THEN GOSUB Header_initial
3950          PRINT LIN(1),"n1 = ";N1;TAB(48);"n3 = ";N3
3960            IF (Grin$="C") OR (Grin$="c") THEN 4010
3970            IF Linecount>Linemax THEN GOSUB Header_initial
3980          PRINT LIN(1),"Nmin = ";Nmin;TAB(48);"Nmax = ";Nmax
3990          Linecount=Linecount+1
4000          GOTO 4040
4010            IF Linecount>Linemax THEN GOSUB Header_initial
4020          PRINT LIN(1),"Nmin = ";Nmin
4030          Linecount=Linecount+1
4040            IF Linecount>Linemax THEN GOSUB Header_initial
4050          PRINT
4060          Linecount=Linecount+1
4070          FOR I=1 TO Nray STEP 3
4080              IF Linecount>Linemax THEN GOSUB Header_initial
4090              IF I>=250 THEN PRINT USING 4140;I,N(I)
4100              IF I>=250 THEN 4120
4110            PRINT USING 4130;I,N(I),I+1,N(I+1),I+2,N(I+2)
4120            Linecount=Linecount+1
4130            IMAGE 3(5X,"n2("3D") = ",2D.5D)
4140            IMAGE 5X,"n2("3D") = ",2D.5D
4150          NEXT I
4160            IF Linecount>Linemax THEN GOSUB Header_initial
4170          PRINT LIN(1),"R0_max = ";R0_max;" inches"
4180          Linecount=Linecount+1
4190            GOTO Grin3
4200 Grin2:     IF Linecount>Linemax THEN GOSUB Header_initial
4210          PRINT LIN(1),"n1 = ";N1;TAB(24);"n2 = ";N2;TAB(48);"n3 = ";N3
4220          Linecount=Linecount+1
4230 Grin3:   FIXED 3
4240            IF Linecount>Linemax THEN GOSUB Header_initial
4250          PRINT LIN(1),"Aperture = ";Aperture;TAB(24);"Ya = ";Ya;" inch ";TA
B(48);"Yb = ";Yb;" inch "
4260            IF (G$="Y") OR (G$="y") THEN Grin4
4270          Linecount=Linecount+1
4280          FIXED 2
4290            IF Linecount>Linemax THEN GOSUB Header_initial
4300          PRINT LIN(1),"Z3 = ";Z3;"inches";TAB(48);"Z4 = ";Z4;" inches"
4310          Linecount=Linecount+1
4320 Grin4:   FIXED 0
4330            IF Linecount>Linemax THEN GOSUB Header_initial
4340          PRINT LIN(1),"Number of Rays = ";Nray
4350          Linecount=Linecount+1
4360            IF (G$="Y") OR (G$="y") THEN Grin5
4370            IF Linecount>Linemax THEN GOSUB Header_initial
4380          PRINT LIN(1),"Number of Increments = ";N_increment
4390          Linecount=Linecount+1
4400 Grin5:   FIXED 2
4410            IF Linecount>Linemax THEN GOSUB Header_initial
```

```
4420          PRINT
4430          Linecount=Linecount+1
4440            GOSUB Header_end
4450 Crt:       IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN DUMP GRAPHICS
4460            IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINT CHR$(27)&"&136T"
4470          Change$="N"          ! RESET CONDITION
4480          Dump_crt$="N"
4490          Y_hard$="N"
4500        RETURN
4510 Yhard: IF (Y_hard$="Y") OR (Y_hard$="y") THEN Flag$="1"
4520        IF (Y_hard$="N") OR (Y_hard$="n") THEN Flag$="0"
4530        IF (Y_hard$="Y") OR (Y_hard$="y") THEN PRINTER IS 0
4540        IF Flag$="1" THEN PRINT CHR$(27)&"&100T"
4550      RETURN
4560 Yhard_end: IF (Flag$="1") OR (Flag$="0") THEN PRINT CHR$(27)&"&136T"
4570            IF (Flag$="1") OR (Flag$="0") THEN PRINTER IS 16
4580          RETURN
4590 Header_coef:   IF Linecount>Linemax THEN GOSUB Header_coef_end
4600              Linecount=0
4610              PRINT PAGE;LIN(5)
4620                GOSUB Char_short
4630              PRINT LIN(1);SPA(6);"COEFFICIENTS";LIN(1)
4640                GOSUB Char_short
4650              PRINT LIN(2)
4660            RETURN
4670 Header_coef_end: PRINT LIN(2)
4680                    GOSUB Char_short
4690               RETURN
4700 Header_c:   IF Linecount>Linemax THEN GOSUB Header_end
4710           Linecount=0
4720           PRINT PAGE;LIN(5)
4730             GOSUB Char
4740           PRINT LIN(1),"      I          n(I)          r(I)         Phi(I)
     Sin(Phi)        c(I)";LIN(1)
4750                   DISP "      I          n(I)          r(I)         Phi(I)
     Sin(Phi)        c(I)"
4760             GOSUB Char
4770           PRINT LIN(2)
4780         RETURN
4790 Header_c_end:   GOSUB Header_end
4800             RETURN
4810 Header_initial:   IF Linecount>Linemax THEN GOSUB Header_end
4820                 Linecount=0
4830                 PRINT PAGE;LIN(5);TAB(28);"INITIAL PARAMETER VALUES";LIN(1)
4840                   GOSUB Char
4850                 PRINT LIN(2)
4860               RETURN
4870 Char_short: Char_max=25
4880             FOR Z=0 TO Char_max
4890                 IF Z=Char_max THEN PRINT CHR$(228)
4900                 IF Z=Char_max THEN 4920
4910               PRINT CHR$(228);
4920             NEXT Z
4930          RETURN
4940 Header:   IF Linecount>Linemax THEN GOSUB Header_end
4950         Linecount=0
4960         PRINT PAGE,LIN(5)
4970           GOSUB Char
4980         PRINT LIN(1),"    (X0,Y0)      (X1,Y1)      (X2,Y2)        (Xc,Yc)
     (X3,Y3)         RHO";LIN(1)
4990           GOSUB Char
5000         PRINT LIN(2)
5010                 DISP "    (X0,Y0)      (X1,Y1)      (X2,Y2)        (Xc,Yc)
     (X3,Y3)         RHO"
5020     RETURN
5030 Header_grin:   IF Linecount>Linemax THEN GOSUB Header_end
5040              Linecount=0
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```
5050              PRINT PAGE,LIN(5)
5060                GOSUB Char
5070              PRINT LIN(1),"       (X0,Y0)     (X1,Y1)       (X3,Y3)
 (Xc,Yc)     N2       RHO";LIN(1)
5080                      DISP " .     (X0,Y0)     (X1,Y1)       (X3,Y3)
 (Xc,Yc)     N2       RHO"
5090                GOSUB Char
5100              PRINT LIN(2)
5110           RETURN
5120 Header_end: PRINT LIN(2)
5130               GOSUB Char
5140           RETURN
5150 Char: FOR Zz=0 TO 79
5160           IF Zz=79 THEN PRINT CHR$(228)
5170           IF Zz=79 THEN 5190
5180         PRINT CHR$(228);
5190       NEXT Zz
5200   !   PRINT RPT$("=",80)     ! THIS IS A LOT FASTER, BUT NOT AS PRETTY
5210     RETURN
5220 Finished: DISP "FINISHED"
5230      END
5240 SUB Slope(Y)
5250   OPTION BASE 0
5260   !
5270   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
5280   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
5290   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
5300   !
5310   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
5320   !
5330  DEG
5340    IF Surf_no=2 THEN Two
5350 One: Alpha2=Alpha
5360      Norm=Alpha2-90
5370   SUBEXIT
5380 Two: IF Y<0 THEN Negative
5390 Positive: Tan_m2=0             ! SLOPE WRT Y-AXIS  [dx/dy]
5400           FOR I=1 TO Np
5410             Tan_m2=Tan_m2+I*Apos(I)*Y^(I-1)
5420           NEXT I
5430           M2=ABS(ATN(Tan_m2))  ! THE ANGLE IS POSITIVE CW WRT Y-AXIS
5440           Alpha2=90-M2  ! THE ANGLE IS POSITIVE CCW WRT X-AXIS
5450           Norm=Alpha2-90
5460      SUBEXIT
5470 Negative: Tan_m2=0
5480           FOR I=1 TO Nn
5490             Tan_m2=Tan_m2+I*Aneg(I)*Y^(I-1)
5500           NEXT I
5510           M2=ABS(ATN(Tan_m2))
5520           Alpha2=90-M2
5530           Tnorm=-1/TAN(Alpha2)
5540           Norm=ATN(Tnorm)-180
5550         SUBEND
5560 SUB Snell(X,Y)
5570   OPTION BASE 0
5580   !
5590   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
5600   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
5610   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
5620   !
5630   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
5640   COM INTEGER Linecount,Linemax
5650   COM INTEGER Pselect,Hpib
5660   !
5670   COM REAL Nmin,Nmax,R0_max
5680   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
5690   COM X(*),Y(*),Xc(*),N(*),R(*)
```

```
5700   !
5710  DEG
5720     IF Surf_no=2 THEN Two
5730 One: Nn1=N1
5740      Nn2=N2
5750      Thetai=90-(Alpha+ABS(Rho))
5760        CALL Slope(Y)
5770        IF Thetai<0 THEN Neg_thetai
5780      GOTO Same
5790 Neg_thetai: Flag=5
5800              Thetai=ABS(Thetai)
5810              Sip=N1/N2*Sin(Thetai)
5820                IF ABS(Sip)>1 THEN Wrong
5830              Thetar=ASN(Sip)
5840              Rho=Norm-Thetar
5850                IF (ABS(Rho)<89.99999) OR (ABS(Rho)>90.00001) THEN Trho=TAN(R
ho)
5860                IF (ABS(Rho)>89.99999) AND (ABS(Rho)<=90) THEN Trho=-9999999
5870                IF (ABS(Rho)>90) AND (ABS(Rho)<90.00001) THEN Trho=9999999
5880           SUBEXIT
5890 Two: Nn1=N2
5900      Nn2=N3
5910        IF Y<0 THEN Negative
5920        CALL Slope(Y)
5930      Thetai=90-ABS(Rho)-Alpha2
5940        IF Thetai<0 THEN Neg_thetai2
5950 Same:     IF (G$="Y") OR (G$="y") THEN GOSUB Grin
5960       Sin_iprime=Nn1/Nn2*SIN(Thetai)
5970         IF ABS(Sin_iprime)>1 THEN Wrong
5980       Thetar=ASN(Sin_iprime)
5990       Rho=Alpha2+Thetar-90      ! USE ALPHA2 INSTEAD OF NORM BECAUSE THE
6000                                 ! SIGN OF THE ANGLE IS DESIRED
6010       Trho=TAN(Rho)
6020       Phi=Alpha-Rho       ! USED FOR GRIN
6030     SUBEXIT
6040 Neg_thetai2: Thetai=ABS(Thetai)
6050              Sip=Nn1/Nn2*SIN(Thetai)
6060                IF Abs(Sip)>1 THEN Wrong
6070              Thetar=ASN(Sip)
6080              Rho=Norm-Thetar
6090                IF ABS(Rho)>=90 THEN Skip   ! THE RAY WILL NOT INTERCEPT
6100                                            ! THE IMAGE PLANE
6110                IF ABS(Rho)<89.99999 THEN Trho=TAN(Rho)
6120                IF (ABS(Rho)>89.99999) AND (ABS(Rho)<90) THEN Trho=-9999999
6130           SUBEXIT
6140 Skip: Flag=15
6150        IF ABS(Rho)>=90.000001 THEN Trho=TAN(Rho)
6160        IF ABS(Rho)<90.000001 THEN Trho=9999999
6170       Rhop=180-ABS(Rho)
6180       Trhop=TAN(Rhop)
6190       Xc(Y0_loop)=X-Y/Trhop
6200    SUBEXIT
6210 Wrong: Flag=10  ! TOTAL INTERNAL REFLECTION AT THE SECOND SURFACE
6220     SUBEXIT
6230 Negative:   CALL Slope(Y)
6240           Thetai=90+ABS(Rho)-Alpha2
6250           Sip=Nn1/Nn2*SIN(Thetai)
6260             IF ABS(Sip)>1 THEN Wrong
6270           Thetar=ASN(Sip)
6280           Rho=Norm-Thetar
6290           Trho=TAN(Rho)
6300         SUBEXIT
6310 Grin: N2=0
6320        IF (Grin$="P") OR (Grin$="p") THEN P
6330 C:     CALL Index_c(X,Y)
6340  .    N(Y0_loop)=N2
```

```
6350         IF Surf_no=1 THEN Nn2=N2
6360         IF Surf_no=2 THEN Nn1=N2
6370       RETURN
6380 P:      CALL Index_p(X,Y)
6390       N(Y0_loop)=N2
6400         IF Surf_no=1 THEN Nn2=N2
6410         IF Surf_no=2 THEN Nn1=N2
6420       RETURN
6430 SUBEND
6440 SUB Binary_search(X,Y)
6450   OPTION BASE 0
6460   !
6470   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
6480   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
6490   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
6500   !
6510   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
6520   COM INTEGER Linecount,Linemax
6530   COM INTEGER Pselect,Hpib
6540   !
6550   COM REAL Nmin,Nmax,R0_max
6560   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
6570   COM X(*),Y(*),Xc(*),N(*),R(*)
6580   !
6590   COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
6600   !
6610  DEG
6620  DEF FNA(X,Y)=X-Y/Trho
6630  DEF FNAi(X,Y)=X-Y/Talpha
6640  DEF FNXn(A)=A-Y_bullet/Trho     ! FINDS THE INTERCEPT OF THE RAY AND Y=-Y_b
ullet
6650  R=1
6660    IF Surf_no=1 THEN One
6670    IF Surf_no=2 THEN Two
6680    IF Surf_no=3 THEN Three
6690 Four: Dy=(Z4-X)*Trho
6700       Y=Y+Dy
6710       X=Z4
6720     SUBEXIT
6730 Three:   IF (Y>0) AND (Trho>0) THEN Skip !THE RAY WILL NOT CROSS THE X-AXIS
6740        Xc(Y0_loop)=FNA(X,Y)
6750 Back: Dy=(Z3-X)*Trho
6760       Y=Y+Dy
6770       X=Z3
6780     SUBEXIT
6790 One:   CALL X1pos(X,Y)
6800     SUBEXIT
6810 Two: A=FNA(X,Y)
6820        IF Flag=5 THEN Search2  ! Theta I < 0 FROM SNELL
6830      Flag=0
6840      Flag2=0
6850        IF (G$="Y") OR (G$="y") THEN Grin
6860        IF A<Z1 THEN Negative  ! THIS IS A SEPARATE AND UNRELATED TEST
6870      GOTO Positive            ! FROM THAT PERFORMED IN SEARCH 2
6880 Search2: !   THIS ROUTINE IS ENTERED WHEN THE ANGLE OF INCIDENCE ON THE
6890          ! FIRST SURFACE IS < 0 DEGREES.
6900          !   THIS MEANS ThetaI HAS DECREASED FROM ITS AXIAL VALUE TO 0
6910          ! (ie. NORMAL TO THE FIRST SURFACE) THEN CONTINUED TO ROTATE PAST
6920          ! THE NORMAL. THIS CORRESPONDS TO A TARGET AT OR NEAR CLOSEST
6930          ! POINT OF APPROACH (CPA).
6940          !   WHEN THIS HAPPENS RHO(1) CAN BE LARGE ENOUGH TO CAUSE THE RAY
6950          ! TO MISS THE FIRST AND SECOND SURFACE.
6960          !   THE ROUTINE FIRST CHECKS TO SEE IF THE RAY CROSSES THE SECOND
6970          ! SURFACE. IF SO, CONTROL SWITCHES TO FASTER BISECTION ROUTINE.
6980          ! IF NOT THEN THE ROUTINE MARCHES SLOWLY DOWN THE RAY TO FIND THE
6990          ! INTERCEPT (IF ANY).
7000 !
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```
7010           IF A>Z1 THEN Positive
7020 !
7030          Delta=.01
7040          X12=X
7050          Y12=Y
7060 !    EVEN THOUGH THE X-INTERCEPT APPEARS TO BE < Z1, THE SECOND SURFACE
7070 ! COULD BE IRREGULAR ENOUGH TO CAUSE THIS SIMPLE CHECK TO BE DECEIVING.
7080 !    THE RAY IS ASSUMED TO HAVE INTERCEPTED THE SECOND SURFACE IF THE
7090 ! DIFFERENCE BETWEEN THE RAY POSITION AND THE SECOND SURFACE < TOLERANCE
7100 March: Deltax=Delta*COS(Rho)
7110        Deltay=Delta*SIN(Rho)
7120        X12=X12+Deltax
7130        Y12=Y12+Deltay
7140          IF Y12<0 THEN Missed   ! YOU HAVE REACHED THE X-AXIS
7150        X2=0
7160          CALL X2pos(X2,Y12)
7170        Xsave=X12-X2
7180          IF ABS(Xave)<=Tol THEN Done1   ! FOUND THE SURFACE WITHIN TOL
7190          IF Xsave<0 THEN March          ! KEEP LOOKING
7200          IF Xsave>0 THEN Found_it       ! THE RAY CROSSED X2POS
7210 Done1: X=(X2+X12)/2
7220        Y=Y12
7230      SUBEXIT
7240 Missed: X=A        ! THE RAY HAS MISSED THE POS AND NEG SURFACES
7250         Y=0        ! DRAW THE RAY TO THE X-AXIS
7260         GOTO Blind
7270 Found_it: Xmid=X12      ! THE RAY HAS CROSSED THE SECOND SURFACE. GO TO THE
7280           Xlast=Xmid    ! BISECTION ROUTINE FOR SPEED
7290           Ymid=Y12
7300           Ylast=Ymid
7310           Xsave=0
7320           GOTO Recurse_p
7330 Positive: X1=0
7340           X2=0
7350             CALL X1pos(X1,Y)
7360             CALL X2pos(X2,Y)
7370             IF X2<=X1 THEN Woops
7380           Xmid=A       !   THE RAY HAS CROSSED THE X-AXIS BEHIND THE SECOND
7390           Xmid=A       !   THE RAY HAS CROSSED THE X-AXIS BEHIND THE SECOND
7400           Xlast=Xmid   ! SURFACE. THEREFORE Y>0 AT THE INTERCEPT
7410           Ymid=0
7420           Ylast=Ymid
7430           Xsave=0
7440 Recurse_p: Xmid=(X+R*Xmid)/(1+R)
7450            Ymid=(Y+R*Ymid)/(1+R)
7460            X2=0
7470              CALL X2pos(X2,Ymid)
7480            Xsave=X2-Xmid
7490              IF Xsave<0 THEN First      ! INTERCEPT IS BEHIND THE MIDPOINT
7500              IF Xsave>0 THEN Second     ! INTERCEPT IS AHEAD OF THE MIDPOINT
7510              IF Xsave=0 THEN Done       ! INTERCEPT IS THE MIDPOINT
7520 Recurse_n: Xmid=(X+R*Xmid)/(1+R)
7530            Ymid=(Y+R*Ymid)/(1+R)
7540            X2=0
7550              CALL X2neg(X2,Ymid)
7560            Xsave=X2-Xmid
7570              IF Xsave<0 THEN First      ! INTERCEPT IS BEHIND THE MIDPOINT
7580              IF Xsave>0 THEN Second     ! INTERCEPT IS AHEAD OF THE MIDPOINT
7590              IF Xsave=0 THEN Done       ! INTERCEPT IS THE MIDPOINT
7600 First:   IF ABS(Xsave)<Tol THEN Done
7610        Xlast=Xmid
7620        Ylast=Ymid
7630          IF Flag=1 THEN Recurse_n
7640        GOTO Recurse_p
7650 Second:   IF ABS(Xsave)<Tol THEN Done
7660         X=Xmid
7670         Y=Ymid
```

```
7680         Xmid=Xlast
7690         Ymid=Ylast
7700           IF Flag=1 THEN Recurse_n
7710         GOTO Recurse_p
7720 Negative: X1=0
7730           X2=0
7740           Xc(Y0_loop)=A
7750             CALL X1neg(X1,Y)
7760             CALL X2neg(X2,Y)
7770             IF X2<=X1 THEN Woops
7780           Flag=1        !   THE RAY CROSSES THE X-AXIS AHEAD OF THE SECOND
7790           Y11=-Y_bullet ! SURFACE AND COULD POSSIBLY INTERCEPT X2NEG.
7800           X=A           !   THE SAME TYPE OF TESTS DESCRIBED ABOVE
7810                         ! IN SEARCH 2 ARE PERFORMED HERE.
7820           Y=0
7830           Delta=.01
7840 Search: X11=FNXn(A)
7850         X2=0
7860           CALL X2neg(X2,Y11)
7870           IF X11>X2 THEN Ok    ! X11 > X2max
7880         X11=A
7890         Y11=0
7900         Deltay=Delta*SIN(Rho)
7910         Deltax=Delta*COS(Rho)
7920 Search_n:   IF ABS(Y11)>=Y_bullet THEN Missed2  ! THE SECOND SURFACE WAS MI
SSED
7930           X11=X11+Deltax
7940           Y11=Y11+Deltay
7950           X2=0
7960             CALL X2neg(X2,Y11)
7970           Xsave=X11-X2
7980             IF ABS(Xsave)<=Tol THEN Victory  ! FOUND THE SURFACE
7990             IF Xsave>=0 THEN Ok              !    "    "    "
8000             IF Xsave<0 THEN Search_n    ! KEEP LOOKING
8010 Missed2: X=FNXn(A)    ! DRAW THE RAY UNTIL Y= -Y_bullet
8020          Y=-Y_bullet
8030          GOTO Blind
8040 Victory: Flag2=1
8050          X=(X11+X2)/2
8060          Y=Y11
8070        SUBEXIT
8080 Ok: Flag2=1         !   THE INTERCEPT OF THE RAY AND X2NEG HAS BEEN FOUND.
8090     Xmid=X11        ! GO TO THE BISECTION ROUTINE FOR SPEED.
8100     Xlast=Xmid
8110     Ymid=Y11
8120     Ylast=Ymid
8130     GOTO Recurse_n
8140 Done: X=(X2+Xmid)/2
8150       Y=Ymid
8160     SUBEXIT
8170 Skip: Xc(Y0_loop)=0
8180       GOTO Back
8190 Blind:   IF Flag=1 THEN Blind_n
8200        Flag=20
8210        X=A
8220        Y=0
8230        Xc(Y0_loop)=A
8240      SUBEXIT
8250 Blind_n: Flag=20
8260          Xc(Y0_loop)=A
8270        SUBEXIT
8280 Woops:   Flag=30       ! THE SECOND SURFACE HAS BENT AND CROSSED THE FIRST
8290        X=X2          ! STOP THE RAY TRACE
8300        Y=Y
8310        SUBEXIT
8320 Grin: Ai=Z3
8330       Y=(A-Ai)*(Talpha*Trho)/(Trho-Talpha)  ! INTERCEPT OF TWO LINES
```

```
8340       X=A+Y/Trho
8350       Xc(Y0_loop)=A   ! REQUIRED FOR DRAWING PURPOSES
8360     SUBEND
8370 SUB Graph
8380   OPTION BASE 0
8390   !
8400   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
8410   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
8420   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
8430   !
8440   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
8450   COM INTEGER Linecount,Linemax
8460   COM INTEGER Pselect,Hpib
8470   !
8480   COM REAL Nmin,Nmax,R0_max
8490   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
8500   COM X(*),Y(*),Xc(*),N(*),R(*)
8510   !
8520   COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
8530   !
8540   !
8550  DEG
8560  PRINTER IS 16
8570    IF (Grinc$="Y") OR (Grinc$="y") THEN Grinc
8580    IF (Digitize$="Y") OR (Digitize$="y") THEN Digit
8590 Draw_axes:     CALL Plot
8600 First_surf: FIXED 2
8610             LINE TYPE 8
8620             Y=Y_bullet
8630             MOVE 0,0
8640             X=Y/Talpha
8650             DRAW X,Y
8660             DRAW 100,Y
8670             MOVE 0,0
8680             DRAW X,-Y
8690             DRAW 100,-Y
8700               IF (G$="Y") OR (G$="y") THEN Grin_image
8710 X2pos: MOVE Apos(0),0
8720        FOR Y=0 TO Y_bullet STEP .01*Y_bullet
8730          X2pos=0
8740            FOR I=0 TO Np
8750              X2pos=X2pos+Apos(I)*Y^I
8760            NEXT I
8770          DRAW X2pos,Y
8780        NEXT Y
8790 X2neg: MOVE Apos(0),0
8800        FOR Y=0 TO -Y_bullet STEP -.01*Y_bullet
8810          X2neg=0
8820            FOR I=0 TO Nn
8830              X2neg=X2neg+Aneg(I)*Y^I
8840            NEXT I
8850          DRAW X2neg,Y
8860        NEXT Y
8870        EXIT GRAPHICS
8880        Y$="N"
8890        INPUT "DO YOU WANT A HARD COPY OF THE PLOT WITHOUT RAYS ? Y/N",Y$
8900          IF (Y$="Y") OR (Y$="y") THEN GOSUB Dump_it
8910        GRAPHICS
8920 First_image: LINE TYPE 5
8930              MOVE Z3,Y_bullet
8940              DRAW Z3,-Y_bullet
8950 Second_image: LINE TYPE 6
8960               MOVE Z4,Y_bullet
8970               DRAW Z4,-Y_bullet
8980                 IF (G$="N") OR (G$="n") THEN First_
8990 Grin_image: LINE TYPE 7
9000             Ym=-Y_bullet+.5*Y_bullet
```

```
9010             Xm=Z3+Ym/Talpha
9020               MOVE Xm,Ym
9030             Ym=-Ym
9040             Xm=Z3+Ym/Talpha
9050               DRAW Xm,Ym
9060 REM DRAW THE RAYS
9070 First_:  LINE TYPE 1
9080          CLIP 0,100,-Y_bullet,Y_bullet
9090          I=0
9100          FOR Ray_first=1 TO Nray
9110            MOVE X(I),Y(I)
9120            DRAW X(I+1),Y(I+1)
9130            I=I+5
9140          NEXT Ray_first
9150 Second_: I=1
9160          FOR Ray_second=1 TO Nray
9170            MOVE X(I),Y(I)
9180            DRAW X(I+1),Y(I+1)
9190            I=I+5
9200          NEXT Ray_second
9210   IF (G$="Y") OR (G$="y") THEN Wait_
9220 First_image_p: I=2
9230                Nr=-1
9240                FOR Ray_image=1 TO Nray
9250                  Nr=Nr+3
9260                  MOVE X(I),Y(I)
9270                    IF X(I)<>0 THEN Ok
9280                  I=I+5
9290                  GOTO Next_ray_image
9300 Ok:                IF X(I+1)=0 THEN Reflect
9310                  DRAW X(I+1),Y(I+1)
9320                  I=I+5
9330                  GOTO Next_ray_image
9340 Reflect: ! IF THE ANSWERE TO THE QUESTION IS NO, THE RAY WAS TOTALLY
9350          ! REFLECTED AND DO NOT DRAW A LINE
9360          !  IF A LINE IS DRAWN, THEN ABS(RHO) ≥ 90 DEGREES
9370                    IF (Xc(Ray_image)=0) OR (Rho(Nr)=0) THEN 9400
9380                  Trho=-1*Y(I)/(Xc(Ray_image)-X(I))
9390                  DRAW X(I)-(Y(I)+Y_bullet)/Trho,-Y_bullet   ! RHO ≥  90 DE
GREES
9400                  I=I+5
9410 Next_ray_image: NEXT Ray_image
9420 Decision: WAIT 2000
9430           EXIT GRAPHICS
9440           BEEP
9450           Y$="N"
9460           INPUT "DO YOU WANT TO PLOT TO THE SECOND IMAGE PLANE ? Y/N",Y$
9470           GRAPHICS
9480             IF (Y$="N") OR (Y$="n") THEN Wait_
9490 Second_image_p: I=3
9500                 Nr=-1
9510                 FOR Ray_image2=1 TO Nray
9520                   Nr=Nr+3
9530                   MOVE X(I),Y(I)
9540                     IF X(I)<>0 THEN Ok2
9550                   I=I+5
9560                   GOTO Next_ray_image2
9570 Ok2:                IF X(I+1)=0 THEN Reflect2
9580                   DRAW X(I+1),Y(I+1)
9590                   I=I+5
9600                   GOTO Next_ray_image2
9610 Reflect2:           IF (Xc(Ray_image2)=0) OR (Rho(Nr)=0) THEN 9640
9620                  Trho=-1*Y(I)/(Xc(Ray_image2)-X(I))
9630                  DRAW X(I)-(Y(I)+Y_bullet)/Trho,-Y_bullet   ! RHO ≥  90 DE
GREES
9640                  I=I+5
9650 Next_ray_image2: NEXT Ray_image2
```

```
9660                  GOTO Wait_
9670 Grinc:   CALL Plot
9680  !     R1=C(1)/(N(1)*SIN(Phi(1)))
9690  !       MOVE R1,C(1)
9700          MOVE R(1),C(1)
9710        FOR I=2 TO Nray
9720    !     R=C(I)/(N(I)*SIN(Phi(I)))
9730    !     DRAW R,C(I)
9740          DRAW R(I),C(I)
9750        NEXT I
9760 Wait_: ! SETGU                     !    THIS ROUTINE PRESENTS THE PLOT TO THE
9770   !    LINE TYPE 1               ! OPERATOR TO STUDY UNTIL READY TO CONTINUE
9780   !    LDIR 0
9790   !    MOVE 2,2
9800   !    CSIZE 2
9810   !    LABEL "PRESS CONT"
9820   !    CSIZE 15/4.54
9830        SETUU
9840        BEEP
9850        WAIT 250
9860        BEEP
9870        PAUSE
9880        EXIT GRAPHICS
9890      SUBEXIT
9900 Digit: GRAPHICS
9910        POINTER Z3,0
9920        DIGITIZE Z3,Y
9930        LINE TYPE 5
9940          MOVE Z3,Y_bullet
9950          DRAW Z3,-Y_bullet
9960        SETGU
9970        LDIR 0
9980        LINE TYPE 1
9990        LORG 5
10000       CSIZE 2.5
10010       Centerx=72.5    ! CHANGE IN Laxes also
10020         MOVE 1.5*Centerx,5
10030       LABEL USING 10040;Z3
10040       IMAGE "Image Plane : ",DD.DD," inches"
10050       SETUU
10060       CSIZE 15/4.54
10070       WAIT 2500
10080       EXIT GRAPHICS
10090    SUBEXIT
10100 Dump_it:PRINTER IS 0
10110         PRINT CHR$(27)&"&100T"
10120         DUMP GRAPHICS
10130         PRINT CHR$(27)&"&136T"
10140         PRINTER IS 16
10150       RETURN
10160 SUBEND
10170 SUB Plot
10180  OPTION BASE 0
10190  !
10200  COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
10210  COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
10220  COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
10230  !
10240  COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_total
10250  COM INTEGER Linecount,Linemax
10260  COM INTEGER Pselect,Hptb
10270  !
10280  COM REAL Nmin,Nmax,R0_max
10290  COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
10300  COM X(*),Y(*),Xc(*),N(*),R(*)
10310  !
10320  COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
```

```
10330  !
10340   ! THE ORIGINAL VERSION OF THIS ROUTINE IS LOCATED IN THE HP-9845
10350   ! LIBRARY TAPE No. TWO SER: 09845-10205  PROGRAM "REGPLT"
10360   !
10370   !
10380   !  FNMAX AND FNMIN HAVE BEEN MADE INTO SUBROUTINES
10390   !
10400 Set_up:    IF (Plot$="P") OR (Plot$="p") THEN P9872a
10410 Crt: PLOTTER IS 13,"GRAPHICS"
10420      GOTO 10440
10430 P9872a: PLOTTER IS Pselect,Hpib,"9872A"
10440         GCLEAR
10450         LIMIT 0,184,0,140           ! ALL UNITS IN MILLIMETERS
10460         LINE TYPE 1
10470         LDIR 0
10480         LORG 5
10490         !
10500         DATA -2,-1,1,2
10510         READ Um,Dm,Md,Mu
10520         DATA .39794,.69897,.87506
10530         READ Log2,Log5,Log7
10540         !
10550           IF (Grinc$="Y") OR (Grinc$="y") THEN Grinc
10560           IF (G$="Y") OR (G$="y") THEN Grin
10570           IF (Digitize$="Y") OR (Digitize$="y") THEN Digit
10580           IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray
10590         !
10600 Ray:Xmin=Xorg=Yorg=0
10610     Xmax=10
10620     Ymax=Y_bullet
10630     Ymin=-Ymax
10640     PRINT PAGE,"  Xmax is the maximum length along the GLM axis which you"
10650     PRINT "want to be displayed."
10660     PRINT LIN(1),"  A rule of theumb is to pick a typical value of Xc (the
 X-intercept"
10670     PRINT "from the printed output for the ray trace and add one inch to"
10680     PRINT "that value. For example: if the majority of the Xc values liste
d"
10690     PRINT "have values around 5.6, then key in 6.6 and PRESS CONT."
10700 Ray1:INPUT "WHAT IS THE VALUE OF Xmax (DEFAULT VALUE IS 10.0 INCHES ) ?",X
max
10710       IF Xmax<=Xmin THEN BEEP
10720       IF Xmax<=Xmin THEN DISP "Xmax MUST BE > Xmin = 0. PLEASE RE-ENTER
Xmax."
10730       IF Xmax<=Xmin THEN WAIT 2500
10740       IF Xmax<=Xmin THEN Ray1
10750 Ray_scale: LOCATE 15,130,25,100           ! ALL UNITS IN GDU'S
10760              GOSUB Same
10770  !         SCALE Xmin-.5*Xfudge,Xmax+.5*ABS(Ytic),-Y_bullet-.25*ABS(Ytic),
Y_bullet+.25*ABS(Ytic)
10780            SCALE Xmin-.5*ABS(Ytic),Xmax+.5*ABS(Ytic),-Y_bullet-.25*ABS(Yti
c),Y_bullet+.25*ABS(Ytic)
10790   !         CLIP Xmin-.5*Xfudge,Xmax+.5*ABS(Ytic),-Y_bullet-.25*ABS(Ytic),
Y_bullet+.25*ABS(Ytic)
10800             CLIP Xmin-.8*ABS(Ytic),Xmax+.5*ABS(Ytic),-Y_bullet-.25*ABS(Yti
c),Y_bullet+.25*ABS(Ytic)
10810              GOSUB Same_axes
10820            CLIP Xmin,100,-Y_bullet,Y_bullet
10830          SUBEXIT
10840 Digit: LOCATE 0,130,30,100         ! ALL UNITS IN GDU'S
10850        Xmax=Y_bullet
10860        Xmin=-Xmax
10870        Xorg=Yorg=0
10880        Ymax=1
10890        Ymin=0
10900          GOSUB Same
10910 Digiti_scale: SCALE -Y_bullet-.15*ABS(Ytic),Y_bullet+.15*ABS(Ytic),-.15*AB
```

```
S(Ytic),1+.15*ABS(Ytic)
10920               CLIP -Y_bullet-.15*ABS(Ytic),Y_bullet+.15*ABS(Ytic),-.15*AB
S(Ytic),1+.15*ABS(Ytic)
10930                GOSUB Same_axes
10940             SUBEXIT
10950 Grin:  GOTO Ray
10960 Grinc: LOCATE 18,133,20,101        ! ALL UNITS IN GDU'S
10970        Xorg=Yorg=0
10980          GOSUB Min_c
10990          GOSUB Max_c
11000        Ymin=Cmin=Minc
11010        Ymax=Cmax=Maxc
11020          GOSUB Min_r
11030          GOSUB Max_r
11040        Xmax=Maxr
11050        Xmin=Minr
11060 Grinc_scale: GOSUB Same
11070              SCALE Xmin,Xmax+ABS(Xtic),Ymin,Ymax+ABS(Ytic)
11080               CLIP Xmin,Xmax+ABS(Xtic),Ymin,Ymax+ABS(Ytic)
11090                GOSUB Same_axes
11100            SUBEXIT
11110 Same: Lx=LGT(Xmax-Xmin)
11120       Ly=LGT(Ymax-Ymin)
11130       Xfudge=.20*(Xmax-Xmin)
11140       Yfudge=.20*(Ymax-Ymin)
11150 Ticmarks: Testxtic=FRACT(Lx)+(Lx<0)
11160           Testytic=FRACT(Ly)+(Ly<0)
11170           Xtic=10^(INT(Lx)-1)*(1+1.5*((Testxtic>Log2) AND (Testxtic<Log5))
+4*((Testxtic>=Log5) AND (Testxtic<=Log7))+6.5*(Testxtic>Log7))
11180           Ytic=10^(INT(Ly)-1)*(1+1.5*((Testytic>Log2) AND (Testytic<Log5))
+4*((Testytic>=Log5) AND (Testytic<=Log7))+6.5*(Testytic>Log7))
11190         RETURN
11200 Same_axes:  CALL Laxes(Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize,Xmin,Xmax,
Ymin,Ymax,Minn,Maxx)
11210        RETURN
11220 Min_r: Minr=R(0)
11230        FOR I=1 TO Nray
11240          Minr=MIN(Minr,R(I))
11250        NEXT I
11260      RETURN
11270 Max_r: Maxr=R(0)
11280        FOR I=1 TO Nray
11290          Maxr=MAX(Maxr,R(I))
11300        NEXT I
11310      RETURN
11320 Min_c: Minc=C(0)
11330        FOR I=1 TO Nray
11340          Minc=MIN(Minc,C(I))
11350        NEXT I
11360      RETURN
11370 Max_c: Maxc=C(0)
11380      FOR I=1 TO Nray
11390        Maxc=MAX(Maxc,C(I))
11400      NEXT I
11410     RETURN
11420    SUBEND
11430 SUB Laxes(Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize,Xmin,Xmax,Ymin,Ymax,Min
n,Maxx)
11440   OPTION BASE 0
11450   !
11460   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
11470   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
11480   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
11490   !
11500   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
l
11510   COM INTEGER Linecount,Linemax
```

```
11520   COM INTEGER Pselect,Hpib
11530   !
11540   COM REAL Nmin,Nmax,R0_max
11550   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
11560   COM X(*),Y(*),Xc(*),N(*),R(*)
11570   !
11580   COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
11590   !
11600   DEG
11610   LINE TYPE 1
11620   LDIR 0
11630   LORG 5
11640     IF (Xmin>=Xmax) OR (Ymin>=Ymax) THEN SUBEXIT
11650   GRAPHICS
11660   Xfudge=.02*(Xmax-Xmin)
11670   Yfudge=.02*(Ymax-Ymin)
11680   Xmaj=1
11690   Ymaj=1
11700   Minticsize=2
11710   !
11720     IF (Grinc$="Y") OR (Grinc$="y") THEN Grinc
11730     IF (G$="Y") OR (G$="y") THEN Grin
11740     IF (Digitize$="Y") OR (Digitize$="y") THEN Digit
11750     IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray
11760   !
11770 Grinc: LINE TYPE 3
11780        GRID Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,2*Minticsize
11790        LINE TYPE 1
11800        FRAME
11810        GOTO Labelx_grinc
11820 Grin: GOTO Ray
11830 ! Digit: AXES .1,.1,0,0,1,1,2
11840 Digit: AXES Xtic,.1,0,0,1,1,2
11850        GOTO Labelx_d
11860 Ray: AXES Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize
11870 Labelx: LDIR 90
11880         LORG 8
11890         FOR A=Xorg TO Xmax STEP ABS(Xtic)
11900           MOVE A,Yorg-Yfudge
11910             IF A=0 THEN LABEL USING 11920;A
11920             IMAGE #,K
11930             IF A=0 THEN GOTO 11960
11940           LABEL USING 11950;A
11950           IMAGE DDD.D,K
11960         NEXT A
11970 Labely: LDIR 0
11980         LORG 8
11990         FOR A=-Y_bullet TO Y_bullet STEP ABS(Ytic)
12000           MOVE Xorg-Xfudge,A
12010             IF A=0 THEN LABEL USING 11920;A
12020             IF A=0 THEN GOTO 12040
12030           LABEL USING 11950;A
12040         NEXT A
12050         GOTO Label_
12060         !
12070 Labelx_d: LDIR 90
12080           LORG 8
12090  !        FOR A=-Y_bullet TO Y_bullet STEP .1
12100           FOR A=-Y_bullet TO Y_bullet STEP ABS(Xtic)
12110             MOVE A,Yorg-Yfudge
12120               IF A=0 THEN LABEL USING 11920;A
12130               IF A=0 THEN GOTO 12150
12140             LABEL USING 11950;A
12150           NEXT A
12160 Labely_d: LDIR 0
12170           LORG 8
12180           FOR A=0 TO Ymax STEP .1
```

```
12190            MOVE Xorg-Xfudge,A
12200              IF A=0 THEN LABEL USING 11920;A
12210              IF A=0 THEN GOTO 12230
12220            LABEL USING 11950;A
12230          NEXT A
12240          GOTO Label_
12250          !
12260 Labelx_grinc: LDIR 90
12270              LORG 8
12280              FOR A=Xorg TO Xmax+ABS(Xtic) STEP ABS(Xtic)
12290                  MOVE A,Yorg-Yfudge
12300                LABEL USING 12310;A
12310                IMAGE #,K
12320              NEXT A
12330 Labely_grinc: LDIR 0
12340              LORG 8
12350              FOR A=Yorg TO Ymax+ABS(Ytic) STEP ABS(Ytic)
12360                  MOVE Xorg-Xfudge,A
12370                LABEL USING 12310;A
12380              NEXT A
12390              !
12400 Label_: LDIR 0
12410         LORG 5
12420           MOVE -10*Xmax,0
12430         LABEL "RESET CR/LF"
12440         SETGU
12450         CSIZE 2.5
12460           IF (Grinc$="Y") OR (Grinc$="y") THEN Grinc_label
12470           IF (G$="Y") OR (G$="y") THEN Grin_label
12480           IF (Digitize$="Y") OR (Digitize$="y") THEN Digit_label
12490           IF (Ray_trace$="Y") OR (Ray_trace$="y") THEN Ray_label
12500           !
12510 Ray_label: Centerx=72.5     ! CHANGE IN Graph ALSO
12520            Centery=62.5
12530              MOVE 1.5*Centerx,13
12540            LABEL USING 12550;Nray
12550            IMAGE "Number of Rays : ",K
12560              MOVE .5*Centerx,9
12570            LABEL USING 12580;Rho_initial
12580            IMAGE "Incident Ray Angle : ",3D.DD," deg"
12590              MOVE .5*Centerx,5
12600            LABEL USING 12610;N2
12610            IMAGE "n2 : ",D.DDDDD
12620              MOVE 1.5*Centerx,9
12630            LABEL USING 12640;Aperture
12640            IMAGE "Aperture : ",D.DDD," inch"
12650              MOVE Centerx,20
12660            LABEL "Distance Along the GLM Axis (inches)"
12670            !
12680            CSIZE 3
12690              MOVE .5*Centerx,13
12700            LABEL USING 12710;Curve
12710            IMAGE "Curve Number : ",K
12720            LDIR 90
12730              MOVE 1.5,Centery
12740            LABEL "Radial Distance From the Axis (inches)"
12750            LDIR 0
12760            CSIZE 15/4.54
12770            GOTO Label_end
12780            !
12790 Digit_label: Centerx=65
12800              Centery=65
12810                MOVE 1.5*Centerx,13
12820              LABEL USING 12550;Nray
12830                MOVE .5*Centerx,9
12840              LABEL USING 12850;Hit_total
```

```
12850              IMAGE "Number of Hits on the Image Plane : ",K
12860                MOVE 1.5*Centerx,9
12870              LABEL USING 12880;Percent_image
12880              IMAGE "% of Rays to Image Plane : ",3D.DD
12890                MOVE 1.5*Centerx,5
12900              LABEL USING 12910;Z3
12910              IMAGE "Image Plane : ",DDD.DD," inches"
12920                MOVE .5*Centerx,5
12930              LABEL USING 12940;N_increment
12940              IMAGE "Number of Increments [0,1.1] : ",K
12950              !
12960              CSIZE 3
12970                MOVE Centerx,20
12980              LABEL "Normalized Number of Hits  vs  Distance from the Axis"
12990                MOVE .5*Centerx,13
13000              LABEL USING 12710;Curve
13010              IMAGE "Curve Number : ",K
13020              GOTO Label_end
13030              !
13040 Grin_label: Centerx=72.5
13050             Centery=62.5
13060             CSIZE 2.5
13070               MOVE .5*Centerx,13
13080             LABEL USING 13090;N1
13090             IMAGE "n1 : ",DD.DDDDD
13100               MOVE .5*Centerx,9
13110             LABEL USING 13120;N3
13120             IMAGE "n3 : ",DD.DDDDD
13130               MOVE 1.5*Centerx,13
13140             LABEL USING 13150;Alpha
13150             IMAGE "Alpha : ",DD.DD," degrees"
13160               MOVE 1.5*Centerx,9
13170             LABEL USING 13180;Nray
13180             IMAGE "Number of Rays : ",K
13190             CSIZE 3
13200               MOVE Centerx,20
13210             LABEL "GRIN TRACE"
13220             LDIR 90
13230               MOVE 1.5,Centery
13240             LABEL "Radial Distance From the Axis ( inches )"
13250             GOTO Label_end
13260             !
13270 Grinc_label: CSIZE 3
13280              LORG 5
13290              LDIR 0
13300              Centerx=74
13310              Centery=64
13320                MOVE Centerx,5
13330              LABEL "Radius From the Origin (inches)"
13340              LDIR 90
13350                MOVE 1.5,Centery
13360              LABEL "Gradient Index Constant : C(r) (inches)"
13370              !
13380 Label_end: CSIZE 15/4.54
13390            LDIR 90
13400            LORG 5
13410            SETUU
13420          SUBEND
13430 SUB X1pos(X,Y)
13440   OPTION BASE 0
13450   !
13460   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
13470   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
13480   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
13490   !
13500   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
l
```

```
13510   !
13520 DEG
13530 DEF FNY1(Y)=Y*Talpha/(Talpha-Trho)  ! FINDS THE Y-VALUE OF THE INTERCEPT
13540                                     ! OF THE RAY AND THE FIRST SURFACE
13550 DEF FNX1(Y)=Y/Talpha   ! FINDS THE X-VALUE OF THE INTERCEPT OF THE RAY
13560                        ! AND THE FIRST SURFACE
13570       IF Surf_no=2 THEN Woops
13580     Y=FNY1(Y)
13590 Woops:X=FNX1(Y)
13600     SUBEND
13610 SUB X1neg(X,Y)
13620   OPTION BASE 0
13630   !
13640   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
13650   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
13660   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
13670   !
13680   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
1
13690   !
13700 DEG
13710 Alphan=-1*Alpha
13720 Talphan=TAN(Alphan)
13730 DEF FNY1(Y)=Y*Talphan/(Talphan-Trho)  ! FINDS THE Y-VALUE OF THE INTERCEPT
13740                                     ! OF THE RAY AND THE FIRST SURFACE
13750 DEF FNX1(Y)=Y/Talphan  ! FINDS THE X-VALUE OF THE INTERCEPT OF THE RAY
13760                        ! AND THE FIRST SURFACE
13770       IF Surf_no=2 THEN Woops
13780     Y=FNY1(Y)
13790 Woops:X=FNX1(Y)
13800 SUBEND
13810 SUB Dialogue
13820   OPTION BASE 0
13830   !
13840   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
13850   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
13860   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
13870   !
13880   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
1
13890   COM INTEGER Linecount,Linemax
13900   COM INTEGER Pselect,Hpib
13910   !
13920   COM REAL Nmin,Nmax,R0_max
13930   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
13940   COM X(*),Y(*),Xc(*),N(*),R(*)
13950   !
13960   COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
13970   !
13980 DEG
13990 Initialize_top:Alpha=21
14000                Talpha=TAN(Alpha)
14010                Rho_initial=0
14020                Trho_i=TAN(Rho_initial)
14030                !
14040                Tol=.00001
14050                Z3=Z4=50
14060                N1=N3=1
14070                N2=1.5
14080                !
14090                Nray=10  ! IN ORDER TO OUTLINE THE APERTURE AN EXTRA RAY
14100                Add_ray=0  ! MUST BE ADDED IF Ya # 0
14110                Ya=0
14120                Yb=Y_bullet
14130                Aperture=Ya-Yb
14140                !
14150                Y_bullet=1.1
```

```
14160                N_increment=100
14170                !
14180                Linecount=0
14190                Linemax=46
14200                !
14210                Nmax=4
14220                Nmin=2
14230                  Ym2=(1.05*Y_bullet/2)^2   ! THE 1.05 IS A FUDGE FACTOR TO
14240                  Xm2=(1.05*Y_bullet/(2*Talpha))^2 !PREVENT THE RADIUS FROM
14250                R0_max=SQR(Xm2+Ym2)  ! BECOMING TOO SMALL AND CAUSING A
14260                !                  NUMERICAL ERROR IN THE INDEX SUBROUTINES
.
14270                Pselect=7
14280                Hpib=5
14290                !
14300                Grin$="C"
14310                G$="N"
14320                Grinc$="N"
14330                Plot$="C"
14340                Digitize$="N"
14350                Date$=""
14360 REM  *** TEMP ***
14370   GOTO Alpha
14380 REM  *** TEMP ***
14390 Begin: PRINT PAGE,"   The following drawing illustrates the optical proble
m"
14400        PRINT "which this program attempts to solve; focousing monochromati
c "
14410        PRINT "light rays through a conical lens for a Gun-Launched-Missile
 (GLM)."
14420        PRINT "   The picture is also labeled showing the various quantitie
s"
14430        PRINT "Please study the picture and become familiar with it. The va
riables"
14440        PRINT "will be explained as they appear in the program."
14450        PRINT LIN(2),"PRESS CONT"
14460        PAUSE
14470          CALL Picture
14480        PRINT PAGE,"   Z1 is the intercept of the second surface and the"
14490        PRINT "the GLM-axis. Therefore, the value of Z1 is APOS(0)."
14500        PRINT LIN(1),"NOTE: Apos(0) is the constant term in the polynomia
l used"
14510        PRINT "      to describe the second surface of the lens in the "
14520        PRINT "      upper half-plane."
14530        PRINT "   The (min,max) values of Z1 are [0,infinity] inches."
14540        PRINT LIN(2),"PRESS CONT"
14550        PAUSE
14560        PRINT PAGE,"   There are two planes used as image planes. The first
 is "
14570        PRINT "located at X=Z3 (default value = ";Z3;"inches). This locatio
n"
14580        PRINT "is programmable, according to the needs of the particular pr
oblem."
14590        PRINT "You key in the new location when locating the position of "
14600        PRINT "maximum ray density, or when a new run is begun. Directions
will "
14610        PRINT "be provided at the appropriate time."
14620        PRINT LIN(1),"   The second image plane is permanent, located at X=
Z4 = ";Z4;"inches."
14630        PRINT LIN(2),"PRESS CONT"
14640        PAUSE
14650        PRINT PAGE,"   This program will trace mochromatic light rays which
 arrive"
14660        PRINT "at the first surface parallel or at an angle to the GLM axis
"
14670        PRINT "Non-axial rays approach the GLM from 'above', for all rays a
re "
```

```
14680        PRINT "initiated in the upper half of the meridian plane."
14690        PRINT LIN(2),"PRESS CONT"
14700        PAUSE
14710        PRINT PAGE,"   The program uses Snell's Law without approximation."
14720        PRINT "The appropriate variables are:"
14730        PRINT LIN(1),"     1) Norm =  dY/dX =  slope of the normal to the s
urface"
14740        PRINT "     2) Thetai = angle of incidence with respect to the surf
ace normal."
14750        PRINT "     3) Thetar = angle of refraction with respect to the sur
face normal."
14760        PRINT "     4) Rho = angle of the ray with respect to the GLM-axis"
14770        PRINT "              (subscripts indicate the region the ray is in)
"
14780        PRINT LIN(2),"PRESS CONT"
14790        PAUSE
14800  PRINT PAGE,"The sign convention is :"
14810  PRINT LIN(1),"      1) the origin is placed at the vertex of the first su
rface."
14820  PRINT "      2) distances: (a) positive to the right of the origin."
14830  PRINT "                    (b) positive above the origin (Y-axis)."
14840  PRINT "                    (c) positive out of the meridian plane (screen
)."
14850  PRINT "                    (ie. a right hand system.)"
14860  PRINT "      3) angles: slope = Alpha2 is POSITIVE for counter-clockwise"
14870  PRINT "                         rotation from the GLM-axis to the ray"
14880  PRINT "      4) symbols: unprimed = object space."
14890  PRINT "                  primed = image space."
14900  PRINT LIN(2),"PRESS CONT"
14910  PAUSE
14920 Alpha:PRINT PAGE,"    Alpha is the angle of inclination of the "
14930       PRINT "FIRST surface with respect to the GLM-AXIS"
14940       PRINT LIN(3),"   The current value of ALPHA is";Alpha;"degrees"
14950       INPUT "WHAT IS THE NEW VALUE OF ALPHA (DEGREES)?",Alpha
14960 Index:PRINT PAGE,"    The lens system is assumed to be opersting in air."
14970   PRINT "Therefore the first index of refraction = n1= 1.00000"
14980   PRINT LIN(1),"    It is assumed that n1 < n2 for the homogeneous and gra
dient cases."
14990   PRINT LIN(1),"   You are free to choose any values for n2 and n3 for the
 homogeneous case. "
15000   PRINT LIN(2),"PRESS CONT"
15010   PAUSE
15020   PRINT PAGE,"   The program has the capability to use a gradient index of
 refraction (GRIN)."
15030   PRINT "Two GRIN subroutines are available:"
15040   PRINT LIN(1),"          1) Parabola : the index of region 2 (the lens) v
aries "
15050   PRINT "                        as a parabola from a min at the vertex"
15060   PRINT "                        to a maximum (at a radius specified by yo
u)"
15070   PRINT "                        then down to a value determined by the "
15080   PRINT "                        top of the GLM"
15090   PRINT "                        The equation is [ n(r) = k * r^2 ]"
15100   PRINT LIN(1),"          2) Circle : the index varies as a circle whose c
enter is located"
15110   PRINT "                      at a specified radius from the origin"
15120   PRINT "                      The equation is [ n(r) = SQR( R0_max^2 + r^
2 ) ]"
15130   PRINT LIN(1),"The radius (for circle)  has a minimum value which is half
 the "
15140   PRINT "length of the first surface; approximately ";R0_max;" inches."
15150   PRINT LIN(1),"   You are free to choose any value for n3 for the GRIN ca
se."
15160   PRINT LIN(2),"PRESS CONT"
15170   PAUSE
15180   G$="N"
15190   INPUT "ARE YOU GOING TO USE GRIN ? Y/N",G$
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15200     IF (G$="N") OR (G$="n") THEN N2
15210    PRINT PAGE,"  Required data for grin :"
15220    PRINT LIN(1),"           1) Nmin = minimum value of the index of"
15230    PRINT "                     of refraction ( default = ";Nmin;")"
15240    PRINT "          2) Nmax = maximum value of the index (not required for
circle.)"
15250    PRINT "                     ( Default = ";Nmax;")"
15260    PRINT "          3) R0_max = radius from the origin at which the maximum
"
15270    PRINT "                       value of the index occurs. ( MINIMUM = ";R0
_max;")"
15280    INPUT "WHICH ONE : CIRCLE (C) OR PARABOLA (P) ( DEFAULT = C ) ? C/P",Gri
n$
15290    INPUT "WHAT IS Nmin ?",Nmin
15300     IF (Grin$="C") OR (Grin$="c") THEN N3
15310    INPUT "WHAT IS Nmax ?",Nmax
15320    INPUT "WHAT IS R0_max ?",R0_max
15330 N2:INPUT "WHAT IS n2 ?",N2
15340 N3:INPUT "WHAT IS n3 ?",N3
15350      IF (G$="N") OR (G$="n") THEN Rho_initial
15360      PRINT LIN(2),"NOTE: When the parameter data is printed, the values
of "
15370      PRINT "      the index at the first surface intercept will be "
15380      PRINT "      printed in the folowing manner:"
15390      PRINT "        n(8) = 2.3543     n(9) = 2.3567   n(10) = 1.9876"
15400      PRINT "      The subscript indicates the number of the ray intercepti
ng"
15410      PRINT "      the first surface."
15420      PRINT LIN(2),"PRESS CONT"
15430      PAUSE
15440 Rho_initial:PRINT PAGE,"   The rays incident on the first surface are "
15450            PRINT "initially assumed to be parallel to the X-axis. "
15460            PRINT LIN(2),"   If you want to trace rays which are at an ang
le "
15470            PRINT "other than zero (0) degrees with respect to the X-axis,
"
15480            PRINT "then key in the value of the angle (in degrees) with "
15490            PRINT "respect to the X-axis."
15500            INPUT "RHO ?",Rho_initial
15510            Rho_initial=-1*ABS(Rho_initial)
15520            Trhoi=TAN(Rho_initial)
15530 Y_bullet: PRINT PAGE,"   The GLM is assumed to be assumed to be symmetric
about the"
15540           PRINT "longitudinal axis. The maximum value of the GLM radius is
"
15550           PRINT "called Y_bullet whose default value is ";Y_bullet;" inche
s."
15560           PRINT LIN(2),"   If your design requires a different value, plea
se"
15570           PRINT "enter that value now."
15580           INPUT "Y_BULLET (INCHES) ?",Y_bullet
15590           Yb=Y_bullet
15600 Aperture: PRINT PAGE,"   The aperture is the difference in radial distance
 from the GLM axis"
15610           PRINT "projected onto the first surface,into which light is allo
wed to enter"
15620           PRINT "the GLM optical system. The parameters are :"
15630           PRINT LIN(1),"      1) Ya : the minimum Y-value of the aperture"
15640           PRINT "           (default value =";Ya;"inch )"
15650           PRINT "       2) Yb : the maximum Y-value of the aperture"
15660           PRINT "           (default value =";Yb;"inch)  and is ≤ Y_bulle
t."
15670           INPUT "WHAT IS Ya (INCHES) ?",Ya
15680           INPUT "WHAT IS Yb (INCHES) ?",Yb
15690             IF Ya>=Yb THEN BEEP
15700             IF Ya>=Yb THEN PRINT "Ya must be < Yb. Please enter again."
15710             IF Ya>=Yb THEN GOTO 15670
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15720             Aperture=Yb-Ya
15730               IF Ya<>0 THEN Add_ray=1
15740 Nray: PRINT PAGE,"   The program will trace";Nray;" rays through the syste
m usless "
15750        PRINT "you specify another value"
15760        INPUT " How many rays do you want the program to trace?",Nray
15770        Nray=INT(ABS(Nray))
15780          IF Nray<1 THEN Nray=10
15790          IF (G$="Y") OR (G$="y") THEN Date
15800 Digitize_: PRINT PAGE,"   The interval between (0 ≤ Y ≤ ;";Y_bullet;")
 and (-";Y_bullet;" ≤ Y ≤ 0) have been divided"
15810             PRINT "into";N_increment;" increments to determine ray density.
"
15820             PRINT LIN(1),"If you want to change the number of increments, p
lease do so now."
15830             PRINT LIN(2),"The amximum number of increments is 1000."
15840             INPUT "NUMBER OF INCREMENTS ?",N_increment
15850             N_increment=INT(ABS(N_increment))
15860               IF N_increment<1 THEN N_increment=100
15870 Date:PRINT PAGE
15880        INPUT "What is today's date ?",Date$
15890 Start:    IF (G$="Y") OR (G$="y") THEN Gymnastics
15900          PRINT PAGE,"   The second surface must be input as polynomial of de
gree "
15910          PRINT "16 or less, defined as a function of Y [ x=f(y) for (x,y) sy
stem ]"
15920          PRINT "If you have not fit a curve to your data, then go to the uti
lities "
15930          PRINT "library,tape two and load REGD. "
15940          PRINT "NOTE : 1) Use printer code 0 for a hard copy"
15950          PRINT "       2) Invert the X and Y values for the positive branch"
15960          PRINT "       3) Key in (Y,-X) for the negative branch"
15970          PRINT LIN(1),"NOTE : The negative branch is the second surface in
"
15980          PRINT "                 the lower half plane"
15990          PRINT LIN(1),"   If this is the case, then PRESS STOP, then REWIND
T15 "
16000          PRINT "fit your curve using the utilties library."
16010          PRINT LIN(1),"  If not, then PRESS CONT."
16020          PAUSE
16030 Coef_data:PRINT PAGE           ! THE FOLLOWING DATA ARE ARRANGED AS FOLLOWS:
16040                                ! DATA CURVE No.
16050                                ! DATA NP,NN,N
16060                                ! DATA APOS COEFICIENTS X=f(Y) FOR Y>0
16070                                ! DATA ANEG COEFICIENTS X=f(Y) FOR Y<0
16080                                ! DATA Y=F(X) COEFICIENTS
16090                                !
16100    !        DATA 1    ! CURVE No. ONE
16110    !        DATA 4,4,4
16120    !        DATA .5003067966,1.15219319,3.0138759,-3.4182912,1.39217781
16130    !        DATA .5003067966,-1.15219319,3.0138759,3.4182912,1.39217781
16140    !        DATA  -.45216186,1.18544007,-.66358797,.25054923,-.034174838
16150             !
16160             !
16170    !        DATA 2    ! CURVE No. 10
16180    !        DATA 3,3,3
16190    !        DATA 1.5193756399,2.27918152,-2.5230983,1.63047031
16200    !        DATA 1.5193756399,-2.27918152,-2.5230983,-1.63047031
16210    !        DATA 3.3554752,-5.6849365,2.959523,-.4412878
16220             !
16230             !
16240    !        DATA 3      ! CURVE No. 11
16250    !        DATA 6,6,3
16260    !        DATA 2.006223634,.30397,12.96206,-62.1508,125.7966,-114.0765,38
.06104
16270    !        DATA 2.006223634,-.30397,12.96206,62.1508,125.7966,114.0765,38.
06104
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```
16280   !         DATA -11.205773,13.18159,-5.328168,.767801
16290             !
16300             !
16310             DATA 4        ! SURFACE NR 1 FOR n2=4, F=4 INCHES
16320             DATA 2,2,2
16330             DATA .3184601721,2.7540136449,-.3640728143
16340             DATA .3184601721,-2.7540136449,-.3640728143
16350             DATA -.1071362125,.3341667828,.02751837081
16360             !
16370             !
16380 Explain: PRINT "   Since this program is intended to be used  to evaluate"
16390           PRINT "various second surfaces a method of labeling the current"
16400           PRINT "design surface has been incorporated into the algorithm."
16410           PRINT "   The program will ask you for a curve number. At that"
16420           PRINT "time, key in any number (integer or real) you wish to use"
16430           PRINT "identify the surface."
16440           PRINT LIN(2),"PRESS CONT"
16450           PAUSE
16460 Coef_pos:  PRINT PAGE,"The current values of Curve and Np (the degree"
16470            PRINT "of the positive branch x = f(y) ) are:"
16480            READ Curve,Np,Nn,N
16490            PRINT LIN(1),"            Curve = ";Curve
16500            PRINT "                 Np = ";Np
16510            INPUT "WHICH CURVE ARE YOU USING (CURVE No.) ?",Curve
16520            INPUT "WHAT IS THE DEGREE OF THE POLYNOMIAL FOR THE POSITIVE BR
ANCH (Np) ?",Np
16530            PRINT LIN(2)
16540 Quick_loop_p: PRINT "NOTE: These are the current values of the positive
branch coeficients:"
16550               PRINT
16560               FOR I=0 TO Np
16570                 READ Apos(I)
16580                 PRINT USING 16590;I,Apos(I)
16590                 IMAGE "Apos("4D") = ",K
16600               NEXT I
16610               PRINT
16620    Z1=Apos(0)
16630    Any$="NONE"
16640    INPUT "ARE YOU GOING TO CHANGE ANY OF ALL OF THE COEFICIENTS ? SOME/ALL/
NONE ",Any$
16650      IF (Any$="NONE") OR (Any$="none") THEN None_p
16660      IF (Any$="SOME") OR (Any$="some") THEN Some_p
16670 All_p:PRINT
16680        PRINT "Please key in the coeficients, beginning with the constant:"
16690        FOR I=0 TO Np
16700          INPUT "",Apos(I)
16710          PRINT USING 16590;I,Apos(I)
16720        NEXT I
16730 None_p:  PRINT
16740          Y$="N"
16750          INPUT "DO YOU NEED TO MAKE ANY CORRECTIONS ? Y/N",Y$
16760            IF (Y$="N") OR (Y$="n") THEN Coef_neg
16770 Some_p:  INPUT "WHAT IS THE NUMBER OF THE COEFICIEINT ? (0,1,2, etc)",Npp
16780          INPUT "KEY IN THE CORRECT VALUE, THEN CONT",Apos(Npp)
16790          PRINT USING 16590;Npp,Apos(Npp)
16800          Y$="Y"
16810          INPUT "IS THAT ALL",Y$
16820            IF (Y$="y") OR (Y$="y") THEN Coef_neg
16830            IF (Any$="SOME") OR (Any$="some") THEN Some_p
16840          GOTO Some_p
16850 Coef_neg:  PRINT PAGE,"The current values of Curve and Nn (the degree"
16860            PRINT "of the negative branch x = f(-y) ) are:"
16870            PRINT LIN(1),"            Curve = ";Curve
16880            PRINT "                 Nn = ";Nn
16890            INPUT "WHAT IS THE DEGREE OF THE NEGATIVE BRANCH POLYNOMIAL (Nn)
 ?",Nn
16900 Quick_loop_n:PRINT LIN(2),"NOTE: These are the current values of the neg
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ative branch coeficients:"
16910             PRINT
16920             FOR I=0 TO Nn
16930               READ Aneg(I)
16940               PRINT USING 16950;I,Aneg(I)
16950               IMAGE "Aneg("4D") = ",K
16960             NEXT I
16970             PRINT
16980   Any$="NONE"
16990   INPUT "ARE YOU GOING TO CHANGE ANY OF ALL OF THE COEFICIENTS ? SOME/ALL/
NONE ",Any$
17000     IF (Any$="NONE") OR (Any$="none") THEN None_n
17010     IF (Any$="SOME") OR (Any$="some") THEN Some_n
17020 All_n:PRINT
17030       PRINT "Please key in the coeficients, beginning with the constant:"
17040       FOR I=0 TO Nn
17050         INPUT "",Aneg(I)
17060         PRINT USING 16950;I,Aneg(I)
17070       NEXT I
17080 None_n: PRINT
17090        Y$="N"
17100        INPUT "DO YOU NEED TO MAKE ANY CORRECTIONS ? Y/N",Y$
17110          IF (Y$="N") OR (Y$="n") THEN Gymnastics
17120 Some_n: INPUT "WHAT IS THE NUMBER OF THE COEFICIEINT ? (0,1,2, etc)",Nnn
17130        INPUT "KEY IN THE CORRECT VALUE, THEN CONT",Aneg(Nnn)
17140        PRINT USING 16950;Nnn,Aneg(Nnn)
17150        Y$="Y"
17160        INPUT "IS THAT ALL",Y$
17170          IF (Y$="y") OR (Y$="y") THEN Gymnastics
17180          IF (Any$="SOME") OR (Any$="some") THEN Some_n
17190        GOTO Some_n
17200 Gymnastics:SUBEND
17210 !
17220 !
17230                       SUB Char
17240     FOR I=0 TO 79
17250         IF I=79 THEN PRINT CHR$(228)
17260         IF I=79 THEN 17280
17270       PRINT CHR$(228);
17280     NEXT I
17290  SUBEND
17300 SUB X2pos(X2,Y)
17310   OPTION BASE 0
17320   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
17330   !
17340   FOR I=0 TO Np
17350     X2=X2+Apos(I)*Y^I
17360   NEXT I
17370 SUBEND
17380 SUB X2neg(X2,Y)
17390   OPTION BASE 0
17400   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
17410   !
17420   FOR I=0 TO Nn
17430     X2=X2+Aneg(I)*Y^I
17440   NEXT I
17450 SUBEND
17460 SUB Index_p(X,Y)
17470   OPTION BASE 0
17480   !
17490   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
17500   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
17510   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
17520   !
17530   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
l
17540   COM INTEGER Linecount,Linemax
```

```
17550   COM INTEGER Pselect,Hpib
17560   !
17570   COM REAL Nmin,Nmax,R0_max
17580   !
17590   DEG
17600   Xm2=.60/Talpha         ! FROM y=mx + b  AND Ymax = 1.2
17610   Ym2=Xm2*Talpha
17620     IF R0_max<SQR(Xm2^2+Ym2^2) THEN Ro_max=SQR(Xm2^2+Ym2^2)
17630   Radius=SQR(X^2+Y^2)
17640   R2=R0_max^2
17650   K=(Nmax-Nmin)/R2
17660   B=(Radius-R0_max)^2
17670   N2=Nmax+K*B
17680 SUBEND
17690 SUB Index_c(X,Y)
17700   OPTION BASE 0
17710   !
17720   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
17730   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
17740   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
17750   !
17760   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
l
17770   COM INTEGER Linecount,Linemax
17780   COM INTEGER Pselect,Hpib
17790   !
17800   COM REAL Nmin,Nmax,R0_max
17810   !
17820   DEG
17830   Xm2=.60/Talpha         ! FROM y=mx + b  AND Ymax = 1.2
17840   Ym2=Xm2*Talpha
17850     IF R0_max<SQR(Xm2^2+Ym2^2) THEN Ro_max=SQR(Xm2^2+Ym2^2)
17860   Radius=SQR(X^2+Y^2)
17870   A=R0_max^2
17880   B=(Radius-R0_max)^2
17890   C2=A-B
17900   N2=Nmin+SQR(C2)
17910 SUBEND
17920 SUB Density
17930   OPTION BASE 0
17940   !
17950   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
17960   COM Alpha,Talpha,Rho,Trho,Rho_initial,Trhoi,Phi,Tol,Norm,Alpha2
17970   COM N1,N2,N3,Percent_image,Ya,Yb,Aperture,Y_bullet
17980   !
17990   COM INTEGER Y0_loop,Nray,Add_ray,Surf_no,Flag,Flag2,N_increment,Hit_tota
l
18000   COM INTEGER Linecount,Linemax
18010   COM INTEGER Pselect,Hpib
18020   !
18030   COM REAL Nmin,Nmax,R0_max
18040   COM Ray_trace$,Digitize$,Grin$,G$,Grinc$,Plot$,Date$
18050   COM X(*),Y(*),Xc(*),N(*),R(*)
18060   !
18070   COM Rho(*),Phi(*),C(*),Y3(*),Hit(*)
18080   !
18090   INTEGER Inc,K,L,M,Lines
18100   DEG
18110   ! THIS ROUTINE COMPUTES THE RAY DENSITY IN ONE DIMENSION.
18120   ! ASSUMPTIONS:
18130   !     1) THE POINT OF MAXIMUM RAY DENSITY HAS BEEN FOUND VIA A
18140   !        DIGITIZE STATEMENT.
18150   !     2) THE VALUES OF Y3 HAVE NOT BEEN CHANGED TO REFLECT THE
18160   !        NEW VALUE OF Z3
18170   !
18180   PRINTER IS 16
18190 Initialize_: Nr=-1
```

```
18200                Ns=-3
18210 Re_draw: FOR I=1 TO Nray
18220            Nr=Nr+3
18230            Ns=Ns+5
18240            A=Xc(I)
18250            X=X(Ns)
18260            Y=Y(Ns)
18270            Rho=Rho(Nr)
18280            Trho=TAN(Rho)
18290              IF A<Z1 THEN Negative
18300 Positive:    IF Rho=0 THEN Next_i   ! TOTAL INTERNAL REFLECTION AT X2POS
18310            GOTO Same
18320 Negative:    IF Y<=-Y_bullet THEN Next_i  ! Y2=-Y_bullet =>THE RAY MISSED
X2 POS/NEG
18330              IF Rho=0 THEN Next_i   ! TOTAL INTERNAL REFLECTION AT X2NEG
18340 Same:      Deltay=(Z3-X)*Trho
18350            Y3=Y+Deltay
18360            X(Ns+1)=Z3
18370            Y(Ns+1)=Y3
18380 Next_i:  NEXT I
18390 Y3_array: J=0
18400           FOR I=1 TO Nray
18410             Y3(J)=Y(5*I-2)
18420             J=J+1
18430           NEXT I
18440 Hit_zero: MAT Hit=ZER(2001)    ! ZERO THE ARRAY
18450           Inc=0                ! FIND THE No. OF HITS IN THE
18460           Ylast=-Y_bullet      ! INTERVAL [ -Y_BULLET,-Y_BULLET+DELTA/2)
18470           Delta=Y_bullet/N_increment
18480           Ynext=Ylast+Delta/2
18490           Hit(Inc)=0
18500           J=0
18510           FOR I=1 TO Nray
18520               IF (Xc(I)<Z1) AND (Y3(J)=0) THEN Next_j   ! IGNORES NEGATIVE
18530                                                         ! BRANCH RAYS
18540               IF (Y3(J)>=Ylast) AND (Y3(J)<Ynext) THEN Hit(Inc)=Hit(Inc)+1
18550 Next_j:     J=J+1
18560           NEXT I
18570 Hit_middle: ! FIND THE No. OF HITS IN THE INTERVAL
18580             ! [-Y_BULLET+DELTA/2, Y_BULLET-DELTA/2)
18590             FOR L=1 TO 2*N_increment-1
18600               Inc=Inc+1
18610               Hit(Inc)=0
18620               Ylast=Ynext
18630               Ynext=Ylast+Delta
18640               K=0
18650                 FOR I=1 TO Nray
18660                     IF (Xc(I)<Z1) AND (Y3(K)=0) THEN Next_k   ! IGNORES
18670                                                         ! NEGATIVE BRANCH RAYS
18680                     IF (Y3(K)>=Ylast) AND (Y3(K)<Ynext) THEN Hit(Inc)=Hit(
Inc)+1
18690 Next_k:             K=K+1
18700                 NEXT I
18710               NEXT L
18720 Hit_last: Inc=2*N_increment ! FIND THE No. OF HITS IN THE LAST INTERVAL
18730           Hit(Inc)=0        !  [Y_BULLET-DELTA/2, Y_BULLET]
18740           M=0
18750           Ylast=Y_bullet-Delta/2
18760           Ynext=Y_bullet
18770           FOR I=1 TO Nray
18780               IF (Xc(I)<Z1) AND (Y3(M)=0) THEN Next_m   ! IGNORES
18790                                                         ! NEGATIVE BRANCH RAYS
18800               IF (Y3(M)>=Ylast) AND (Y3(M)<=Ynext) THEN Hit(Inc)=Hit(Inc)+
1
18810 Next_m:     M=M+1
18820           NEXT I
18830 Normalize: Hit_total=0                  ! NORMALIZE THE No. OF HITS ON THE
```

```
18840            FOR I=0 TO 2*N_increment       ! IMAGE PLANE (Z3) TO THE TOTAL
18850              Hit_total=Hit_total+Hit(I)  ! No. OF HITS
18860            NEXT I
18870            !
18880              IF Hit_total=0 THEN None
18890            FOR I=0 TO 2*N_increment
18900              Hit(I)=Hit(I)/Hit_total
18910            NEXT I
18920 None:      Pct=Hit_total/Nray                 ! % OF RAYS THAT ARRIVE AT THE
18930            Percent_image=Pct*100              ! IMAGE PLANE
18940 Graph:   CALL Plot                            ! PLOT AND LABEL THE AXES
18950               IF Hit_total=0 THEN Label_
18960 First_bar: Xlast=-Y_bullet                    ! DRAW THE HISTOGRAM
18970            Xnext=Xlast+Delta/2   ! INTERVAL  [-Y_BULLET, -Y_BULLET+DELTA/2)
18980            Inc=0
18990              IF Hit(Inc)=0 THEN Middle_bars
19000            CLIP Xlast,Xnext,0,Hit(Inc)
19010            FRAME
19020            GOSUB Delt
19030 Middle_bars: FOR I=1 TO 2*N_increment-1     ! INTERVAL
19040                Inc=Inc+1                ! [-Y_BULLET+DELTA/2, Y_BULLET-DELTA/2)
19050                Xlast=Xnext
19060                Xnext=Xlast+Delta
19070                  IF Hit(Inc)=0 THEN Next_i_m
19080                CLIP Xlast,Xnext,0,Hit(Inc)
19090                FRAME
19100                GOSUB Delt
19110 Next_i_m:    NEXT I
19120 Last_bar: Inc=2*N_increment  ! INTERVAL [Y_BULLET-DELTA/2, Y_BULLET]
19130           Xlast=Y_bullet-Delta/2
19140           Xnext=Y_bullet
19150             IF Hit(Inc)=0 THEN Label_
19160           CLIP Xlast,Xnext,0,Hit(Inc)
19170           FRAME
19180           GOSUB Delt
19190 Label_: LDIR 0
19200         LORG 2
19210  !      SETGU
19220  !      MOVE 2,2
19230  !      CSIZE 2.5
19240  !      LABEL "PRESS CONT"
19250  !      SETUU
19260         CSIZE 15/4.54
19270         BEEP
19280         WAIT 250
19290         BEEP
19300         PAUSE
19310         EXIT GRAPHICS
19320       SUBEXIT
19330 Delt: Lines=10
19340       Delt=(Xnext-Xlast)/Lines
19350       X=Xlast-Delt
19360       FOR J=1 TO Lines
19370         X=X+Delt
19380         MOVE X,0
19390         DRAW X,Hit(Inc)
19400       NEXT J
19410     RETURN
19420   SUBEND
19430 SUB Picture
19440 Scale_: GCLEAR
19450         PLOTTER IS 13,"GRAPHICS"
19460         GRAPHICS
19470         LIMIT 0,184,0,140
19480         SCALE -20,152.4,-55,55
19490         AXES 5,5,0,0,10,10
19500 Label_: MOVE 20,45
```

```
19510         LABEL "THE PROBLEM"
19520         MOVE 20,44
19530         DRAW 50,44
19540 Draw_body: MOVE 0,0
19550            DRAW 74,30
19560            DRAW 184,30
19570            MOVE 0,0
19580            DRAW 74,-30
19590            DRAW 184,-30
19600 Draw_inlet: MOVE 70,35
19610             DRAW 80,38
19620             DRAW 184,38
19630             MOVE 70,-35
19640             DRAW 80,-38
19650             DRAW 184,-38
19660 Second_surf: MOVE 30,0
19670              DRAW 35,5
19680              DRAW 40,9
19690              DRAW 45,13
19700              DRAW 50,15
19710              DRAW 74,30
19720              MOVE 30,0
19730              DRAW 35,-5
19740              DRAW 40,-9
19750              DRAW 45,-13
19760              DRAW 50,-15
19770              DRAW 74,-30
19780 Ray: MOVE 0,15
19790      DRAW 40,15
19800      DRAW 45,13
19810      DRAW 184,-14
19820 Image_plane: MOVE 80,30
19830              LINE TYPE 4
19840              DRAW 80,-30
19850 Label_picture: MOVE 65,-50
19860                LINE TYPE 1
19870                LABEL "5/54 GLM"
19880                MOVE 85,15
19890                LABEL "IMAGE PLANE"
19900                MOVE 15,2
19910                LABEL "ALPHA"
19920                MOVE 10,0
19930                DRAW 8,2.5
19940                MOVE 5,18
19950                LABEL "INCIDENT RAY"
19960                MOVE 35,2
19970                LABEL "Z1"
19980                MOVE 35,2
19990                DRAW 30,0
20000                MOVE 113,2
20010                LABEL "Xc"
20020                MOVE 113,2
20030                DRAW 110,0
20040                MOVE 82,2
20050                LABEL "Z3"
20060                MOVE 82,2
20070                DRAW 80,0
20080                MOVE 130,3
20090                LABEL "C"
20100                MOVE 131.5,1.5
20110                LABEL "L"
20120                MOVE 5,-10
20130                LABEL "n1"
20140                MOVE 30,-10
20150                LABEL "n2"
20160                MOVE 50,-10
```

```
20170                LABEL "n3"
20180 Finished: MOVE 15,-55
20190     !     LABEL "PRESS CONT"
20200           BEEP
20210           WAIT 250
20220           BEEP
20230           PAUSE
20240           EXIT GRAPHICS
20250     SUBEND
20260 SUB X2p(X,Y)
20270   OPTION BASE 0
20280   !
20290   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
20300   !
20310   DEG
20320   FOR I=0 TO N
20330     Y=Y+A(I)*Y^I
20340   NEXT I
20350  SUBEND
20360 SUB X2n(X,Y)
20370   OPTION BASE 0
20380   !
20390   COM INTEGER I,J,Curve,Np,Nn,N,Ns,Nr,REAL Aneg(*),Apos(*),A(*),Z1,Z3,Z4
20400   !
20410   DEG
20420     CALL X2p(X,Y)
20430     Y=-1*Y
20440   SUBEXIT
20450 SUB Min(Xm(*),X,INTEGER M)
20460   OPTION BASE 0
20470   !
20480 ! DIM Xm(M)
20490   !
20500   X=Xm(0)
20510   FOR I=1 TO M
20520     X=Min(X,Xm(I))
20530   NEXT I
20540  SUBEND
20550 SUB Max(Xm(*),X,INTEGER M)
20560   OPTION BASE 0
20570   !
20580 ! DIM Xm(M)
20590   !
20600   X=Xm(0)
20610   FOR I=1 TO M
20620     X=MIN(X,Xm(I))
20630   NEXT I
20640  SUBEND
```

# APPENDIX C

## POLYNOMIALS USED FOR THE SECOND SURFACE

### A. INTRODUCTION

The following discussion compares the attributes of three analytic expressions, in polynomial form, used as second surfaces for a conical lens. The lenses are compared in terms of the individual lens ability to cause some or all of the incident light rays to converge to a point. The points of ray convergence are referred to as "focal" points although they are not focal points in the standard usage of the term. Lens designers have developed a system of standards used as figures of merit to compare one lens to another. These standards include spherical abberation, coma, tilt, astigmatism and others. The figures of merit have not been calculated because the three curves discussed in this appendix are known to be completely unsatisfactory to use as refracting surfaces. The purpose of the thesis is to discover which properties of a curve to investigate in order to lead to usable design. Therefore an intuitive comparison of ray diagrams, histograms of ray distributions and the set of basic design criteria described in chapter III are the basis for the discussion of curves A, B and C in this appendix.

### B. CURVE A

Curve A, shown in figure C-1 was the first analytic expression used as a second surface to produce a ray diagram.

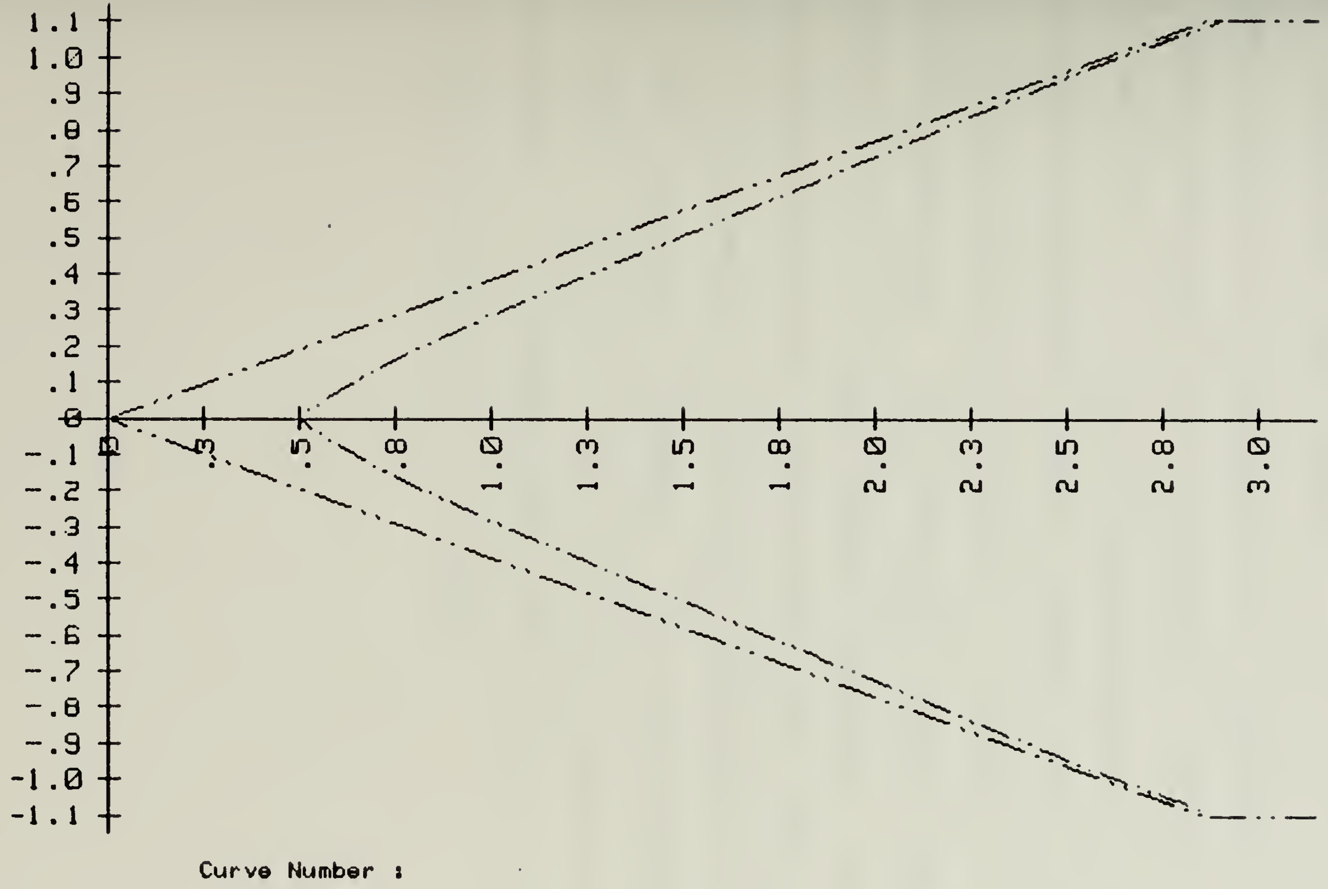


Figure C-1. Conical Lens with the Second Surface Defined by Curve A.

The curve was calculated by otaining the data listed in Table C-1 from an arbitrary hand drawn curve. These data were used as the input to a regression analysis routine in the HP-9845B utilities library. Since TRACE requires the second surface to be expressed as a polynomial function of y, three curves were fit to the data. Table C-II contains the coefficients for each of the polynomials. The first polynomial relates the radial distance from the GLM axis to the distance along the GLM axis, i.e. radius = f (distance along the GLM axis). The remaining two are required by TRACE for ray tracing and graphics purposes. The second expression relates the distance along the GLM axis to the radius from the axis in the upper half-plane; i.e. distance along the GLM axis = f(radius). The third expression relates the distance along the GLM axis to the radius in the lower half-plane; i.e. distance along the GLM axis = f(-radius). All linear dimensions are in inches.

## TABLE C-I

## DATA POINTS FOR CURVE A

| y = f(x)*<br>(Ordinate,Abscissa) | x = f(y)*<br>(Ordinate,Abscissa) | x = f(-6)*<br>(Ordinate,Abscissa) |
|---|---|---|
| 0.500,0.00 | 0.000,0.500 | 0.000,0.500 |
| 0.600,0.069 | 0.069,0.600 | -0.069,0.600 |
| 0.700,0.138 | 0.138,0.700 | -0.138,0.700 |
| 0.800,0.188 | 0.188,0.800 | -0.188,0.800 |
| 0.900,0.238 | 0.238,0.900 | -0.238,0.900 |
| 1.000,0.281 | 0.281,1.000 | -0.281,1.000 |
| 1.500,0.500 | 0.500,1.500 | -0.500,1.500 |
| 2.000,0.731 | 8.731,2.000 | -0.731,2.000 |
| 2.500,0.938 | 0.938,2.500 | -0.938,2.500 |
| 2.900,1.100 | 1.100,2.900 | -1.100,2.900 |

*: For x = distance along the GLM axis and

y = radial distance from the GLM axis.

## TABLE C-II

## POLYNOMIAL COEFFICIENTS FOR CURVE A

| Coefficient Number $a_i$ | $y = f(x)$* | Positive Branch $x = f(y)$* | Negative Branch $x = f(-y)$* |
|---|---|---|---|
| $a_o$ | -0.45216186 | 0.5003067966 | 0.5003067966 |
| $a_i$ | 1.18544007 | 1.15219319 | -1.15219319 |
| $a_2$ | -0.66358797 | 3.0138759 | 3.0138759 |
| $a_3$ | 0.25054923 | -3.4182912 | 3.4182912 |
| $a_4$ | -0.034174838 | 1.39217781 | 1.39217781 |

*: For x = distance along the GLM axis and
y = radial distance from the GLM axis.

A ray diagram using TRACE, which is shown in figure C-2, demonstrates that curve A has the ability to focus some of the rays to a point near the wall of the GLM at a distance of 21.18 inches along the GLM axis. Figure C-3 is the histogram of the ray distribution for figure C-2 and shows a maximum at 1.08 inches. If this distribution were rotated about the GLM axis, the two dimensional distribution would be a set of concentric rings centered on the GLM axis with the most intense ring at a radius of 1.08 inches. Although this ray pattern is unsatisfactory, the fact that some rays converged to a focus was encouraging.

The converging rays were coming from the middle region of the curve. The slope of the middle region of the curve is slightly larger than the slope of the first surface. The slope of the regions above and below the middle region have slopes which are approximately the same as and greater than the slope of the first surface respectively. The ability of the middle region to focus light rays suggested the existence of a range of slopes which would cause convergence of light rays. The light rays which intercept the second surface in the lower half-plane are totally internally reflected because of the large angle of incidence. Total internal reflection occurs when Snell's Law $\sin\theta_R = (n_2/n_3)\sin\theta_I$ results in $\sin_R \geq 1$. The critical angle of incidence $\theta_c$ is where $\sin\theta_R = 1$ and any angle greater than $\theta_c$ results in total internal reflection. The range appears to be in a narrow

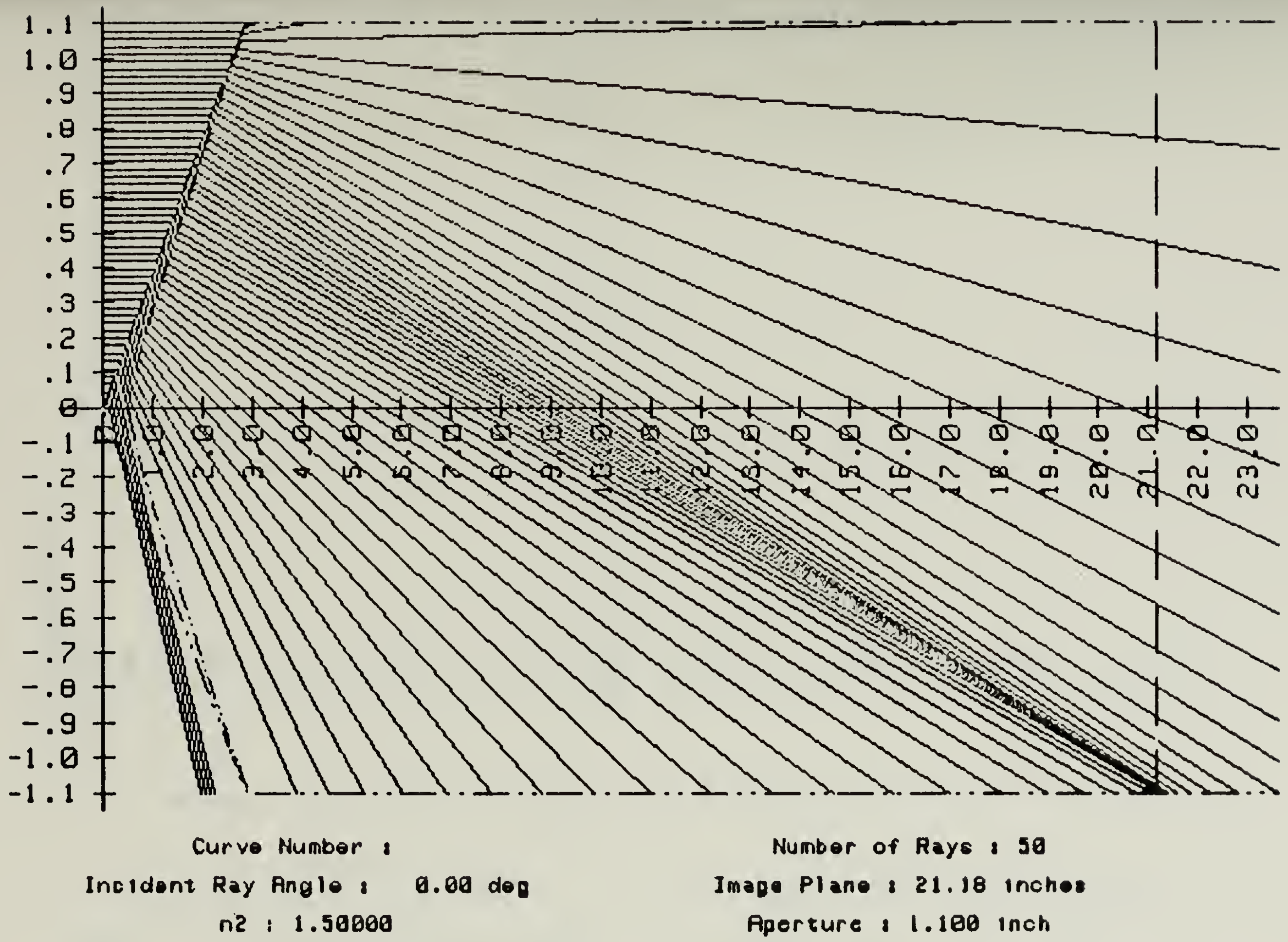


Figure C-2. Ray Diagram for a Conical Lens Using Curve A as the Second Surface.

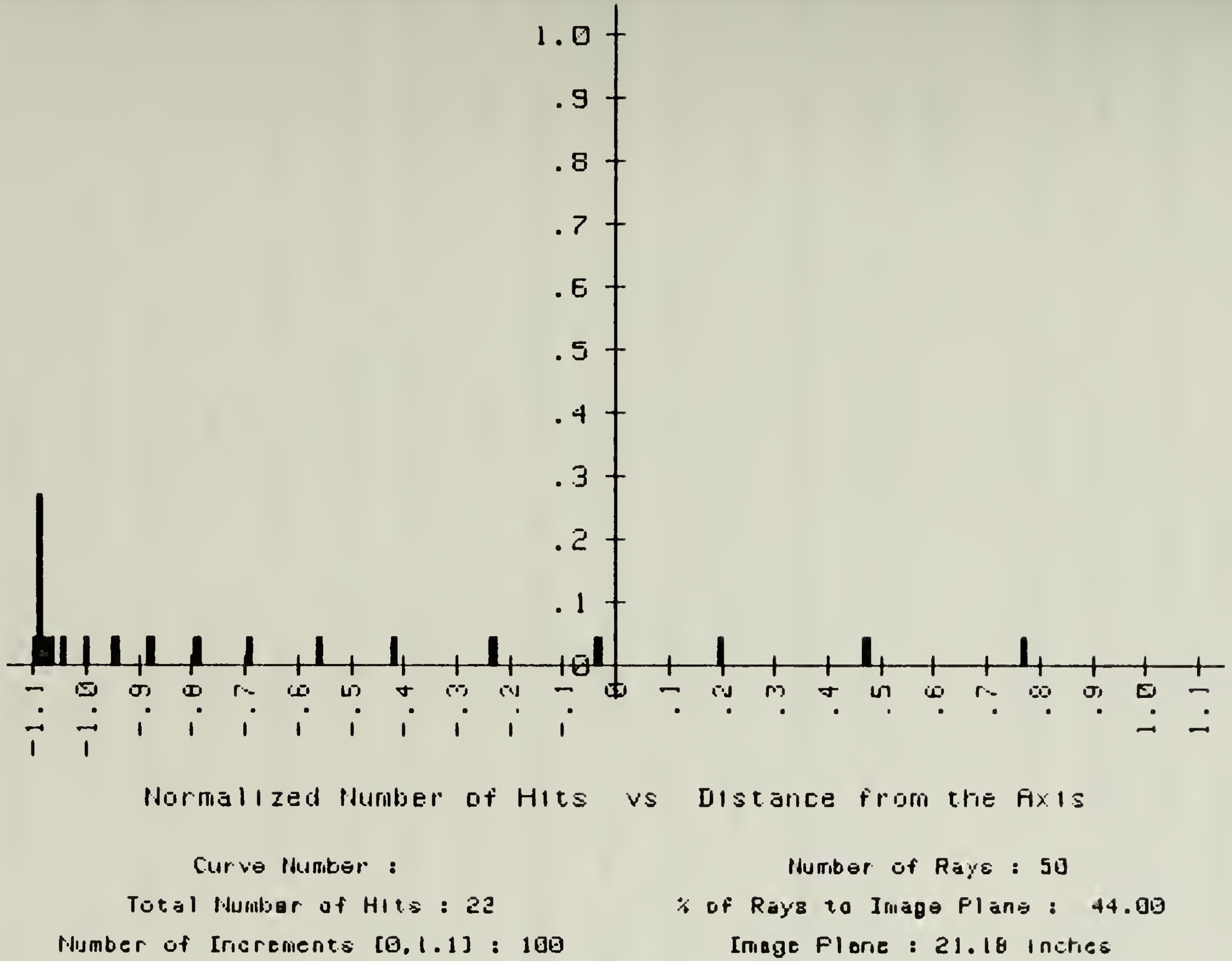


Figure C-3. Histogram of the Ray Distribution on the Image Plane Shown in Figure C-2.

band with the minimum value greater than the value of the cone angle, $\alpha$. Since the curve is concave with respect to the first surface, the next step of the investigation was to study curves with the appropriate slope comparable with curve A and regions concave and convex with respect to the first surface. These two criteria were the basis for choosing curves B and C for further investigation.

## C. CURVE B

Curve B, shown in figure C-4 was chosen because the analytical expression which describes the curve exhibits regions similar to those discussed for curve A. The shape of curve B is smooth with an inflection point providing regions concave and convex with respect to the first surface. The data used to generate the analytical expression for the curve are listed in Table C-III. Three analytic expressions were calculated as for curve A. The coefficients for the three polynomials describing curve B are listed in Table C-IV.

The ray diagram in figure C-5 demonstrates the ability of curve B to cause some of the light rays to converge to a "focal" point. A ray diagram with an image plane located at the point of maximum ray density is shown in figure C-6. Although this ray distribution is not satisfactory the distribution of rays is relatively better than the ray distribution for curve A shown in figure C-2.

The ray distribution of curve B is better than that for curve A because the point of ray convergence is closer to

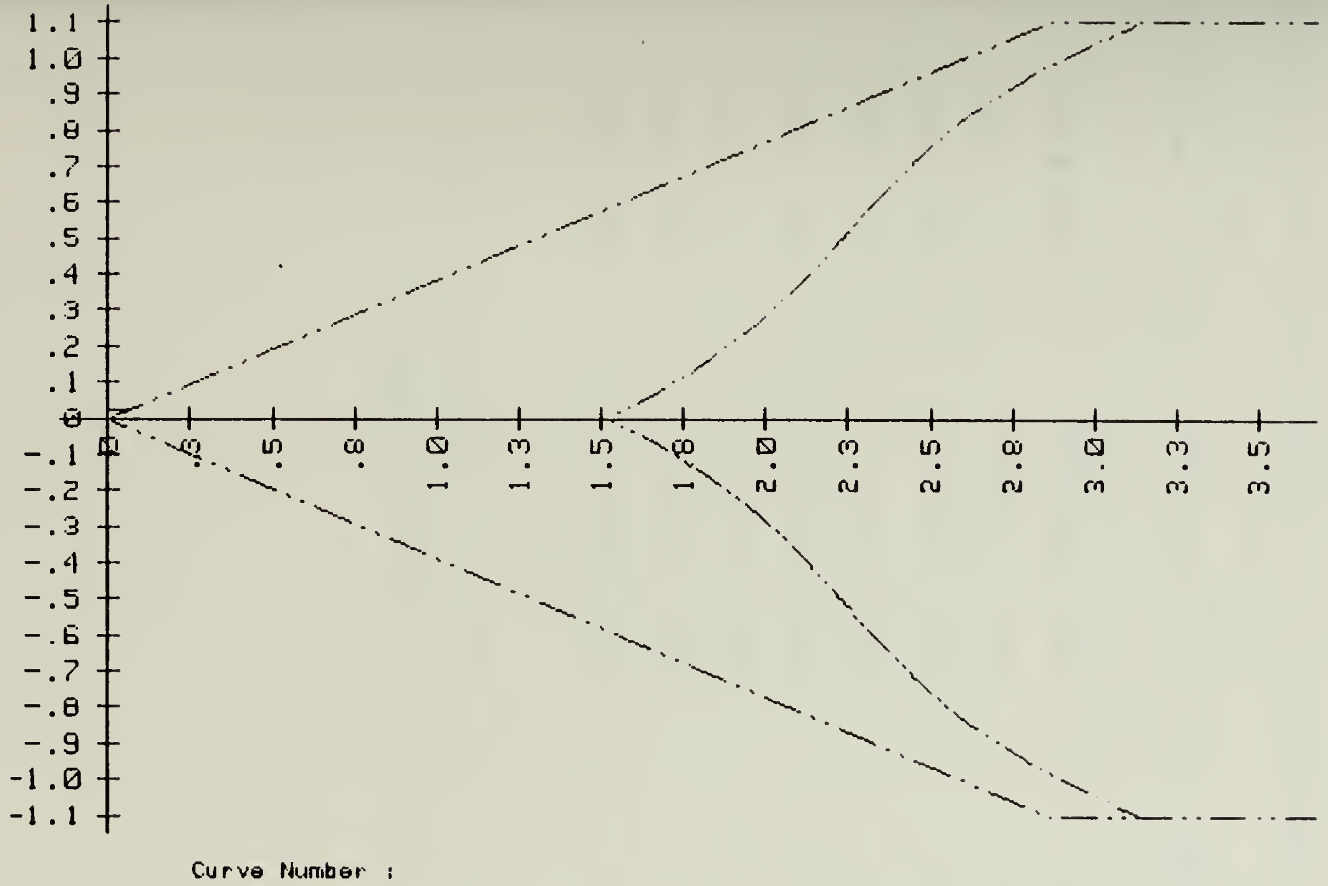


Figure C-4. Conical Lens with the Second Surface Defined by Curve B.

## TABLE C-III

## DATA POINTS FOR CURVE B

| y = f(x)*<br>(Ordinate,Abscissa) | x = f(y)*<br>(Ordinate,Abscissa) | x = f(-y)*<br>(Ordinate,Abscissa) |
|---|---|---|
| 1.500,0.000 | 0.000,1.500 | 0.000,1.500 |
| 1.700,0.075 | 0.075,1.700 | -0.075,1.700 |
| 1.900,0.200 | 0.200,1.900 | -0.200,1.900 |
| 2.100,0.394 | 0.394,2.100 | -0.394,2.100 |
| 2.300,0.569 | 0.569,2.300 | -0.569,2.300 |
| 2.500,0.744 | 0.744,2.500 | -0.744,2.500 |
| 2.700,0.888 | 0.888,2.700 | -0.888,2.700 |
| 2.900,1.000 | 1.000,2.900 | -1.000,2.900 |

*: For x = distance along the GLM axis and
y = radial distance from the GLM axis.

## TABLE C-IV

## POLYNOMIAL COEFFICIENTS FOR CURVE B

| Coefficient Number $a_i$ | $y = f(x)$* | Positive Branch $x = f(y)$* | Negative Branch $x = f(-y)$* |
|---|---|---|---|
| $a_o$ | 3.3554752 | 1.5193756399 | 1.5193756399 |
| $a_i$ | -5.6849365 | 2.279198152 | -2.279198152 |
| $a_2$ | 2.959523 | -2.5230983 | -2.5230983 |
| $a_3$ | -0.4412878 | 1.63047031 | -1.63047031 |

*: For x = distance along the GLM axis and
y = radial distance from the GLM axis.

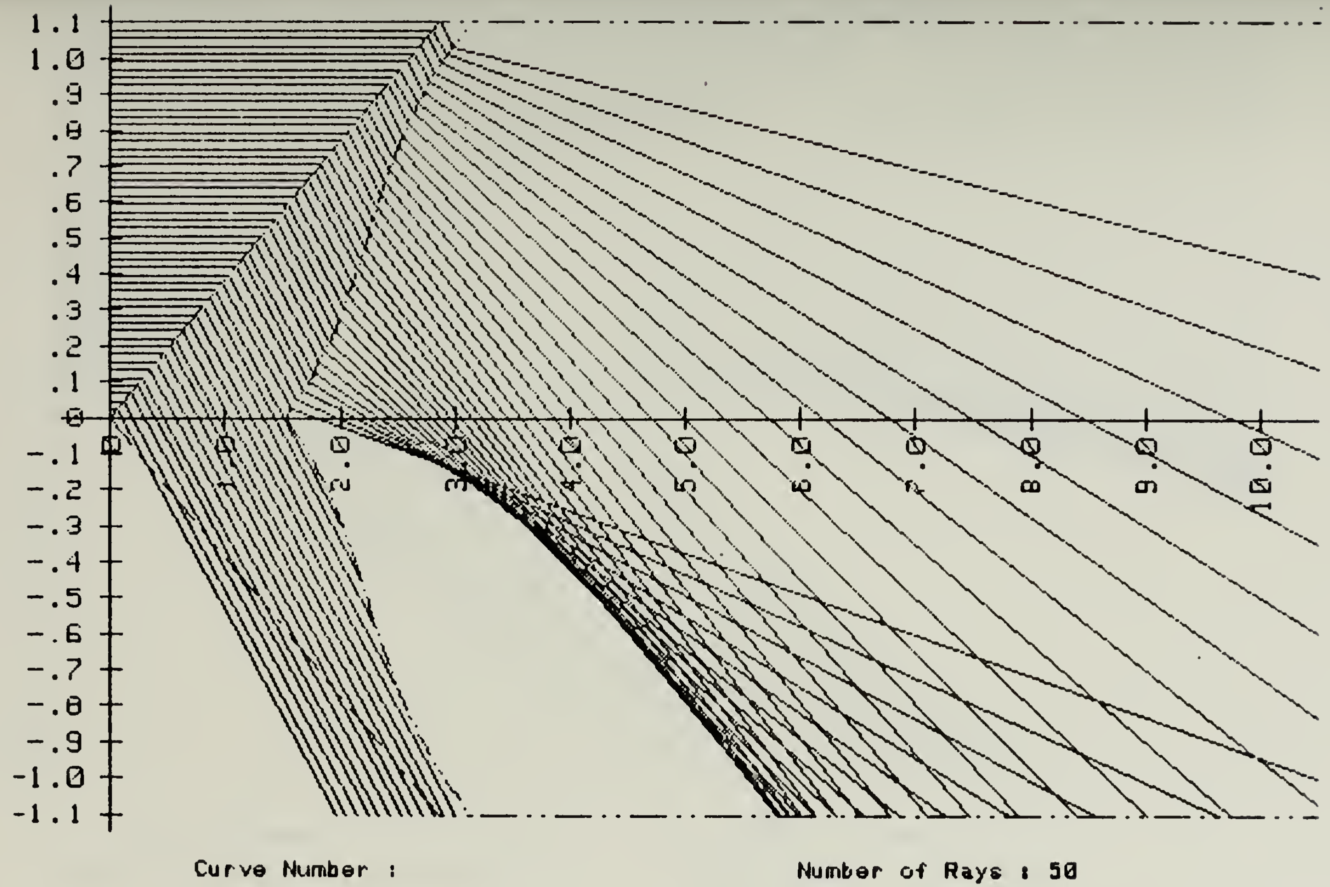
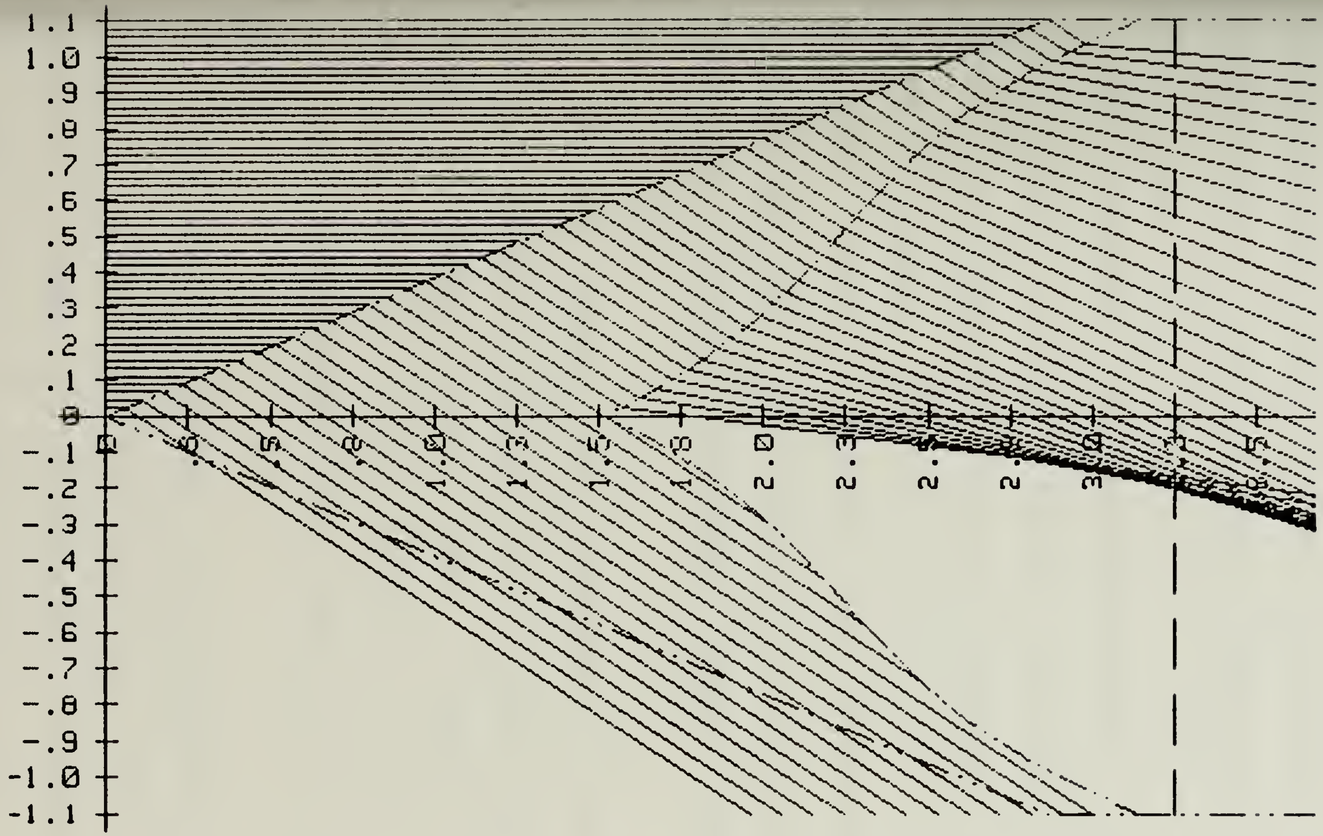


Figure C-5. Ray Diagram for a Conical Lens Using Curve B as the Second Surface.



Curve Number :

Incident Ray Angle : 0.00 deg

n2 : 1.50000

Number of Rays : 50

Image Plane : 3.25 inches

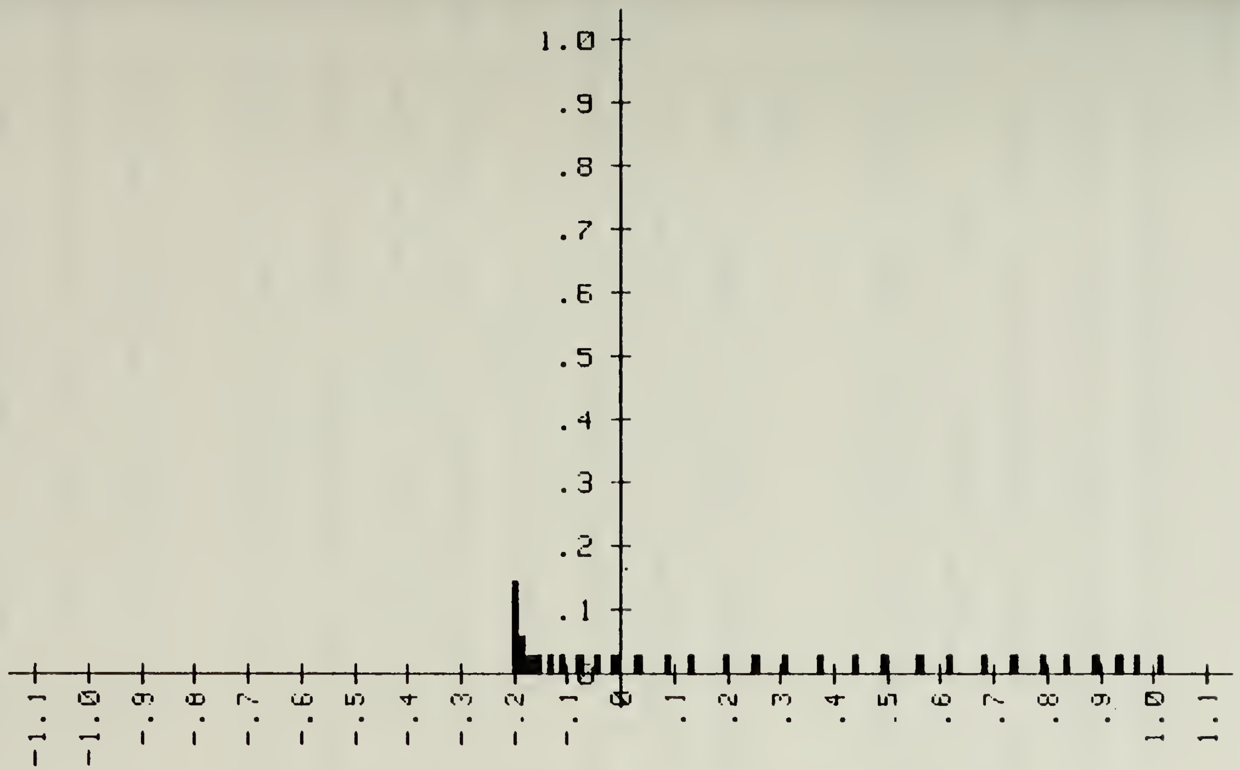
Aperture : 1.100 inch

Figure C-6. Ray Diagram Using Curve B with Image Plane at 3.25 Inches.

both the lens and the GLM axis. The histogram for curve B in figure C-7 shows the point of maximum ray density is located at a radius of approximately 0.2 inch from the GLM axis. The point of maximum ray density for curve A is at a radius of approximately 1.08 inch from the GLM axis. Although curve A causes more of the rays which strike the image plane to focus than curve B does, the ability to achieve a short focal length is more desirable and therefore curve B is judged to be better than curve A. A second factor which must be taken into account is the fraction of rays transmitted through the lens. Curve A transmits 84% (42 of 50) of the incident rays through the lens. Curve B transmits 68% (34 of 50) of the light rays through the lens. The fraction of transmitted rays could be increased by moving the second surface forward and making the lens thinner.

The ray diagram and histogram for curve B in figures C-6 and C-7 reveal the absence of a large number of rays in the lower half-plane near the lens. Rotating the histogram about the GLM-axis would generate a set of concentric rings with the most intense at a radius of 0.2 inch from the GLM axis.

Using the ray diagram and histogram for curve A in figures C-2 and C-3 as baseline design figures for the GLM lens curve B is judged to be the superior curve of the two due to the capability of curve B to cause light rays to converge in a region relatively close to the lens and near the GLM axis.



Normalized Number of Hits vs Distance from the Axis

Curve Number :

Total Number of Hits : 34

Number of Increments [0,1.1] : 100

Number of Rays : 50

% of Rays to Image Plane : 68.00

Image Plane : 3.25 inches

Figure C-7. Histogram of the Ray Distribution and the Image Plane in Figure C-6.

Curve A exhibited one convex region and one focal point. The rays intercepting the second surface in the lower half-plane were totally internally reflected as they were for curve A.

The investigation was now directed to probe the ability of a convex surface to cause convergence of the light rays at a shorter distance along the GLM axis than a concave region. Curve C was chosen because the analytic expression is a higher order polynomial with several inflection points.

## D. CURVE C

Curve C shown in figure C-8 was chosen because the higher order polynomial contains several inflection points. The inflection points provide regions of concave and convex surfaces. If the results for curve B are valid then curve C should cause rays to converge to several focal points, one for each convex region. The data points for curve C listed in Table C-V were obtained and the polynomial coefficients in Table C-VI were generated in the same manner as for curve A.

The ray diagram in figure C-9 show two distinct areas where rays exiting the lens from a convex region converge. The focal points possess the same characteristics as those of curve B. The focal points are located in a region close to the lens and are formed from rays emerging from a convex portion of the surface. Light rays refracted from concave regions of the lens do cross other light rays, but do not

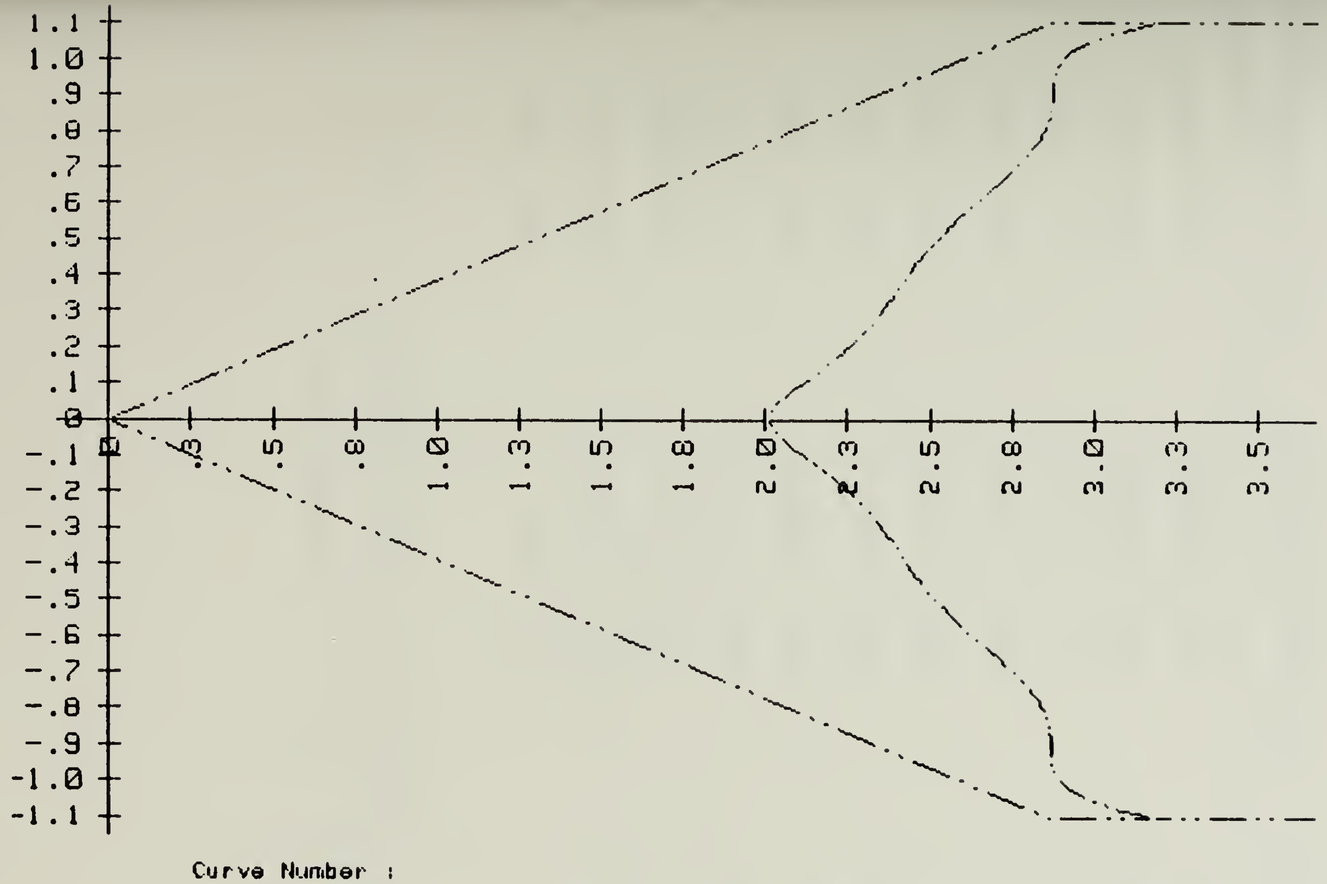


Figure C-8. Conical Lens with the Second Surface Defined by Curve C.

## TABLE C-V

## DATA POINTS FOR CURVE C

| y = f(x)*<br>(Ordinate,Abscissa) | x = f(y)*<br>(Ordinate,Abscissa) | x = f(-y)*<br>(Ordinate,Abscissa) |
|---|---|---|
| 2.000,0.00 | 0.000,2.000 | 0.000,2.000 |
| 2.100,0.081 | 0.081,2.100 | -0.081,2.100 |
| 2.200,0.175 | 0.175,2.200 | -0.175,2.200 |
| 2.300,0.244 | 0.244,2.300 | -0.244,2.300 |
| 2.400,0.331 | 0.331,2.400 | -0.331,2.400 |
| 2.500,0.500 | 0.500,2.500 | -0.500,2.500 |
| 2.600,0.600 | 0.600,2.600 | -0.600,2.600 |
| 2.700,0.612 | 0.612,2.700 | -0.612,2.700 |
| 2.800,0.744 | 0.744,2.800 | -0.744,2.800 |
| 2.850,0.8125 | 0.8125,2.850 | -0.8125,2.850 |
| 2.900,1.000 | 1.000,2.900 | -1.000,2.900 |

*: For x = distance along the GLM axis and

y = radial distance from the GLM axis.

## TABLE C-VI

## POLYNOMIAL COEFFICIENTS FOR CURVE C

| Coefficient Number $a_i$ | $y = f(x)$* | Positive Branch $x = f(y)$* | Negative Branch $x = f(-y)$* |
|---|---|---|---|
| $a_o$ | -11.205773 | 2.003622364 | 2.003622364 |
| $a_1$ | 13.18159 | 0.30397 | -0.30397 |
| $a_2$ | -5.328168 | 12.96206 | 12.96206 |
| $a_3$ | 0.767801 | -62.1508 | 62.1508 |
| $a_4$ | 0.00 | 125.7966 | 125.7966 |
| $a_5$ | 0.00 | -114.0765 | 114.0765 |
| $a_6$ | 0.00 | 38.06104 | 38.06104 |

*: For x = distance along the GLM axis and

y = radial distance from the GLM axis.

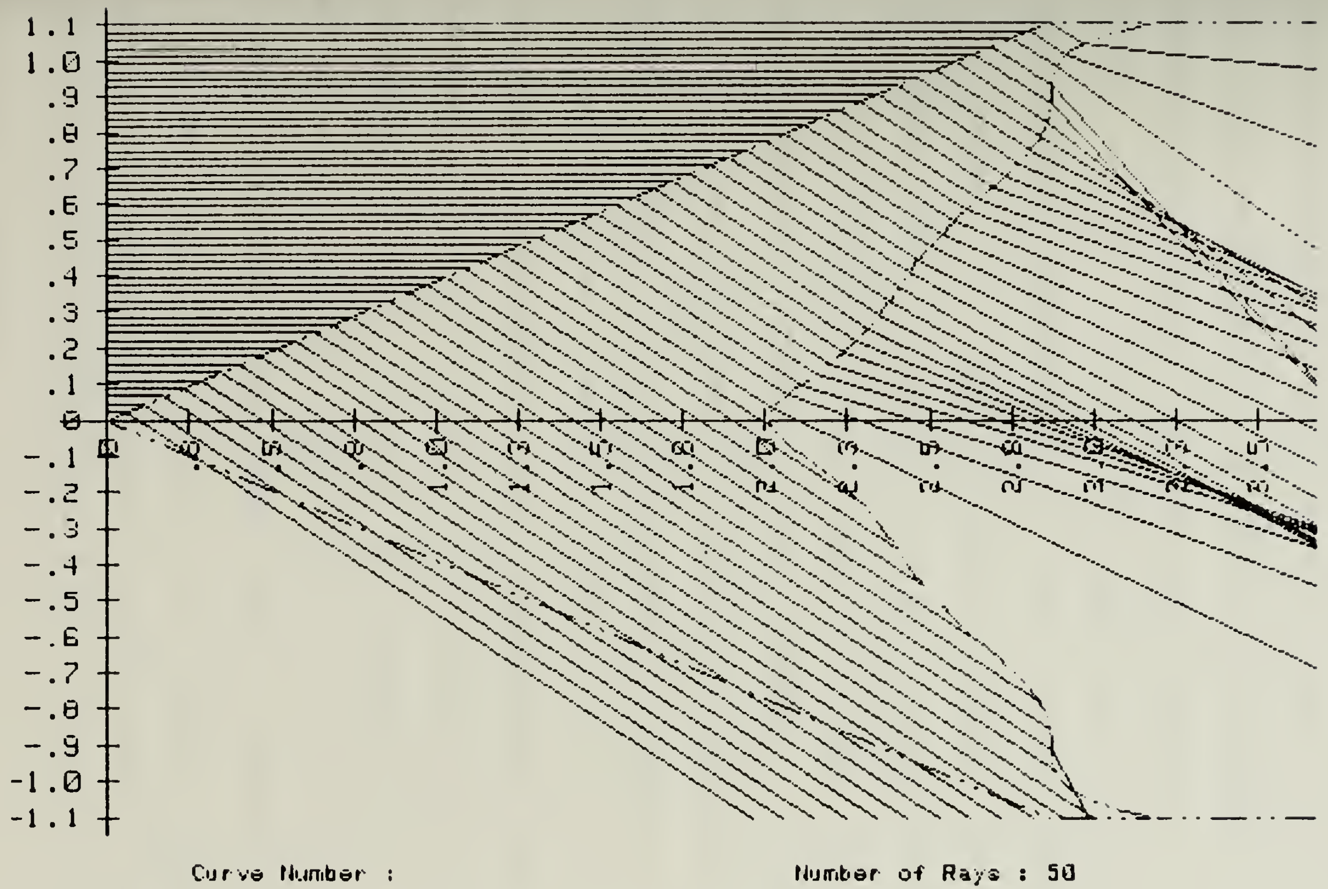
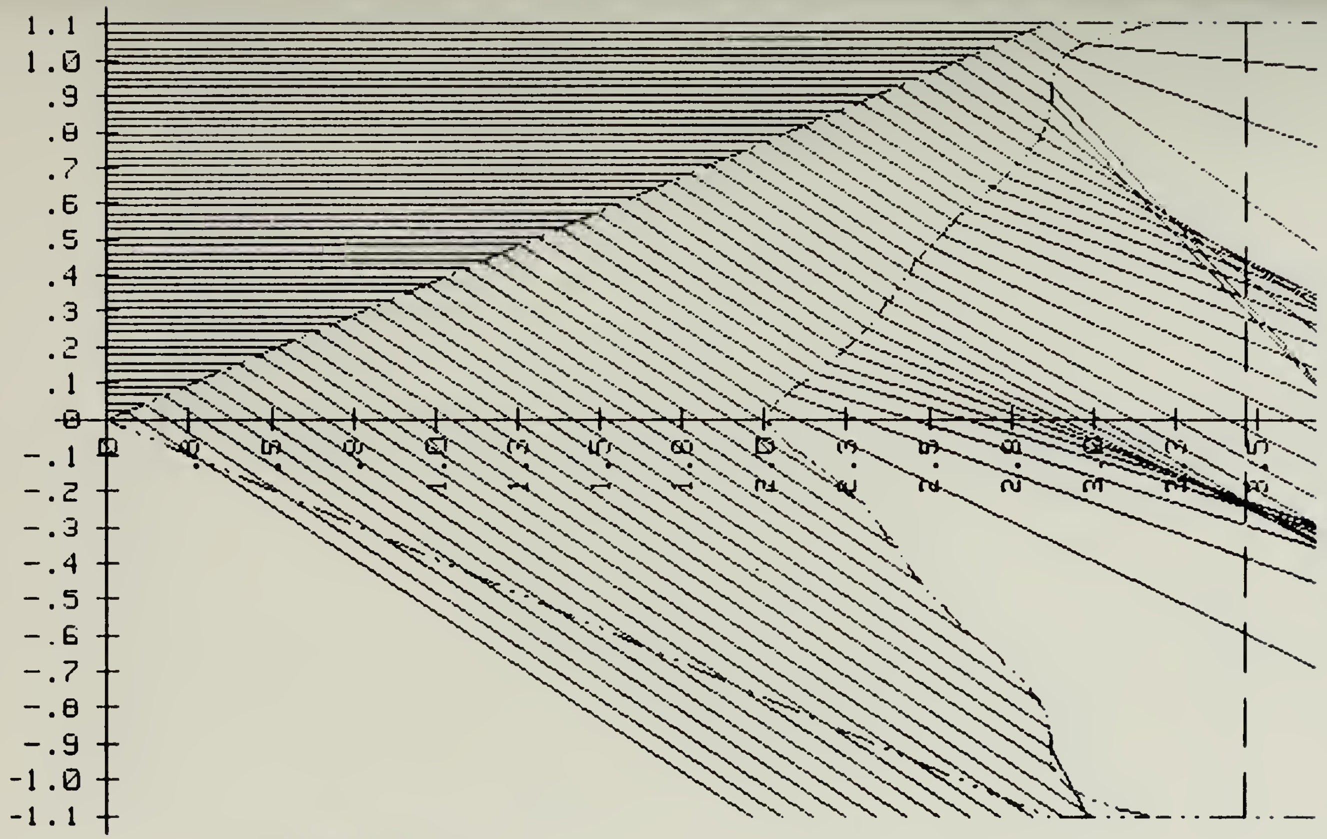


Figure C-9. Ray Diagram for a Conical Lens Using Curve C as the Second Surface.

converge as a group as do the light rays from the convex regions.

Comparing the focal points caused by curve C shown in figures C-10 and 11 with figure C-1 for curve A shows that the focal point caused by curve A is located at a point 21.18 inches along the GLM axis and at a radius of -1.08 inches (21.18, - 1.08) inches. The focal points for curve C are positioned at approximately (3.46, -0.25) inches and (3.25, 0.54) inches. If the histogram in figure C-12, drawn for the image plane in figure C-10 was rotated about the GLM axis a set of concentric rings would be formed. The most intense rings would be located at radii of 0.25, 0.34 and 0.44 inch. Rotating the histogram in figure C-13 drawn for the image plane in figure C-11 would produce a similar set of rings, the most intense located at radii of 0.16 and 0.54 inch. Comparing the number of rays transmitted reveals curve A transmits 84% (42 of 50) of the incident rays and curve C transmits 58% (29 of 50) of the incident rays. However, since curve C causes light rays to converge in a region closer to the lens than does curve A, curve C is judged to be better than curve A. Also, all but one ray which intercepted the second surface of the lens using curve C in the lower half-plane experienced total internal reflection at the second surface.



Curve Number : C

Incident Ray Angle : 0.00 deg

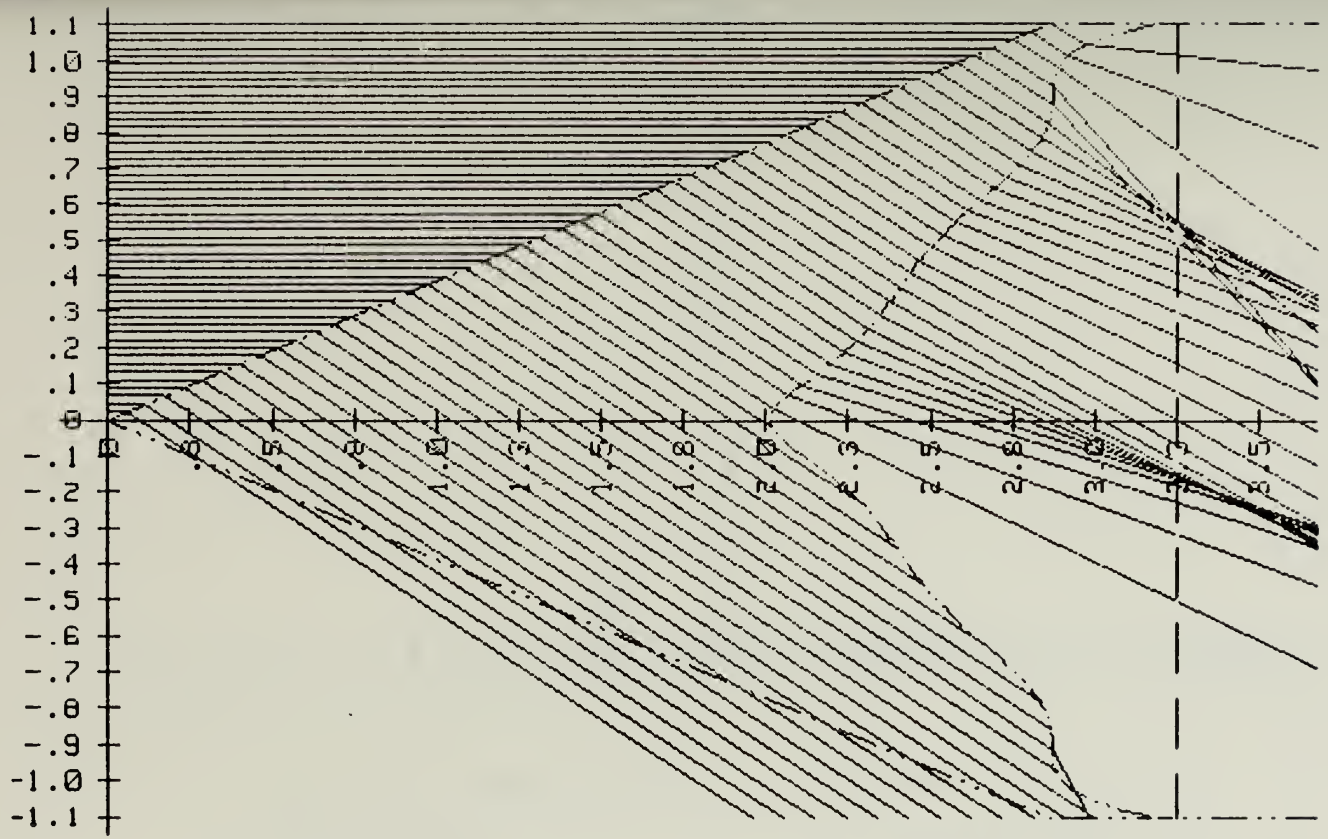
n2 : 1.50000

Number of Rays : 50

Image Plane : 3.46 inches

Aperture : 1.100 inch

Figure C-10. Ray Diagram Using Curve C with the Image Plane at 3.46 Inches.



Curve Number : C

Incident Ray Angle : 0.00 deg

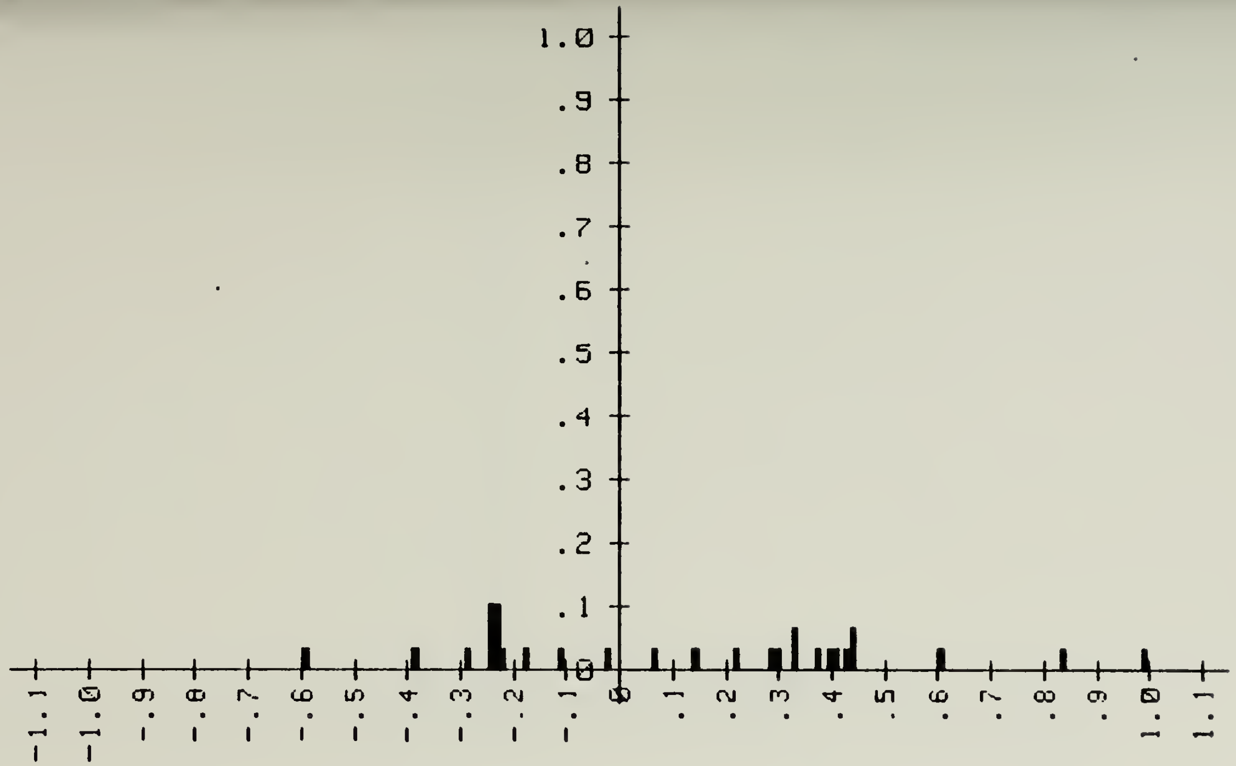
n2 : 1.50000

Number of Rays : 50

Image Plane : 3.25 inches

Aperture : 1.100 inch

Figure C-11. Ray Diagram Using Curve C with the Image Plane at 3.25 Inches.



Normalized Number of Hits vs Distance from the Axis

Curve Number :

Total Number of Hits : 29

Number of Increments [0,1.1] : 100

Number of Rays : 50

% of Rays to Image Plane : 58.00

Image Plane : 3.46 inches

Figure C-12. Histogram of the Ray Distribution on the Image Plane in Figure C-10.

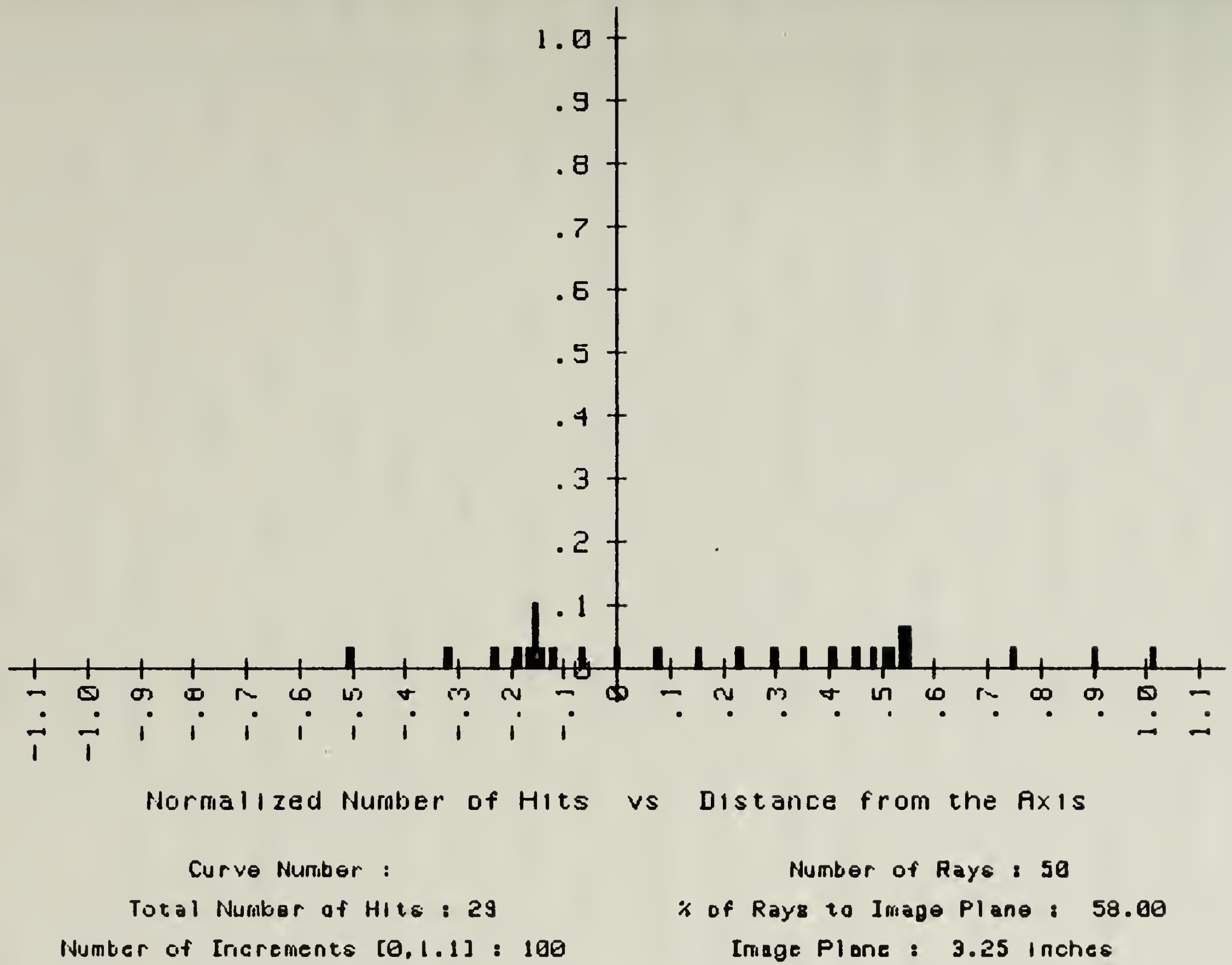


Figure C-13. Histogram of the Ray Distribution on the Image Plane in Figure C-11.

## E. CONCLUSIONS

Three conclusions are drawn from the comparison of curves A, B and C. First, a range of slopes for the second surface exists which will enable the second surface to refract light to a focal point. The ray diagrams for curve A, B and C shown in figures C-2, C-5 and C-9 illustrate this relationship as well as the fact the rays which intercepted the second surface in the lower half-plane were almost all totally internally reflected. The relationship between the slopes of the first surface $\tan\alpha$ and the second surface $\tan\alpha_2$ is shown in figure C-14. Rays are refracted in the region where $\alpha_2 >$ the functional value for a given index of refraction; rays experience total internal reflection when $\alpha_2 <$ the functional value for a given index of refraction. The second conclusion is that a second surface which is convex with respect to the first surface causes light rays to converge in a region much closer to the lens than a concave lens. The third conclusion is that a single polynomial used to describe the second surface of a lens does not satisfactorily refract rays to a focus. CHART calculated surface D shown in figure C-15 for eleven rays. This is surface number one, illustrated in figure 17. A parabola was fitted to the eleven points using the HP-9845B Utilities Package. The fit appears to be quite good, especially when figure C-15 is compared to figure 17. However, when 50 rays are traced through the lens the ray diagram in figure C-15 is the result. The associated

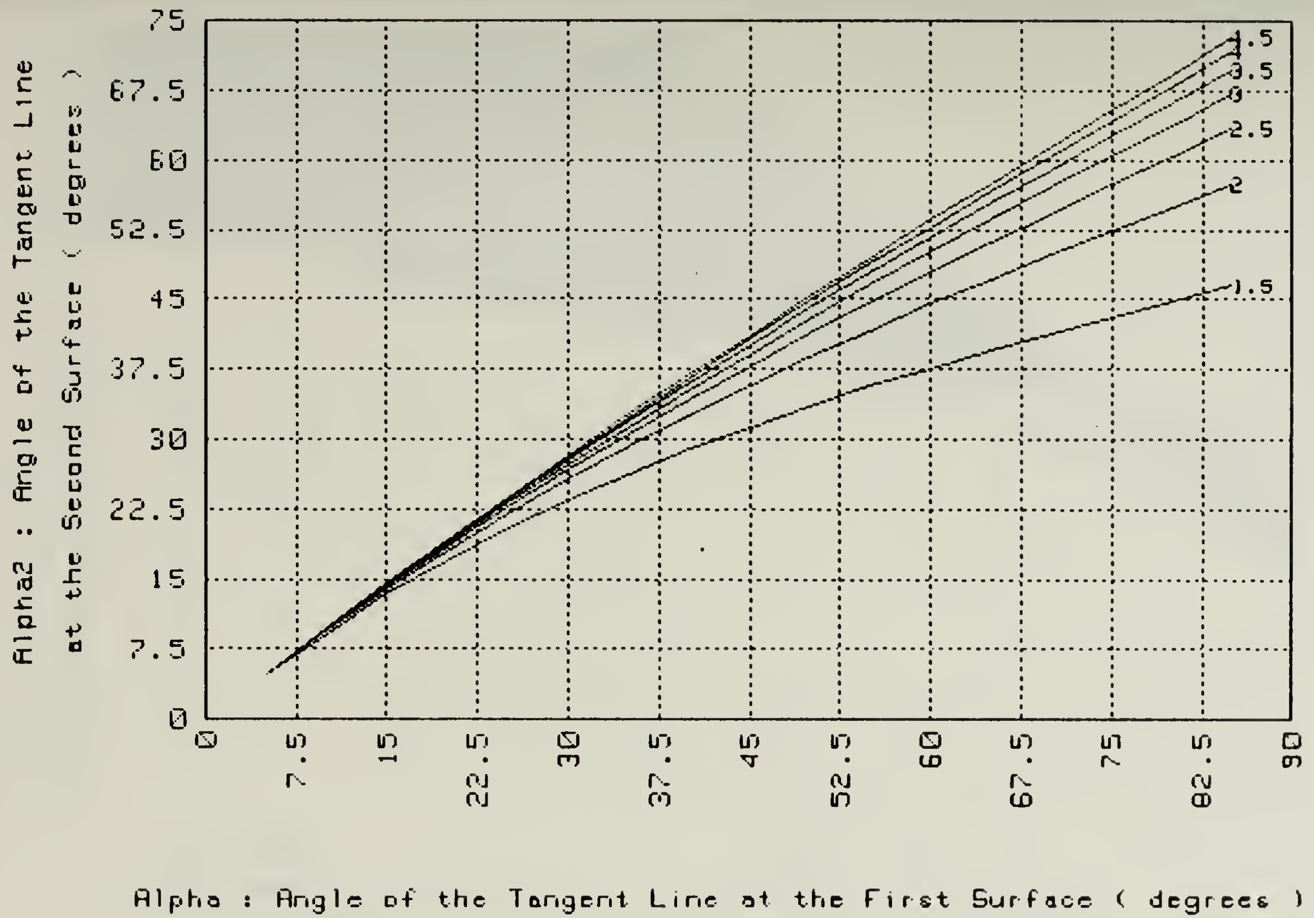
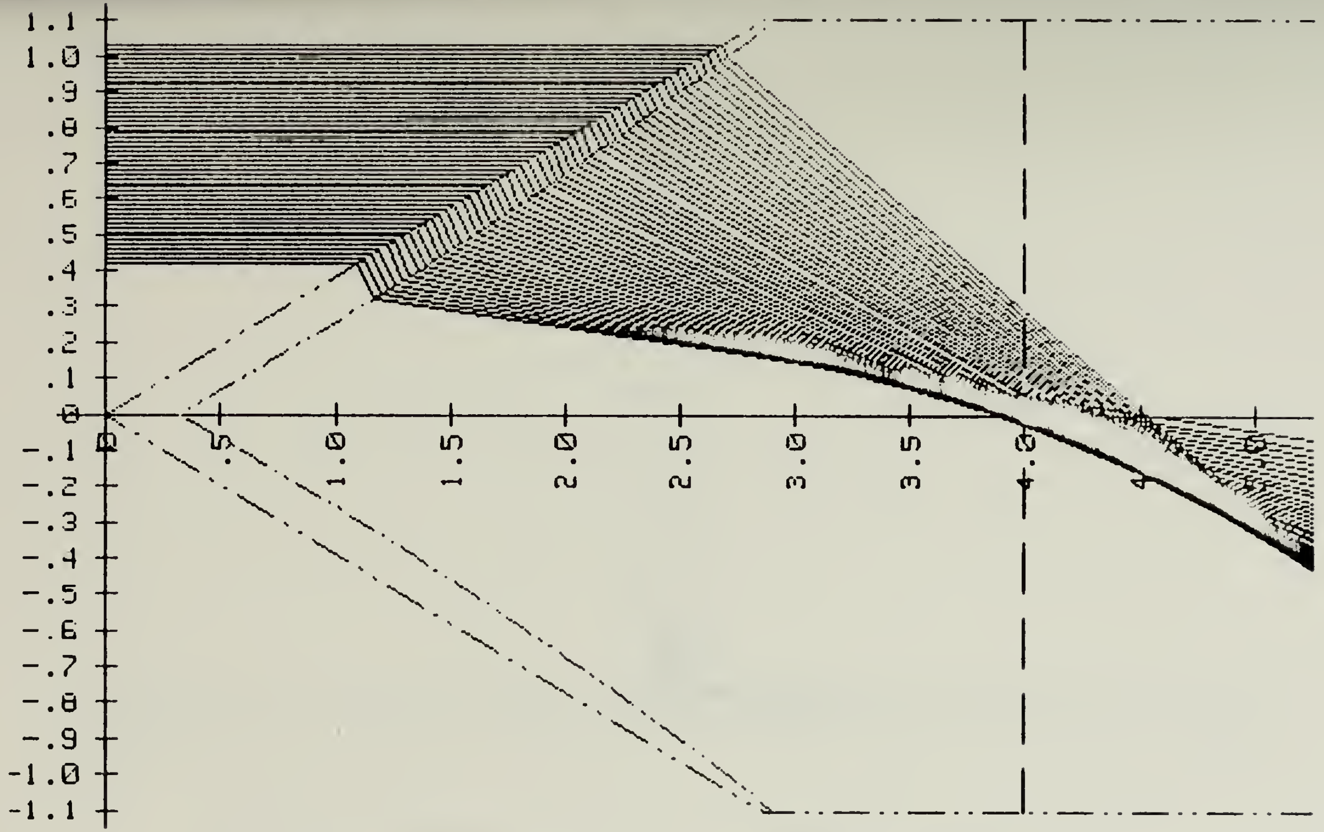


Figure C-14. The Angle of the Tangent Line of the Second Surface $\alpha_2$ as a Function of the Angle of the Tangent Line at the First Surface $\alpha$ for Total Reflection at the Second Surface.



Curve Number :

Incident Ray Angle : 0.00 deg

n2 : 4.00000

Number of Rays : 50

Image Plane : 4.00 inches

Aperture : .610 inch

Figure C-15. Ray Diagram for a Conical Lens Using Curve D as the Second Surface.

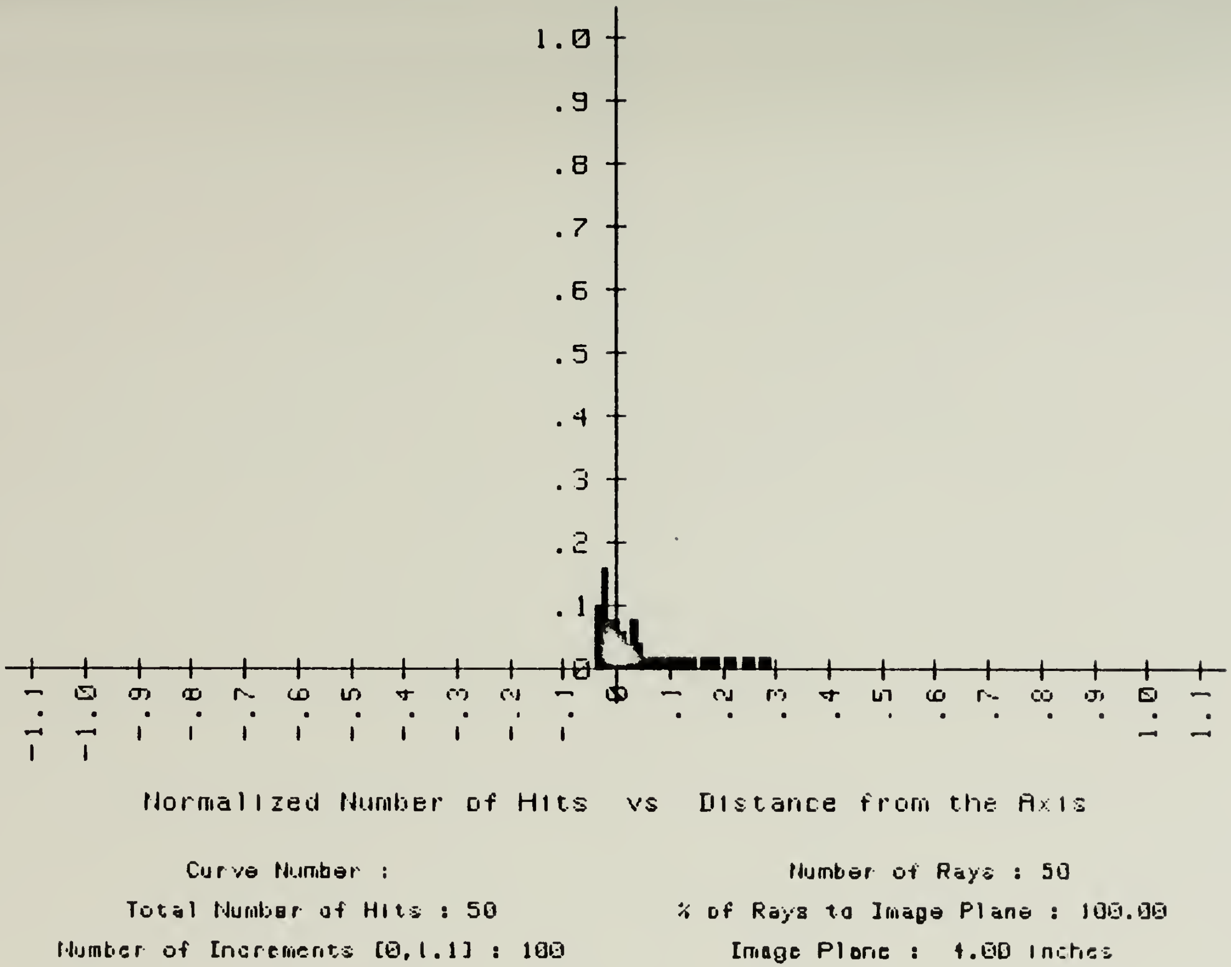


Figure C-16. Histogram of the Ray Distribution on the Image Plane in Figure C-15.

histogram in figure C-16 shows the ray distribution on the image plane at the design focal point of 4.00 inches.

## APPENDIX D

## THE USE OF A GENERAL BLOCK OF MATERIAL AS A LENS

LCDR C.L. Burmaster, Naval Postgraduate School, suggested the use of a rhombus shown in figure D-1 as a refracting lens because this type of lens should transmit light without aberration. The following discussion applies Snell's Law to this lens with the following boundary conditions:

1) Side ① is parallel to side ④.
2) Side ② is parallel to side ③ and symmetrical to the GLM axis.
3) The ray in medium 3 is parallel to the incident ray in medium 1.
4) the index of refraction of the lens is related to the indices of media 1 and 2 in the following manner:

$$n_1 < n_2 \quad \text{and} \quad n_2 > n_3$$

The incident ray in medium 1 makes an angle of incidence $\theta_I$ with $\hat{n}_1$ the normal to the first surface. The ray is refracted into the lens at an angle of refraction, $\theta_{R_1}$ with respect to $\hat{n}$, according to the relation $\sin\theta_{R_1} = (n_1/n_2)\sin\theta_I$ Snell's Law. The ray traverses the lens and forms the angle of incidence $\theta_{I_4}$ with the normal to surface 4. The ray is refracted according to $\sin\theta_{R_4} = (n_3/n_2)\sin\theta_{I_4}$ noting that since surface ④ is parallel to surface ①, $\hat{n}_1$ is parallel to $\hat{n}_4$ and therefore $\theta_{I_4} = \theta_{R_1}$. Since the ray in medium 3 is parallel to the incident ray in medium 1 $\theta_{R_4} = \theta_I$,

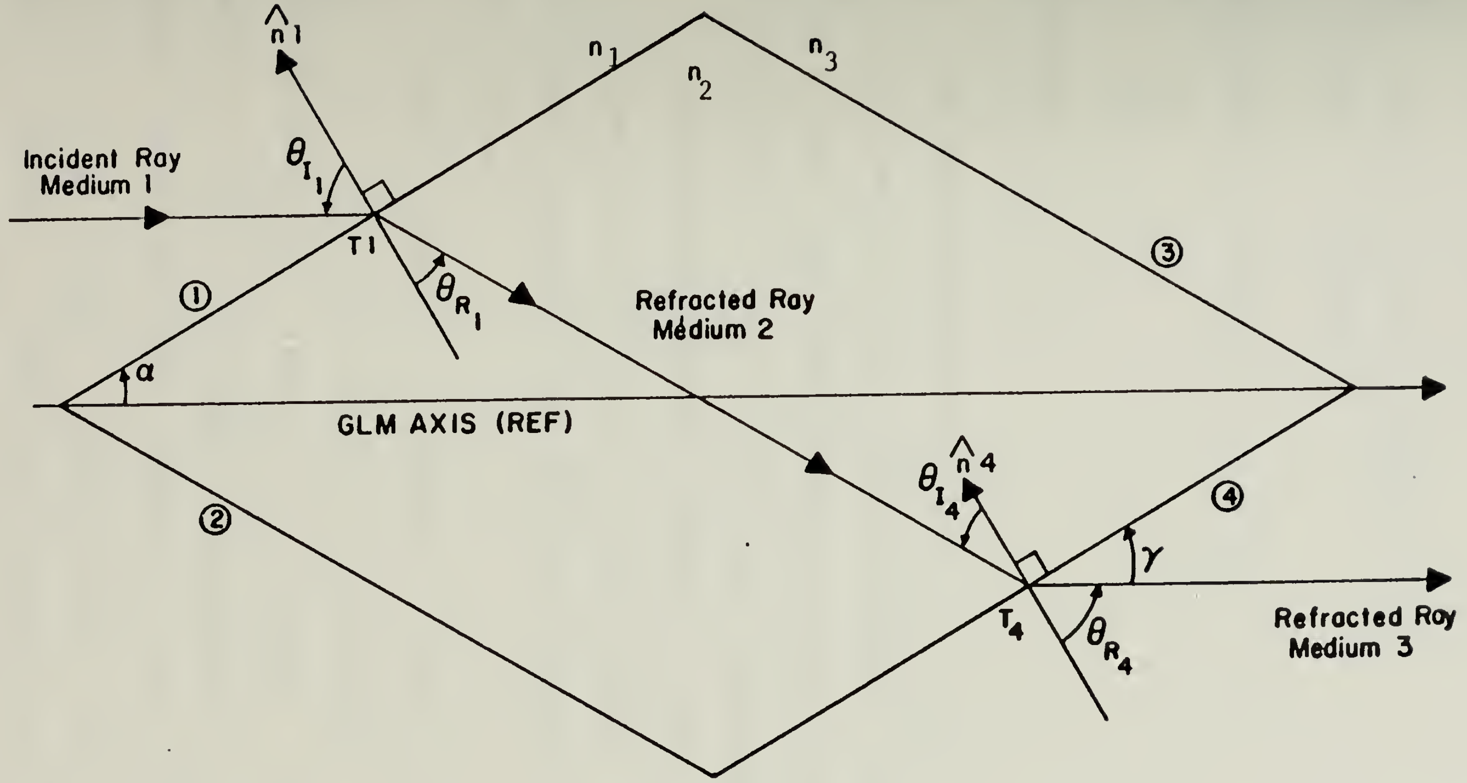


Figure D-1. Rhombus Lens.

and applying Snell's Law at surface number 1

$$\sin\theta_I = \frac{n_2}{n_1} \sin\theta_{R_1}$$

and again at surface number 4

$$\sin\theta_{R_4} = \frac{n_2}{n_3} \sin\theta_{I_4}$$

therefore equating $\theta_{I_1}$ and $\theta_{R_4}$

$$\frac{n_2}{n_1} = \frac{n_2}{n_3}$$

therefore $n_1 = n_3$.

The conclusion for this lens is that the indices of refraction for media 1 and 3 must be equal. The lens causes inversion of the image because the rays are incident in the upper half-plane and exit in the lower half-plane.

A more general lens is shown in figure D-2 which does not have boundary conditions 1) and 2) above. All other boundary conditions apply and in addition the lens is symmetric about the GLM axis.

The incident ray in medium 1 forms an angle of incidence $\theta_{I_1}$ with $\hat{n}_1$, the normal to surface ①. The ray is refracted according to $\sin\theta_{R_1} = (n_1/n_2)\sin\theta_{I_1}$ and traverses the medium, forming the angle of incidence $\theta_{I_4}$ with $\hat{n}_4$, the normal to surface ④. The ray is refracted into medium 3 according to $\sin\theta_{R_4} = (n_2/n_3)\sin\theta_{I_4}$ parallel to the incident ray in medium 1. Noting the angular relationship between $\hat{n}$, and $\hat{n}_4$ in figure D-3 such that $\theta_3 = \theta_2 + \delta$ and $\delta = \alpha - \gamma$, then $\theta_3 = \alpha - \gamma$ where $\alpha$ and $\gamma$ are the angles of sides

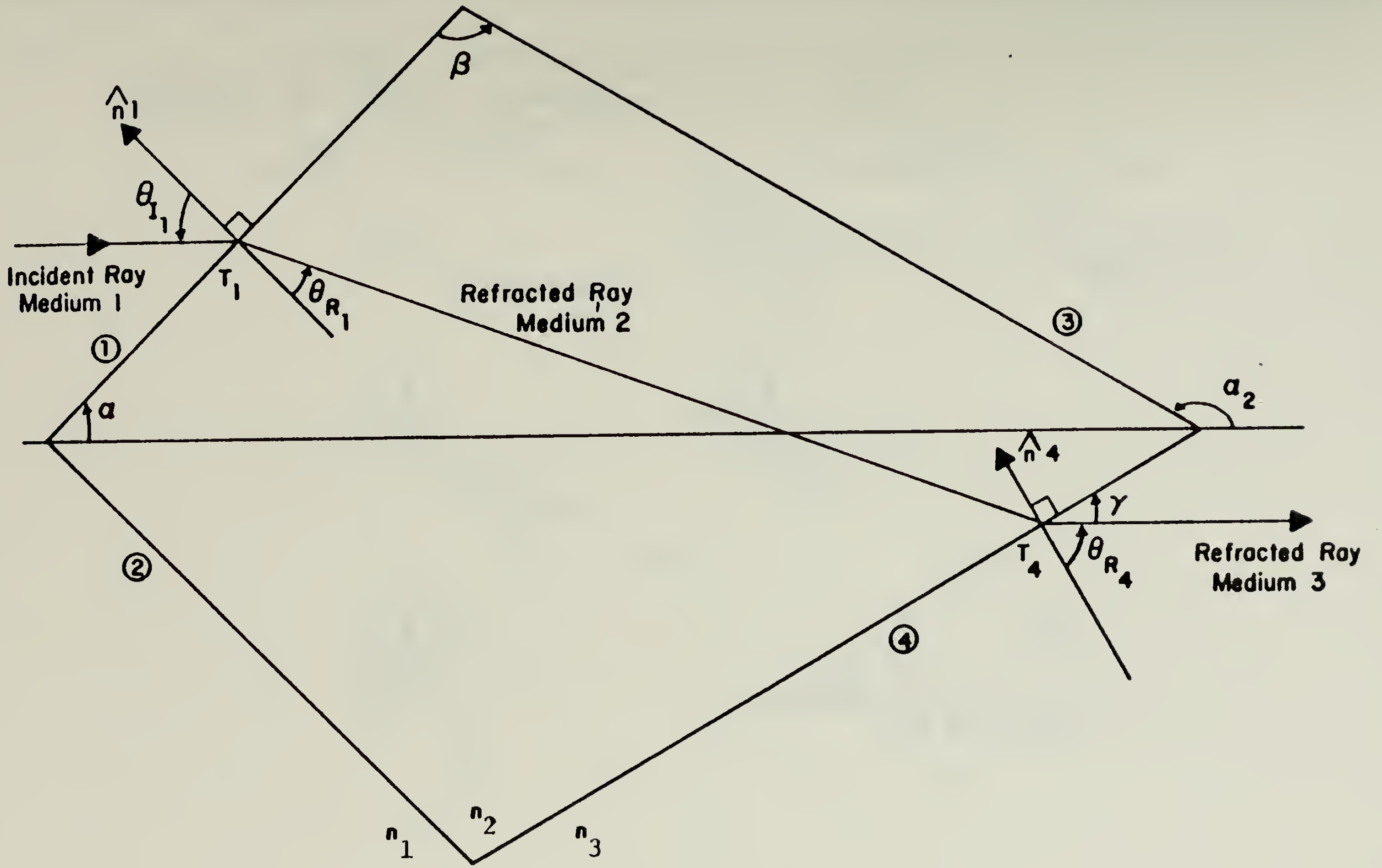


Figure D-2. Shape, Geometry and Symbol Definition for a General Block Lens.

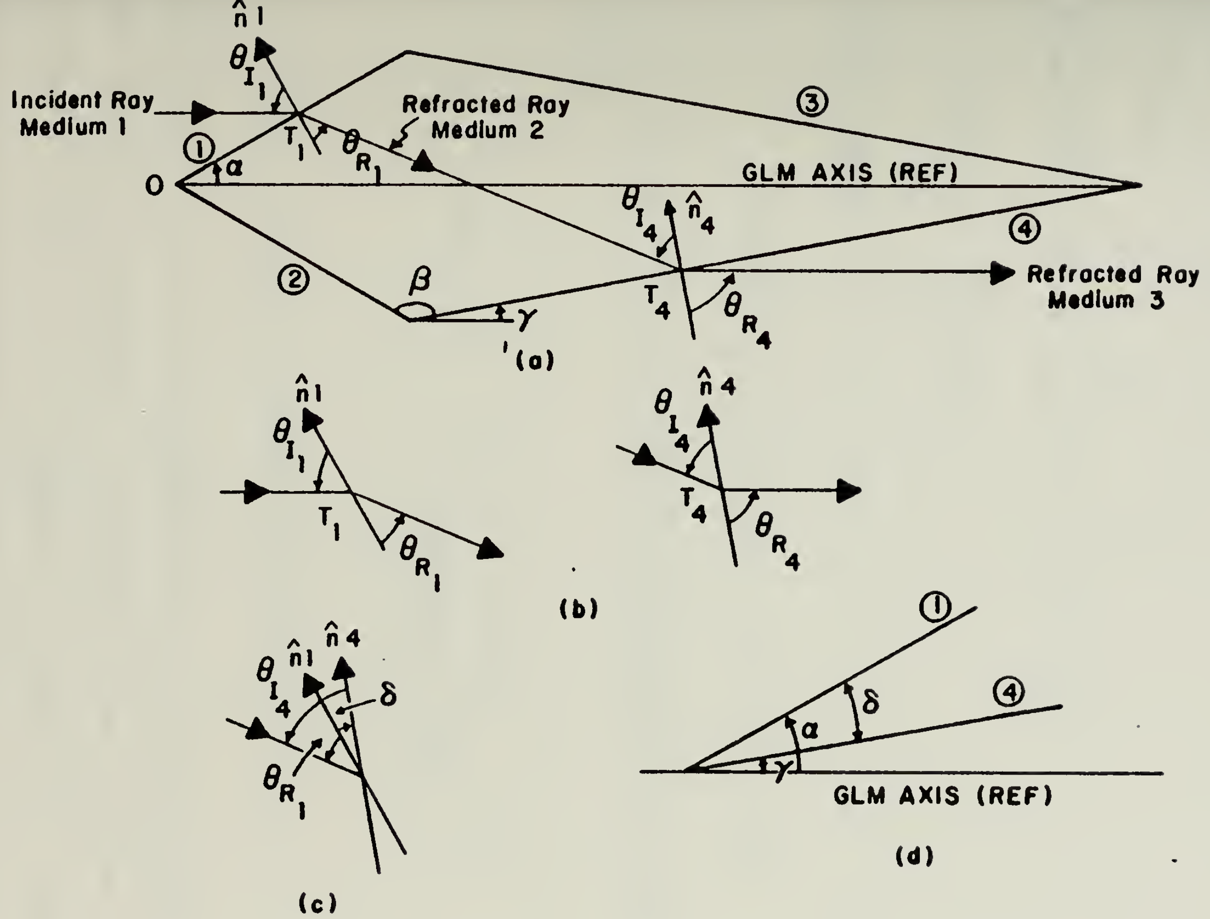


Figure D-3. Angular Relationships for the General Block Lens in Figure D-2.

① and ④ with the GLM axis, respectively.

Now $\theta_{R_4} = \frac{\pi}{2} - \gamma$

Hence $\theta_{R_4} = \frac{\pi}{2} - (\alpha - \gamma)$

Similarly $\theta_{I_4} = \theta_{R_1} + \delta = \theta_{R_1} + \alpha - \gamma$

From Snell's Law applied at side ④ is

$$\sin\theta_{R_4} = \frac{n_2}{n_3} \sin\theta_{I_4} .$$

Noting that $\theta_{I_4} = \theta_{R_1} + \delta$, $\delta = \alpha - \gamma$ and $\gamma = \frac{\pi}{2} \ \theta_{R_4}$

$$\sin\theta_{R_4} = \frac{n_2}{n_3} \sin(\theta_{R_1} + \alpha + \theta_{R_4} - \frac{\pi}{2})$$

Simplifying by trigonometric expansions for sine and cosine yields

$$\tan\theta_{R_4} = -\left[\frac{n_3 + n_2 \sin(\theta_{R_1} + \alpha)}{n_2 \cos(\theta_{R_1} + \alpha)}\right] \qquad \text{(D-1)}$$

$$\text{with} \quad \theta_{I_1} = \arcsin\left[\frac{n_1}{n_2} \sin\theta_{I_1}\right] .$$

Equation (D-1) is a necessary but not sufficient condition for the ray in medium 3 to emerge from surface 4 parallel to to the incident ray in medium 1. The final condition relates the angles of surfaces ① and ④ and $\theta_{I_4} = \theta_{R_I} + \delta$ as shown in figure D-3.

Applying Snell's Law at surface ④ and using trigonometric expansions yields

$$\tan\gamma = \left(\frac{n_2 \sin(\theta_{R_1} + \alpha) - n_3}{n_2 \cos(\theta_{R_1} + \alpha)}\right) \qquad \text{(D-2)}$$

with $\theta_{R_1} = \arcsin\left[ (n_2/n_1) \sin\theta_{I_1} \right]$.

With equations (D-1) and (D-2) rearranged to relate the $\tan(\theta_{R_1} + \alpha)$ as the independent variable the equations become

$$\tan(\theta_{R_1} + \alpha) = -\left(\tan\theta_{R_4} + \frac{n_3}{n_2}\right) \qquad \text{(D-3)}$$

and

$$\tan(\theta_R + \alpha) = -\left(\frac{n_3}{\cos(\theta_{R_1} + \alpha)} + \tan\delta\right) \qquad \text{(D-4)}$$

Since equation (D-3) equals equation (D-4) and solving for $\tan\gamma$ yields

$$\tan\gamma = \tan\theta_{R_4} + \frac{n_3}{\cos(\theta_{R_1} + \alpha)} + \frac{n_3}{n_2} \qquad \text{(D-5)}$$

With the condition that $\gamma + \theta_{R_4} = \frac{\pi}{2}$ and noting that

$\theta_{I_1} = \frac{\pi}{2} - \alpha$, $\delta - \alpha - \gamma$ and $\theta_{I_4} = \theta_{R_1} + \delta$

$$\theta_{R_1} = \arcsin\left[\frac{n_1}{n_2} \sin\left(\frac{\pi}{2} - \alpha\right)\right]$$

and

$$\theta_{R_4} = \arcsin\left[\frac{n_3}{n_2} \sin(\theta_{R_1} + \alpha - \gamma)\right]$$

Equation (D-5) is recognized as a transcendental equation of the form

$$\tan\gamma = \tan\left\{\arcsin\left[\frac{n_3}{n_2}\ \sin(\theta_{R_1} + \alpha-\gamma)\right]\right\} + \frac{n_3}{\cos(\theta_{R_1} + \alpha)} + \frac{n_3}{n_2} \quad \text{(D-6)}$$

which must be solved iteratively. The constraints on the solution are that $(\theta_{R_1} + \alpha) < \frac{\pi}{2}$ and

$(n_3/n_2)\ \sin(\theta_{R_1} + \alpha + \gamma) \leq 1$ and $\theta_{R_4} + \gamma = \frac{\pi}{2}$.

The appearance of the configuration of this lens in 3-dimensions would be two cones fit base to base. The image formed would be inverted with the center missing; i.e. an annulus. Therefore a substantial amount of energy could be lost in the image if the energy distribution of the object is Gaussian with the maximum at the center of the object. Therefore equations (D-1) and (D-6) are necessary and sufficient conditions to insure the incident ray in medium 1 is parallel to the exit ray in medium 3. If the condition that $\theta_{R_2} = \theta_{I_1}$ then the general lens reduces to the rhombus of figure D-1, or that of a parallelopiped in figure D-4. The position a block lens would occupy in the GLM is shown in figure D-5.

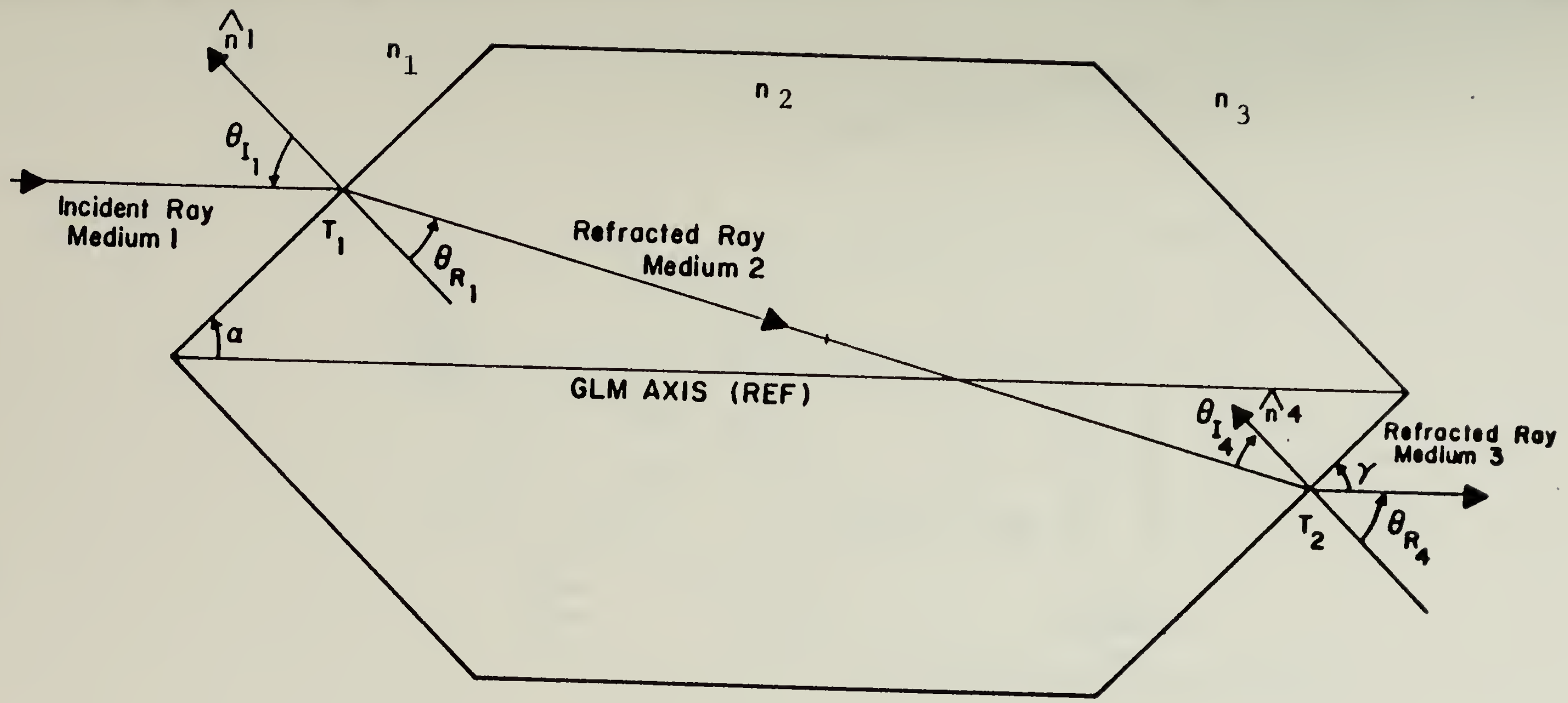


Figure D-4. Parallelopiped Block Lens.

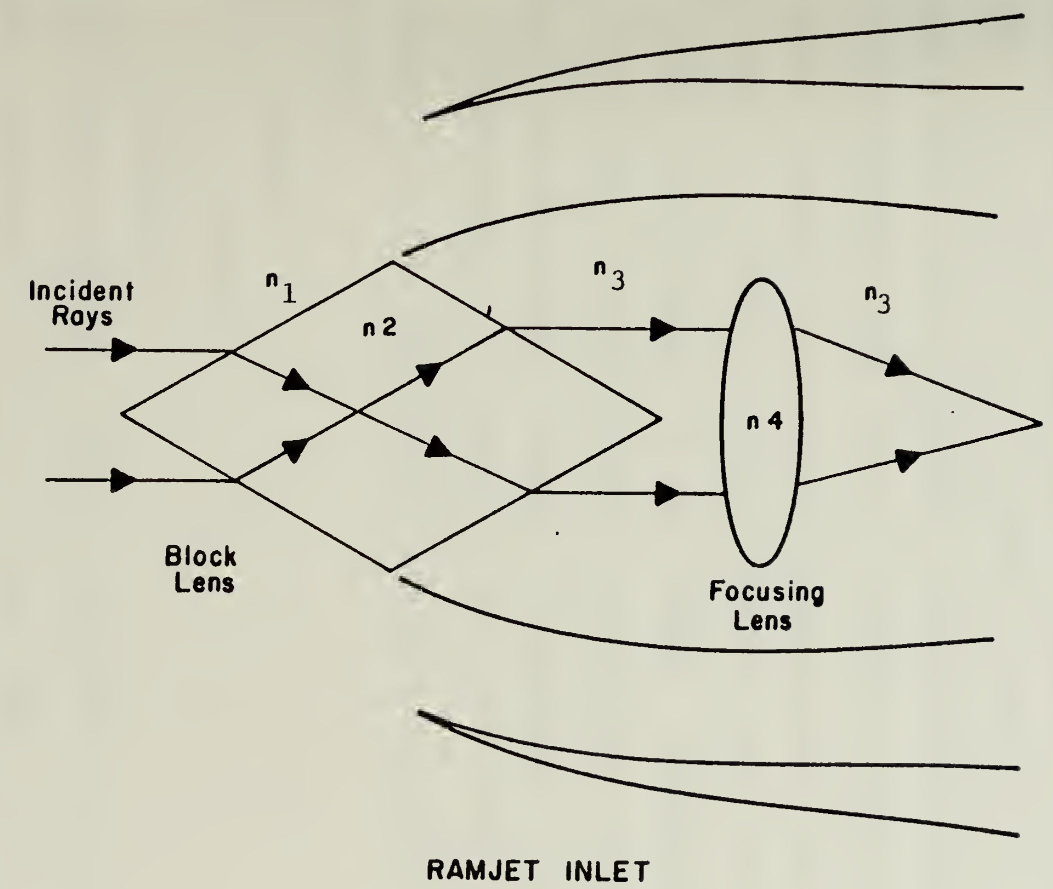


Figure D-5. Position of a Block Lens in the GLM.

# APPENDIX E

## GRADIENT INDEX OPTICS

The development of material which exhibits a continuous change in the optical index of refraction over distance has opened a new area in the field of optics. Gradient index (GRIN) material has been applied in the field of fiber optics communications and integrated circuits with expanding numbers of applications. GRIN has the effect of replacing individual lenses in an optical system. Theoretically GRIN material has the ability to refract light because of the continuous change in index of refraction in the medium. Practically the amount of refraction is proportional to the change in the relative index from macroscopic region to macroscopic region.

GRIN material is produced with one of three types of gradients in the index of refraction referenced to the optical axis:

1. Axial
2. Cylindrical
3. Spherical

Axial GRIN varies the index of refraction along the optical axis. Cylindrical GRIN varies the index of refraction radially from the optical axis. Hence, in three dimensions the surfaces of constant index form concentric cylinders centered on the optical axis. Spherical GRIN varies the index of refraction radially from the origin. The surfaces of constant index form a set of concentric spheres [8].

Born and Wolf [9] discuss the general properties of light rays in GRIN material with spherical symmetry. The trajectory of the light ray is described by Bouguer's relation, $nd\sin\phi$ = constant, where $\phi$ is the angle between the position vector $\vec{r}$ and the tangent at point P as shown in figure E-1 and is illustrated inside the GLM lens in figure E-2. For two dimensions using polar coordinates, the angle can describe explicitly as

$$\sin\phi = \frac{r(\phi)}{\sqrt{r^2(\phi) + \left(\frac{dr}{d\theta}\right)^2}} \tag{E-1}$$

where $dr/d\theta = (r/c)\sqrt{n^2r^2 - c^2}$ ; $\theta$ is the angle between two consecutive radii and $c$ is Bouguer's constant. In three dimensions using spherical coordinates the relation for the change in $\theta$, illustrated in figure E-3, over a range of radius is given by

$$\theta - \theta_o = c\int_{r_o}^{r} \frac{dr}{\sqrt{n^2r^2 - c^2}} \tag{E-2}$$

Given a sphere of GRIN material shown in figure E-4 with a spherical gradient, a GLM lens can be selectively cut from the material. The lens can be customized with an index of refraction profile suitable for the application. A possible profile for the GLM conical lens, is shown in figure E-5, with a minimum and maximum located at a specific radius from the origin. Hence the ability to customize the GLM lens

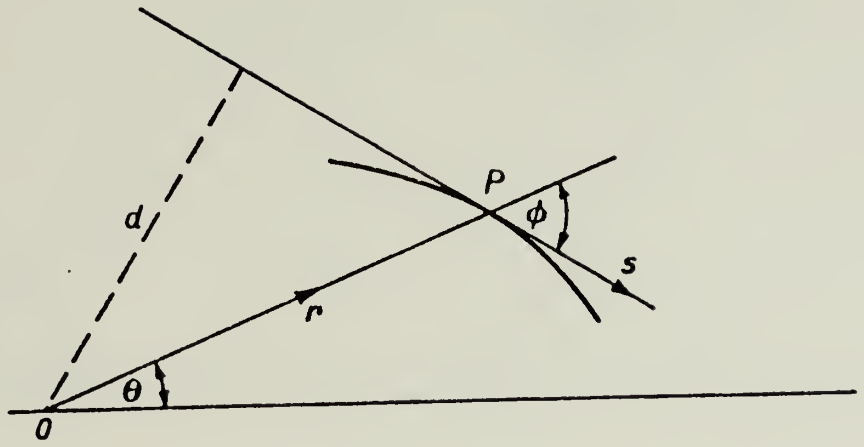


Figure E-1. Illustrating Bouguer's Formula nd = constant, for Rays in a Medium with Spherical Symmetry.

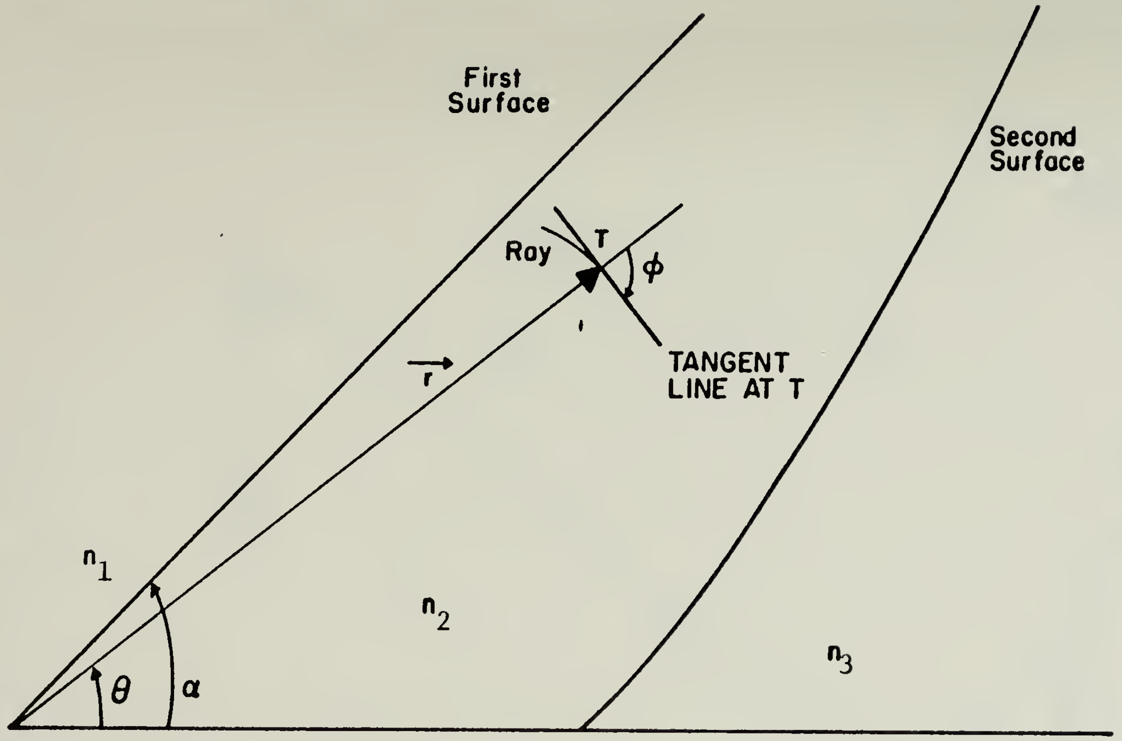


Figure E-2. Illustration of Bouguer's Formula in the GLM Conical Lens.

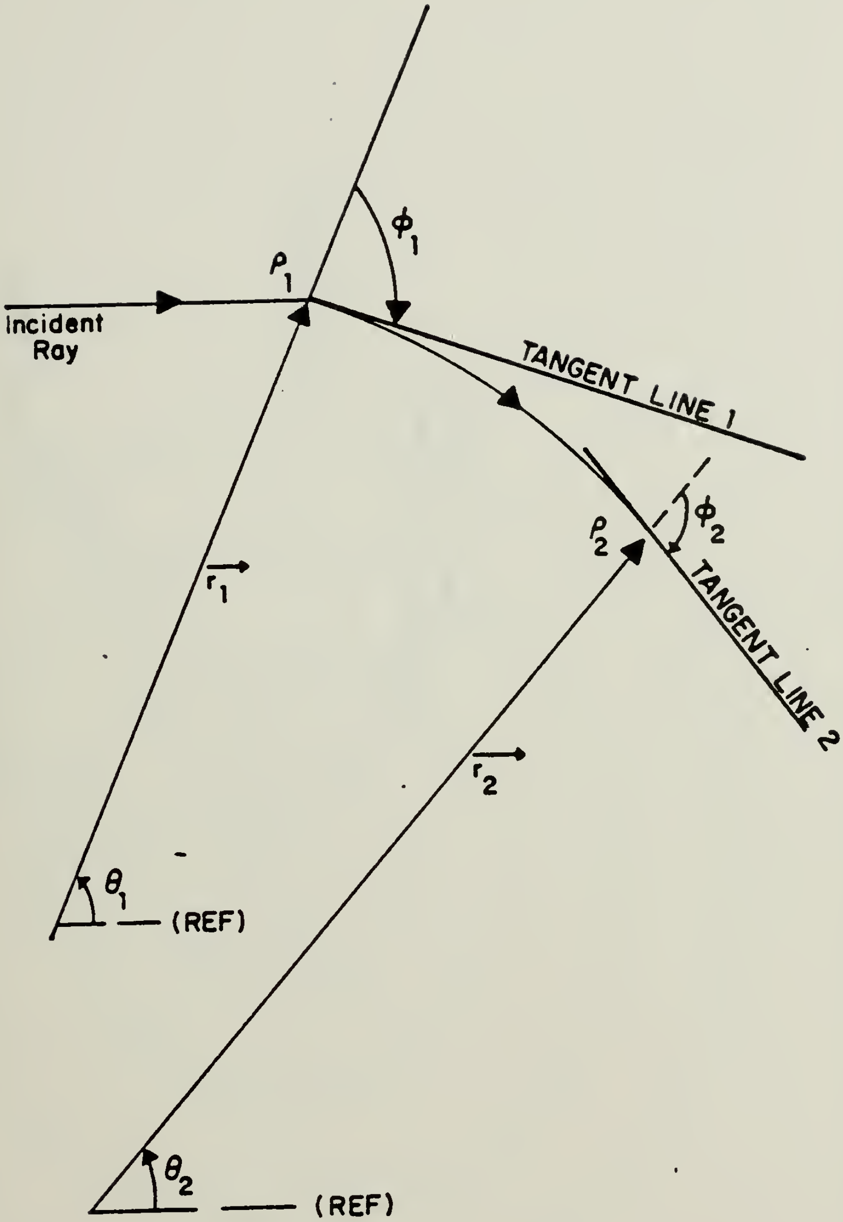


Figure E-3. Illustration of the Relationship Between $\vec{r}$ and $\phi$ Between Adjacent Rays in the GLM GRIN Lens.

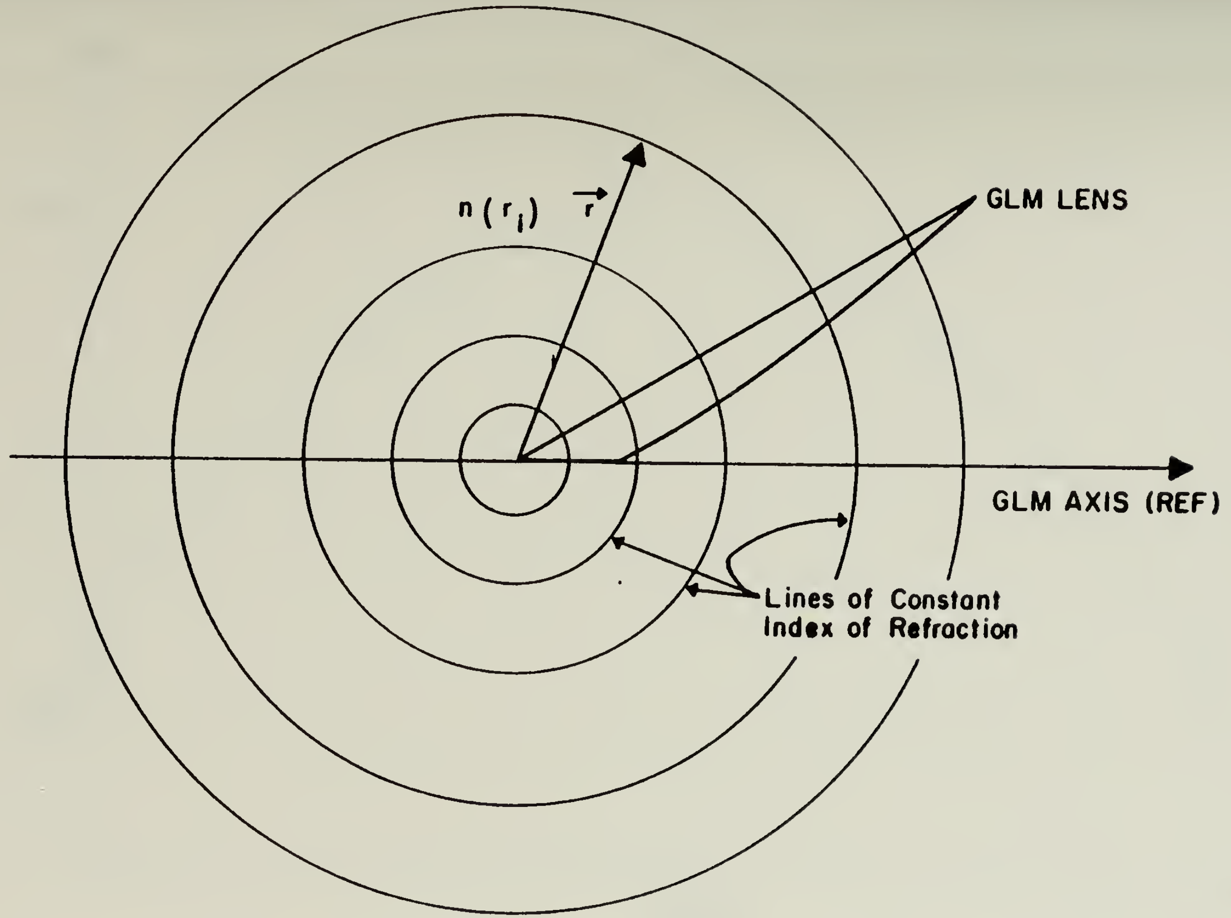


Figure E-4. Illustration of a GLM Lens as a Section of GRIN Material Exhibiting Spherical Symmetry.

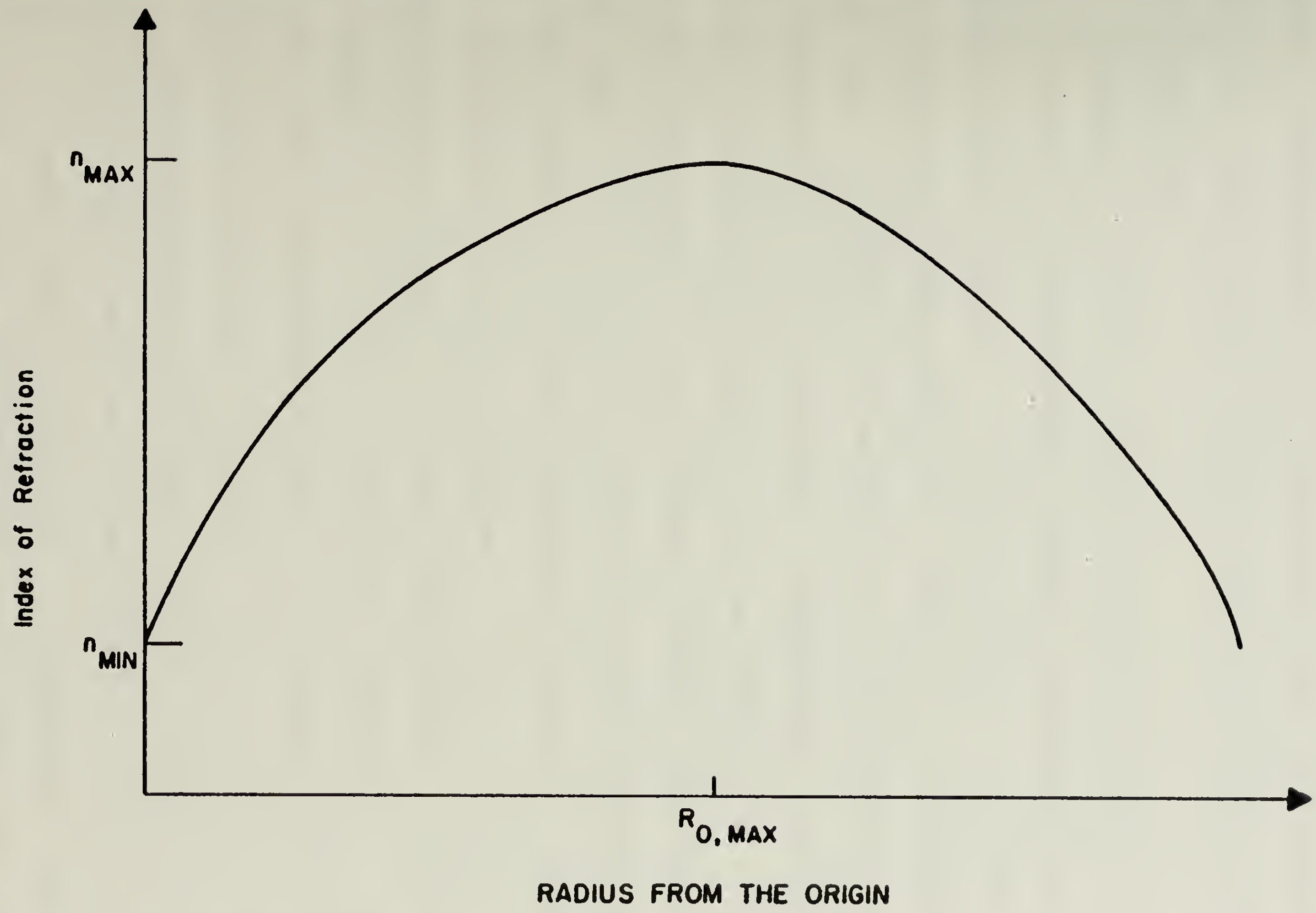


Figure E-5. Index of Refraction Profile as a Function of Radius from the Origin.

enables the lens designer to place the maximum value of the index of refraction at the desired location. The profile shown in figure E-5 can be applied to the GLM conical lens with the index profile varying along the first surface shown in figure E-6.

Bouguer's constant, defined by the relation $nr\sin\phi = c$ is a function of the radius, index of refraction at that radius and the angle $\phi$ defined earlier. However, once the ray enters the GRIN medium, the constant is invariant along the ray. Therefore the constant can theoretically be determined explicitly at any point. Practically, two points exist at which the constant can be determined. These are at the intercept of the incident ray and the first surface and the intercept of the ray in the conical lens and the second surface. The most convenient position to calculate $c$ is the first surface because $n$, $r$ and $\phi$ can all be determined explicitly on the first surface. Table E-I is a tabulation of values of Bouguer's constant $c$. Figure E-7 shows the relationship between the constant $c$ and the radius from the origin along the first surface of the GLM.

Calculating the ray paths through a conical lens composed of GRIN material must be accomplished by solving equation (E-2) at each point in the lens. A complete algorithm was not written because of the time constraints, but TRACE can calculate the value of Bouguer's constant and refract the ray at the first surface shown in figure E-8 using the value of

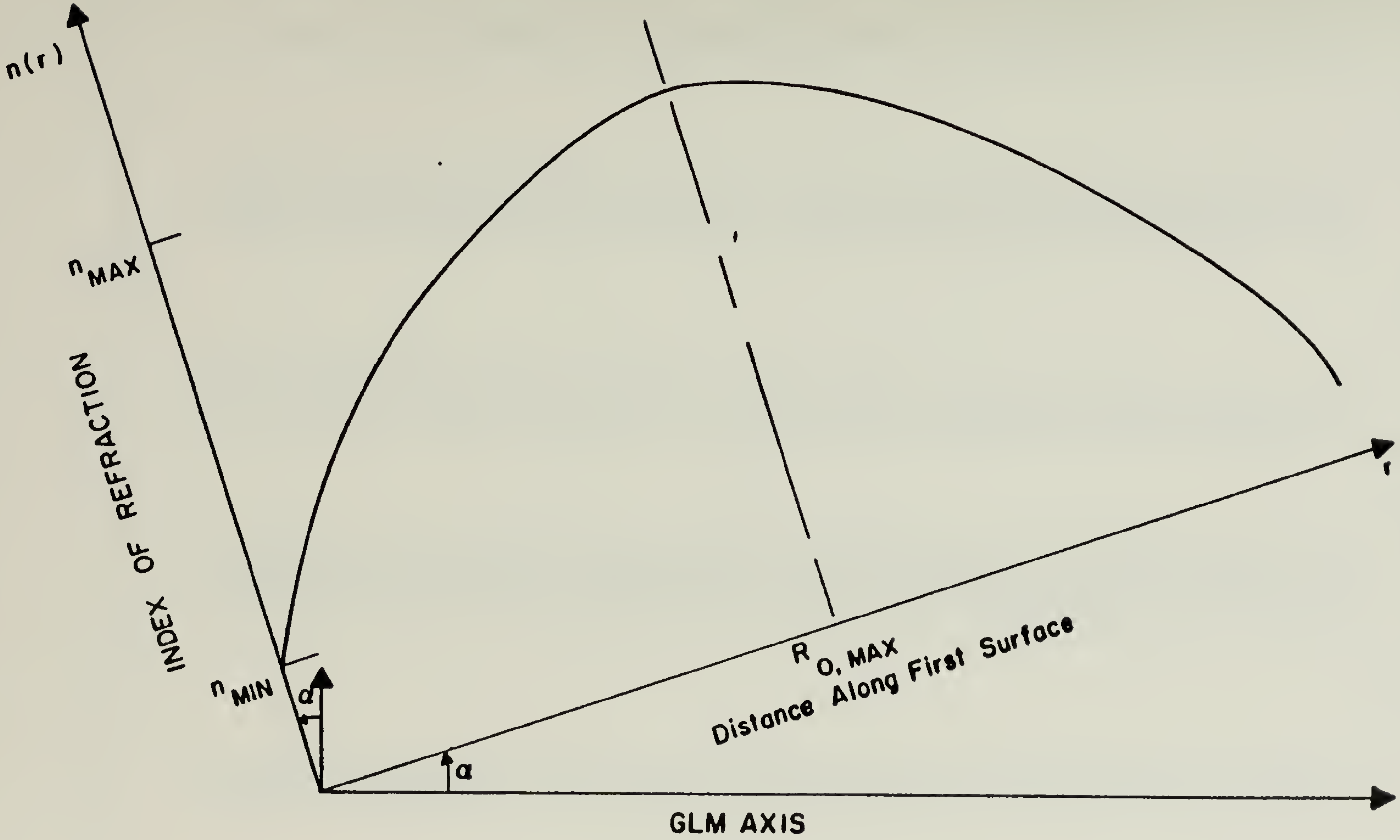


Figure E-6. Index of Refraction as a Function of Distance Along the First Surface of the GLM Lens.

# TABLE E-I

## BOUGUER'S GRIN CONSTANT

| I | n(I) | r(I) | Phi(I) | Sin(Phi) | c(I) |
|---|---|---|---|---|---|
| 1 | 2.4406 | .0614 | 67.5096 | .9239 | .1394 |
| 2 | 2.6170 | .1228 | 69.0997 | .9342 | .3002 |
| 3 | 2.7481 | .1842 | 70.1402 | .9405 | .4760 |
| 4 | 2.8551 | .2456 | 70.9137 | .9450 | .6625 |
| 5 | 2.9461 | .3069 | 71.5251 | .9485 | .8577 |
| 6 | 3.0254 | .3683 | 72.0263 | .9512 | 1.0600 |
| 7 | 3.0956 | .4297 | 72.4472 | .9534 | 1.2683 |
| 8 | 3.1583 | .4911 | 72.8067 | .9553 | 1.4818 |
| 9 | 3.2147 | .5525 | 73.1174 | .9569 | 1.6996 |
| 10 | 3.2656 | .6139 | 73.3882 | .9583 | 1.9210 |
| 11 | 3.3116 | .6753 | 73.6258 | .9594 | 2.1456 |
| 12 | 3.3534 | .7367 | 73.8352 | .9605 | 2.3727 |
| 13 | 3.3911 | .7981 | 74.0200 | .9614 | 2.6017 |
| 14 | 3.4252 | .8595 | 74.1834 | .9621 | 2.8324 |
| 15 | 3.4560 | .9208 | 74.3277 | .9628 | 3.0641 |
| 16 | 3.4835 | .9822 | 74.4549 | .9634 | 3.2965 |
| 17 | 3.5081 | 1.0436 | 74.5664 | .9639 | 3.5291 |
| 18 | 3.5298 | 1.1050 | 74.6637 | .9644 | 3.7616 |
| 19 | 3.5488 | 1.1664 | 74.7477 | .9648 | 3.9935 |
| 20 | 3.5651 | 1.2278 | 74.8193 | .9651 | 4.2245 |
| 21 | 3.5789 | 1.2892 | 74.8792 | .9654 | 4.4541 |
| 22 | 3.5902 | 1.3506 | 74.9279 | .9656 | 4.6820 |
| 23 | 3.5991 | 1.4120 | 74.9659 | .9658 | 4.9078 |
| 24 | 3.6055 | 1.4733 | 74.9935 | .9659 | 5.1310 |
| 25 | 3.6096 | 1.5347 | 75.0109 | .9660 | 5.3514 |
| 26 | 3.6114 | 1.5961 | 75.0184 | .9660 | 5.5683 |
| 27 | 3.6108 | 1.6575 | 75.0159 | .9660 | 5.7815 |
| 28 | 3.6079 | 1.7189 | 75.0034 | .9659 | 5.9904 |
| 29 | 3.6026 | 1.7803 | 74.9809 | .9658 | 6.1946 |
| 30 | 3.5949 | 1.8417 | 74.9482 | .9657 | 6.3936 |
| 31 | 3.5849 | 1.9031 | 74.9049 | .9655 | 6.5869 |
| 32 | 3.5723 | 1.9645 | 74.8506 | .9652 | 6.7738 |
| 33 | 3.5573 | 2.0259 | 74.7850 | .9649 | 6.9539 |
| 34 | 3.5396 | 2.0872 | 74.7073 | .9646 | 7.1265 |
| 35 | 3.5193 | 2.1486 | 74.6168 | .9642 | 7.2908 |
| 36 | 3.4962 | 2.2100 | 74.5125 | .9637 | 7.4461 |
| 37 | 3.4701 | 2.2714 | 74.3933 | .9631 | 7.5915 |
| 38 | 3.4410 | 2.3328 | 74.2578 | .9625 | 7.7261 |
| 39 | 3.4086 | 2.3942 | 74.1042 | .9618 | 7.8488 |
| 40 | 3.3727 | 2.4556 | 73.9304 | .9609 | 7.9583 |
| 41 | 3.3330 | 2.5170 | 73.7338 | .9600 | 8.0533 |
| 42 | 3.2892 | 2.5784 | 73.5108 | .9589 | 8.1319 |
| 43 | 3.2408 | 2.6397 | 73.2573 | .9576 | 8.1921 |
| 44 | 3.1872 | 2.7011 | 72.9675 | .9561 | 8.2315 |
| 45 | 3.1278 | 2.7625 | 72.6337 | .9544 | 8.2467 |
| 46 | 3.0615 | 2.8239 | 72.2454 | .9524 | 8.2337 |
| 47 | 2.9870 | 2.8853 | 71.7873 | .9499 | 8.1867 |
| 48 | 2.9022 | 2.9467 | 71.2357 | .9468 | 8.0975 |
| 49 | 2.8039 | 3.0081 | 70.5519 | .9429 | 7.9532 |
| 50 | 2.6864 | 3.0695 | 69.6639 | .9377 | 7.7317 |

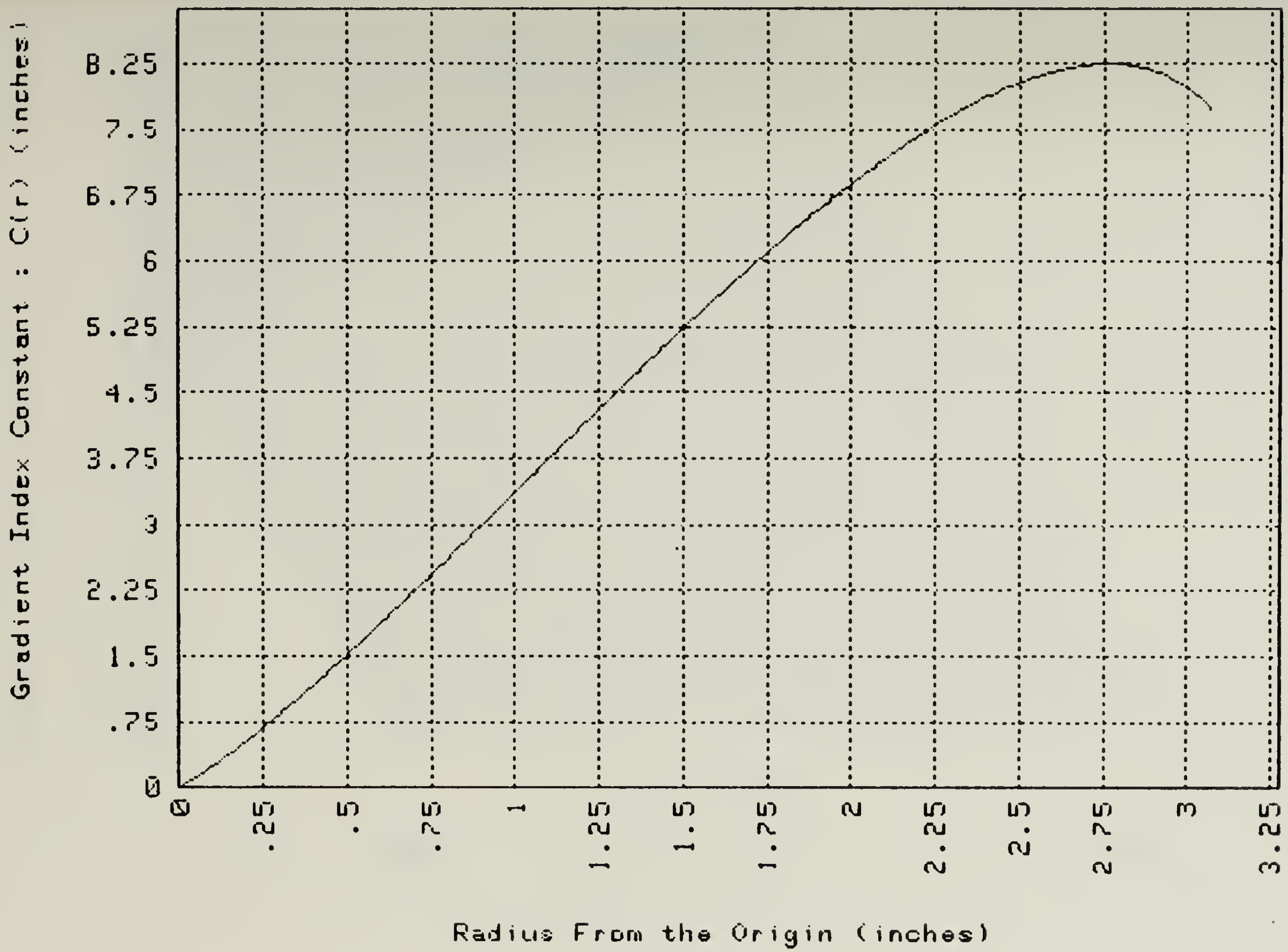


Figure E-7. Bouguer's GRIN Constant as a Function of Radius from the Origin.

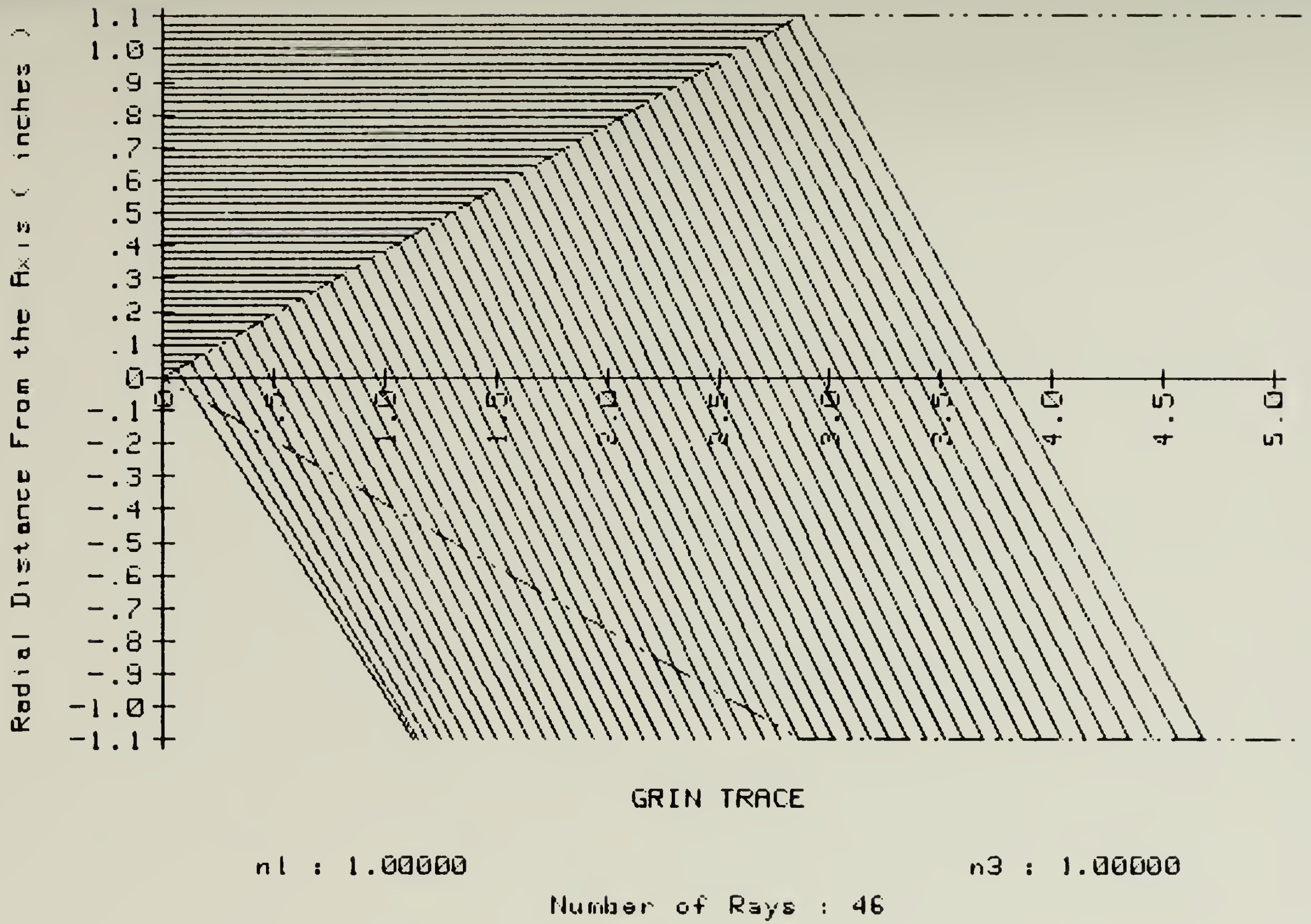


Figure E-8. Ray Diagram Showing the Change in Refracted Ray Angle at the First Surface of the GLM Lens Using GRIN Material for the Lens.

the index of refraction at that point. The calculated data for the ray diagram is tabulated in Tables E-II and E-III.

# TABLE E-II

## INITIAL PARAMETER VALUES

```
RHO(INITIAL) =  0.00 degrees                          Alpha =  21.00 degrees
TAN(RHO-INITIAL) =  0.00                              Tan(Alpha) =  .38

n1 =  1.00000                                         n3 =  1.00000

Nmin =  2.00000

       n2(1) =  2.44494         n2(2) =  2.62297         n2(3) =  2.75522
       n2(4) =  2.86301         n2(5) =  2.95465         n2(6) =  3.03445
       n2(7) =  3.10497         n2(8) =  3.16790         n2(9) =  3.22441
      n2(10) =  3.27535        n2(11) =  3.32137        n2(12) =  3.36296
      n2(13) =  3.40051        n2(14) =  3.43436        n2(15) =  3.46474
      n2(16) =  3.49189        n2(17) =  3.51596        n2(18) =  3.53710
      n2(19) =  3.55543        n2(20) =  3.57106        n2(21) =  3.58406
      n2(22) =  3.59449        n2(23) =  3.60241        n2(24) =  3.60785
      n2(25) =  3.61083        n2(26) =  3.61138        n2(27) =  3.60949
      n2(28) =  3.60516        n2(29) =  3.59836        n2(30) =  3.58907
      n2(31) =  3.57723        n2(32) =  3.56280        n2(33) =  3.54569
      n2(34) =  3.52582        n2(35) =  3.50309        n2(36) =  3.47735
      n2(37) =  3.44845        n2(38) =  3.41619        n2(39) =  3.38033
      n2(40) =  3.34060        n2(41) =  3.29663        n2(42) =  3.24798
      n2(43) =  3.19407        n2(44) =  3.13415        n2(45) =  3.06722
      n2(46) =  2.99185        n2(47) =  2.90593        n2(48) =  2.80612
      n2(49) =  2.68635        n2(50) =  2.68635        n2(51) =  0.00000

R0_max =  1.61147  inches

Aperture =  1.100       Ya =  0.000  inch       Yb =  1.100  inch

Number of Rays =  50
```

# TABLE E-III

## RAY DIAGRAM DATA CALCULATED FOR FIGURE E-8

| (X0,Y0) | | (X1,Y1) | | (X3,Y3) | | (Xc,Yc) | | N2 | RHO |
|---|---|---|---|---|---|---|---|---|---|
| 0.00 | .02 | .06 | .02 | 4.06 | -4.20 | .08 | 0.00 | 2.44 | -46.51 |
| 0.00 | .04 | .11 | .04 | 3.96 | -4.24 | .15 | 0.00 | 2.62 | -48.10 |
| 0.00 | .07 | .17 | .07 | 3.91 | -4.26 | .23 | 0.00 | 2.75 | -49.14 |
| 0.00 | .09 | .23 | .09 | 3.89 | -4.26 | .30 | 0.00 | 2.86 | -49.91 |
| 0.00 | .11 | .29 | .11 | 3.89 | -4.26 | .38 | 0.00 | 2.95 | -50.53 |
| 0.00 | .13 | .34 | .13 | 3.90 | -4.26 | .45 | 0.00 | 3.03 | -51.03 |
| 0.00 | .15 | .40 | .15 | 3.92 | -4.26 | .52 | 0.00 | 3.10 | -51.45 |
| 0.00 | .18 | .46 | .18 | 3.94 | -4.25 | .60 | 0.00 | 3.16 | -51.81 |
| 0.00 | .20 | .52 | .20 | 3.97 | -4.24 | .67 | 0.00 | 3.21 | -52.12 |
| 0.00 | .22 | .57 | .22 | 4.00 | -4.22 | .74 | 0.00 | 3.27 | -52.39 |
| 0.00 | .24 | .63 | .24 | 4.03 | -4.21 | .82 | 0.00 | 3.31 | -52.63 |
| 0.00 | .26 | .69 | .26 | 4.07 | -4.20 | .89 | 0.00 | 3.35 | -52.84 |
| 0.00 | .29 | .75 | .29 | 4.11 | -4.18 | .96 | 0.00 | 3.39 | -53.02 |
| 0.00 | .31 | .80 | .31 | 4.15 | -4.16 | 1.03 | 0.00 | 3.43 | -53.18 |
| 0.00 | .33 | .86 | .33 | 4.19 | -4.15 | 1.11 | 0.00 | 3.46 | -53.33 |
| 0.00 | .35 | .92 | .35 | 4.24 | -4.13 | 1.18 | 0.00 | 3.48 | -53.45 |
| 0.00 | .37 | .97 | .37 | 4.29 | -4.11 | 1.25 | 0.00 | 3.51 | -53.57 |
| 0.00 | .40 | 1.03 | .40 | 4.33 | -4.09 | 1.32 | 0.00 | 3.53 | -53.66 |
| 0.00 | .42 | 1.09 | .42 | 4.38 | -4.08 | 1.40 | 0.00 | 3.55 | -53.75 |
| 0.00 | .44. | 1.15 | .44 | 4.43 | -4.06 | 1.47 | 0.00 | 3.57 | -53.82 |
| 0.00 | .46 | 1.20 | .46 | 4.49 | -4.04 | 1.54 | 0.00 | 3.58 | -53.88 |
| 0.00 | .48 | 1.26 | .48 | 4.54 | -4.02 | 1.61 | 0.00 | 3.59 | -53.93 |
| 0.00 | .51 | 1.32 | .51 | 4.59 | -4.00 | 1.69 | 0.00 | 3.60 | -53.97 |
| 0.00 | .53 | 1.38 | .53 | 4.65 | -3.97 | 1.76 | 0.00 | 3.61 | -53.99 |
| 0.00 | .55 | 1.43 | .55 | 4.70 | -3.95 | 1.83 | 0.00 | 3.61 | -54.01 |
| 0.00 | .57 | 1.49 | .57 | 4.76 | -3.93 | 1.91 | 0.00 | 3.61 | -54.02 |
| 0.00 | .59 | 1.55 | .59 | 4.82 | -3.91 | 1.98 | 0.00 | 3.61 | -54.02 |
| 0.00 | .62 | 1.60 | .62 | 4.88 | -3.89 | 2.05 | 0.00 | 3.61 | -54.00 |
| 0.00 | .64 | 1.66 | .64 | 4.93 | -3.86 | 2.13 | 0.00 | 3.60 | -53.98 |
| 0.00 | .66 | 1.72 | .66 | 5.00 | -3.84 | 2.20 | 0.00 | 3.59 | -53.95 |
| 0.00 | .68 | 1.78 | .68 | 5.06 | -3.82 | 2.27 | 0.00 | 3.58 | -53.90 |
| 0.00 | .70 | 1.83 | .70 | 5.12 | -3.79 | 2.35 | 0.00 | 3.57 | -53.85 |
| 0.00 | .73 | 1.89 | .73 | 5.18 | -3.77 | 2.42 | 0.00 | 3.56 | -53.78 |
| 0.00 | .75 | 1.95 | .75 | 5.25 | -3.74 | 2.50 | 0.00 | 3.54 | -53.71 |
| 0.00 | .77 | 2.01 | .77 | 5.31 | -3.72 | 2.57 | 0.00 | 3.52 | -53.62 |
| 0.00 | .79 | 2.06 | .79 | 5.38 | -3.69 | 2.65 | 0.00 | 3.50 | -53.51 |
| 0.00 | .81 | 2.12 | .81 | 5.45 | -3.67 | 2.73 | 0.00 | 3.47 | -53.39 |
| 0.00 | .84 | 2.18 | .84 | 5.52 | -3.64 | 2.80 | 0.00 | 3.44 | -53.26 |
| 0.00 | .86 | 2.24 | .86 | 5.59 | -3.61 | 2.88 | 0.00 | 3.41 | -53.10 |
| 0.00 | .88 | 2.29 | .88 | 5.66 | -3.58 | 2.96 | 0.00 | 3.37 | -52.93 |
| 0.00 | .90 | 2.35 | .90 | 5.74 | -3.55 | 3.04 | 0.00 | 3.33 | -52.73 |
| 0.00 | .92 | 2.41 | .92 | 5.82 | -3.52 | 3.12 | 0.00 | 3.29 | -52.51 |
| 0.00 | .95 | 2.46 | .95 | 5.90 | -3.49 | 3.20 | 0.00 | 3.24 | -52.26 |
| 0.00 | .97 | 2.52 | .97 | 5.99 | -3.46 | 3.28 | 0.00 | 3.19 | -51.97 |
| 0.00 | .99 | 2.58 | .99 | 6.07 | -3.43 | 3.36 | 0.00 | 3.13 | -51.63 |
| 0.00 | 1.01 | 2.64 | 1.01 | 6.17 | -3.39 | 3.45 | 0.00 | 3.06 | -51.25 |
| 0.00 | 1.03 | 2.69 | 1.03 | 6.27 | -3.35 | 3.54 | 0.00 | 2.99 | -50.79 |
| 0.00 | 1.06 | 2.75 | 1.06 | 6.38 | -3.31 | 3.63 | 0.00 | 2.90 | -50.24 |
| 0.00 | 1.08 | 2.81 | 1.08 | 6.51 | -3.26 | 3.73 | 0.00 | 2.80 | -49.55 |
| 0.00 | 1.10 | 2.87 | 1.10 | 6.65 | -3.20 | 3.83 | 0.00 | 2.69 | -48.66 |

## APPENDIX F

## SECOND SURFACE GENERATION: ITERATIVE SOLUTION

### A. INTRODUCTION

One of the conclusions reached as a result of the investigation of curves A, B and C in Appendix C was that using a single polynomial to describe the second surface of a conical lens cannot satisfactorily refract light to a single, distinct focal point. Hence, a satisfactory image of an object cannot be formed.

Three options for further study remained. First, continue to try and find a suitable polynomial to describe the second surface. This option was not pursued because of the infinite number of trials which would be required to determine if such an analytic expression existed. The second option would be to attempt to optimize a given polynomial by manipulating the coefficients of that polynomial to change the refracting properties of the polynomial such that a single, distinct focal point could be formed. This option was pursued by using Control Program for Engineering Synthesis (COPES) developed by Dr. G.N. Vanderplaats, Naval Postgraduate School. The study was halted because of time constraints. However, initial results concurred with the previously mentioned results of Appendix C.

The third option was provided by an inquiry made by Dr. E.C. Crittenden, Naval Postgraduate School, concerning the capability of the TRACE program to draw rays backward;

i.e. could TRACE begin a set of light rays at a design focal point and draw them through the lens and into the medium ahead of the lens and emerging parallel to the GLM axis? Accomplishing the "backwards" ray trace would design the lens. Conceptually, at least two methods of generating the second surface of a conical lens exist and are available for investigation. The first method consisted of picking the initial point of the second surface to be the GLM axis-intercept of a refracted ray in the lens medium, noting that the ray must be refracted along the GLM axis to pass through the axis. Therefore, the slope of the second surface at the first point must be equal to the slope of the first surface. The derivation of this relationship is performed in Appendix A. The second point on the surface is calculated by extending a straight line by using the slope at the first point to intercept the next ray in the lens. Since the second point is not on the GLM axis, the slope at the second point required to satisfy Snell's Law will not be equal to the slope of the first surface. This process was chosen as the preferred method and is discussed in Chapter III and Appendix A.

The second method of generating a second surface for a conical lens is similar to the first method. The difference is that a parabola is used instead of a straight line to predict the second and succeeding points on the second surface. A different parabola is used for every pair of points. A nonlinear predictor is desired because the shape of the

second surface is nonlinear. The following discussion describes this method of second surface generation for a conical lens.

## B. ITERATIVE SOLUTION

### 1. Derivation of Expressions for $\theta_I$ and $\alpha_2$

The iterative solution is motivated by the ability of a computer to perform a great many calculations in a small amount of time. Referring to figure F-1 for the geometry and symbol definitions, the most significant steps in the iterative solution are:

(1) Initialization:

(a) Choose a focal point on the GLM axis.

(b) Choose an initial lens point T1.

(c) Choose an initial change in $\rho_2$, $\Delta\rho_2$.

(2) Calculate the ray parameters $\theta_I$, $\rho_2$ and $\alpha_2$ for the initial lens point by applying Snell's Law at T1.

(3) Calculate the second lens point T2 by fitting a parabola through T1 and predict the coordinate values of T2. Iterate this prediction until the error is reduced below an acceptable tolerance. The error is defined to be the distance at which T2 intercepts the GLM axis.

(4) Calculate the ray parameters $\theta_I$, $\rho_2$ and $\alpha_2$ for T2.

(5) Continue until the lens has been designed to the GLM axis.

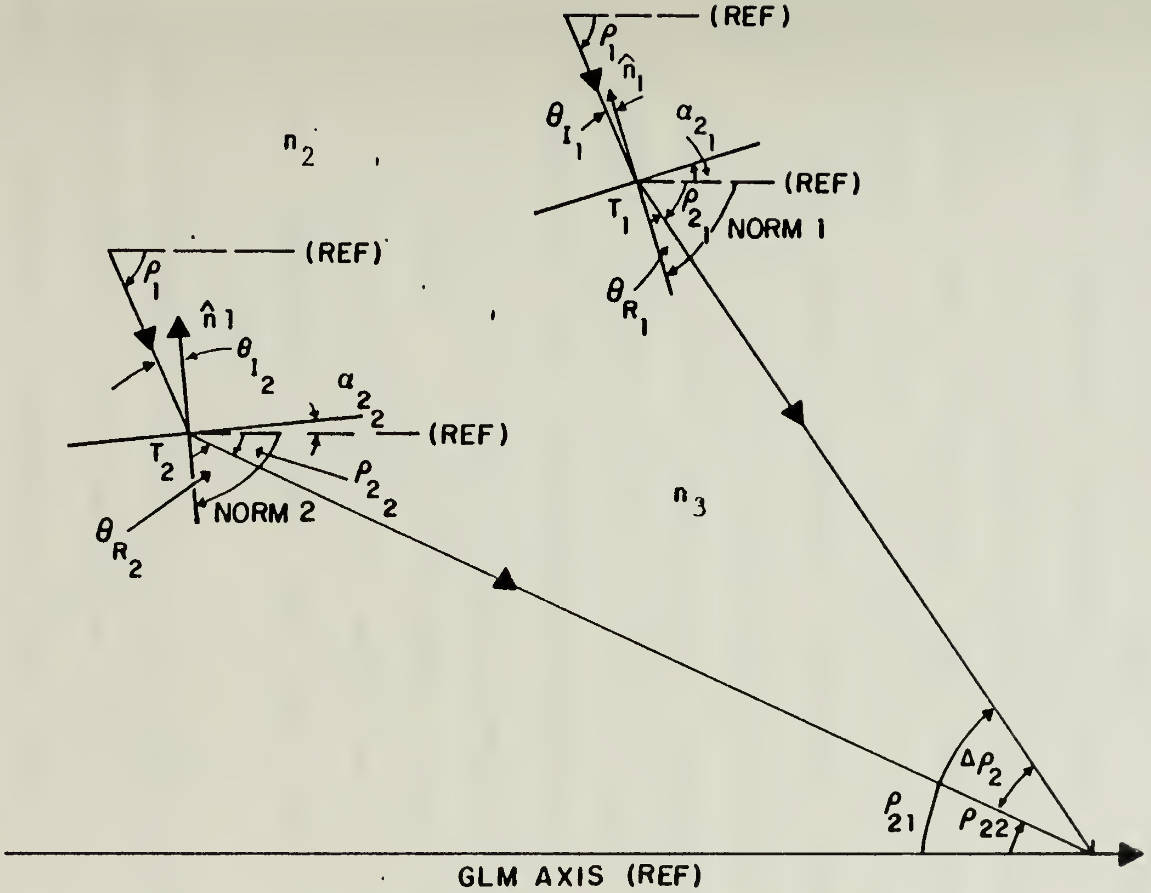


Figure F-1. Geometry and Symbol Definition Used for the Iterative Solution Method of Second Surface Generation.

NOTE: All coordinate values are referenced to a right hand system. All angles are referenced to the GLM axis and are positive for counter-clockwise rotation from the axis to the point in question. The coordinate values (abscissa, ordinate) are referred to as (x,y) with x $\equiv$ the distance along the GLM axis and y $\equiv$ the radius from the GLM axis. The subscripts are either single or double. The single and the first subscript in the double subscripts, the number refers to the surface at which the application of Snell's Law occurred. If the subscript is a letter the subscript refers to the operation of Snell's Law; I for angle of incidence and R for angle of refraction. The second subscript refers to the point number in the lens. For example $\alpha_{2_2}$ is the slope of the second surface at point number two. Any exceptions to this convention for subscripts will be made clear within the context of the discussion.

The investigation consisted of deriving the analytical expressions relating the quantities shown in figure F-2. The primary variables of interest are the angle of incidence $\theta_I$, $\alpha_2$ the angle of the tangent line at T and $\rho_2$ the refracted ray angle in medium 3. The assumptions made in the derivation are that the following quantities are known:

(1) $\rho_1$; the ray angle in the lens medium, measured with respect to the GLM axis.

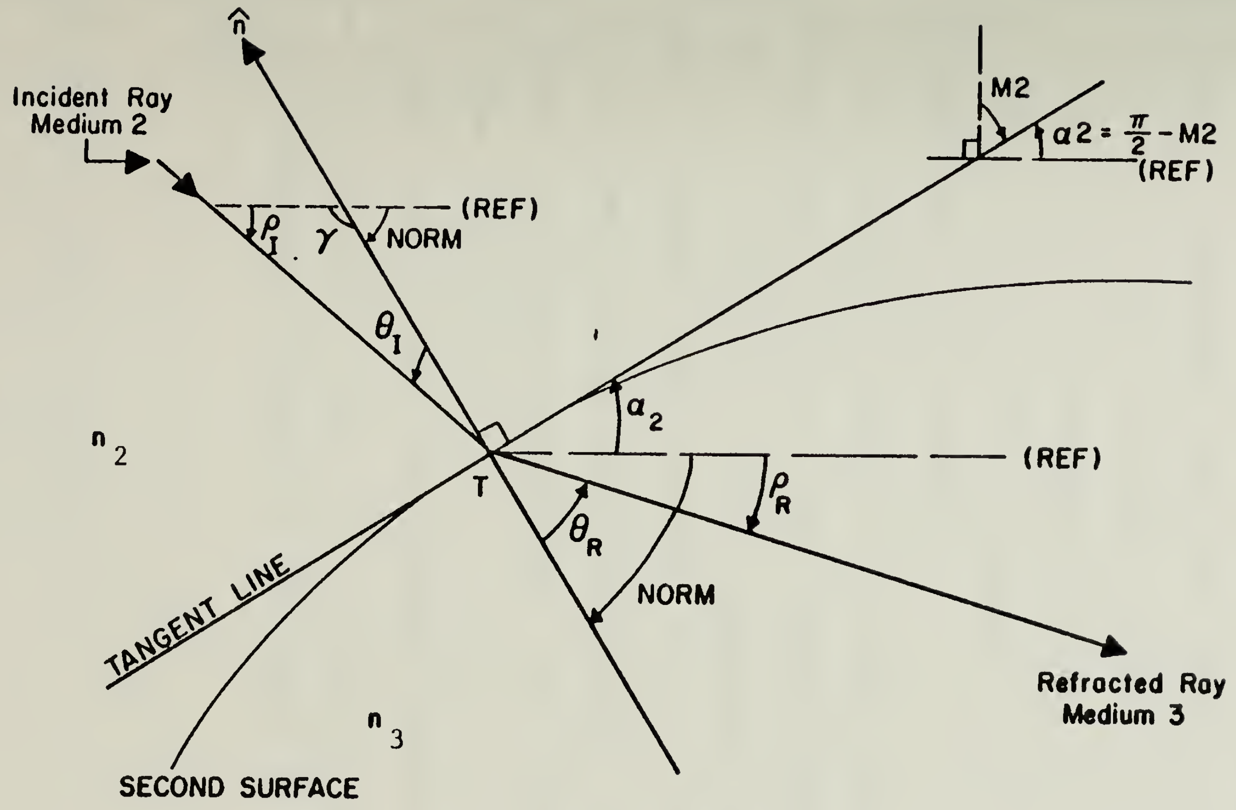


Figure F-2. Geometry and Symbols Used for Snell's Law in the Low Region as Applied to the Iterative Solution Methods for Second Surface Generation.

(2) $\rho_2$; the ray angle in medium 3 measured with respect to the GLM axis.

(3) $n_2$; the index of refraction of the lens medium.

(4) $n_3$; the index of refraction of medium 3.

(5) f; the design focal point on the GLM axis.

(6) (x,y); the coordinate values of the Point T.

(7) the ray is transmitted without loss in all media.

(8) the ray is either refracted or totally reflected at the boundary of two media.

As discussed in Chapter III there exist two regions where the refracted ray in the lens can be refracted at point T. Point T is located in the low region as defined by the area below a line QP shown in figure F-3. In the low region $|\rho_1|$ is always greater than or equal to $|\beta|$. Point T is located in the high region when $|\rho_1|$ is less than $|\beta|$ as shown in figure F-4.

Each point on the lens' second surface must satisfy Snell's Law $\sin\theta_R = n_2/n_3 \sin\theta_I$ or be totally reflected at T. Therefore referring to figure F-2 the angle of incidence in the low region is

$$\theta_I = \left(\frac{\pi}{2} - \alpha_2\right) - |\rho_1| \qquad \text{(F-1)}$$

and the ray angle in medium 3 is

$$|\rho_2| = \frac{\pi}{2} - (\theta_R + \alpha_2) \qquad \text{(F-2)}$$

Substracting equation (F-2) from (F-1) and substituting Snell's Law for $\theta_R$ yields

$$\theta_I = |\rho_2| - |\rho_1| - \arcsin\left[\frac{n_2}{n_3}\sin\theta_I\right] \qquad \text{(F-3)}$$

Now $\alpha_2 = \frac{\pi}{2} - (|\rho_2| - \theta_R)$ hence

$$\alpha_2 = \frac{\pi}{2} - \left[|\rho_2| + \arcsin\left(\frac{n_2}{n_3}\sin\theta_I\right)\right] \qquad \text{(F-4)}$$

Equation (F-3) is a transcendental equation in $\theta_I$ and therefore can be solved graphically or by numerical methods. Therefore, equations (F-3) and (F-4) must both be solved for each iteration of $\theta_I$ until a satisfactory value of $\alpha_2$ is determined. Experience from using the CHART algorithm shows that an initial guess of $\theta_I = \theta_c - 2^{\circ}$ should allow the algorithm to converge quickly. The critical angle $\theta_c$ is the angle of incidence at which $\sin\theta_R = 1$. Changing equations (F-3) and (F-4) into a form suitable for a numerical algorithm yields

$$\theta_{I,n+1} = |\rho_2| - |\rho_1| + \arcsin\left[\frac{n_2}{n_3}\sin\theta_{I,n}\right] \qquad \text{(F-5)}$$

$$\alpha_{2_{1}}{}_{n} = \frac{\pi}{2} - \left[|\rho_2| + \arcsin\left(\frac{n_2}{n_3}\sin\theta_{I,n+1}\right)\right] \qquad \text{(F-6)}$$

for the angle of incidence and angle of the tangent line at Point T in the low region.

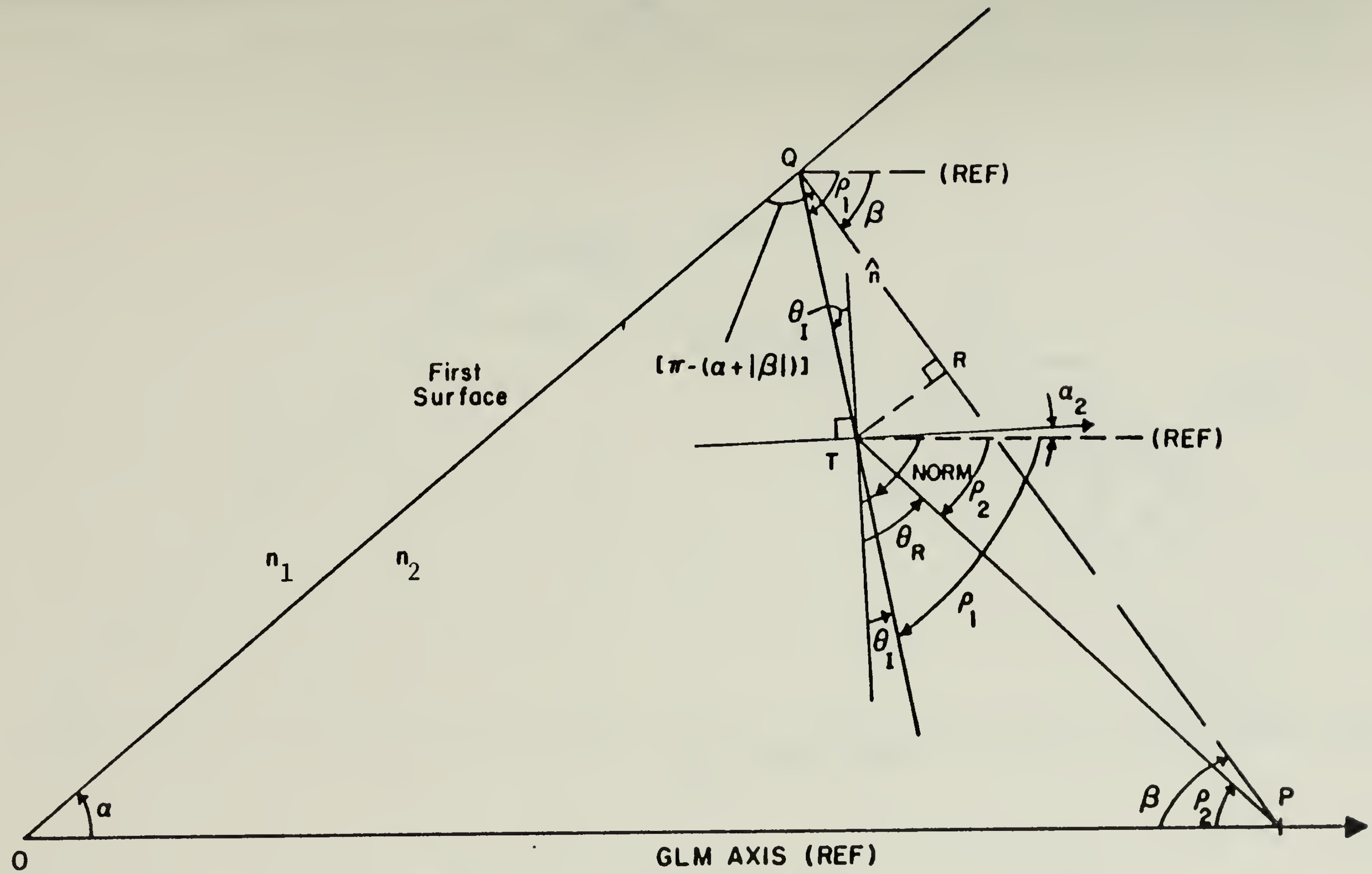


Figure F-3. Geometry and Symbols Used for Snell's Law in the Low Region as Applied to the Iterative Solution Method for Second Surface Generation.

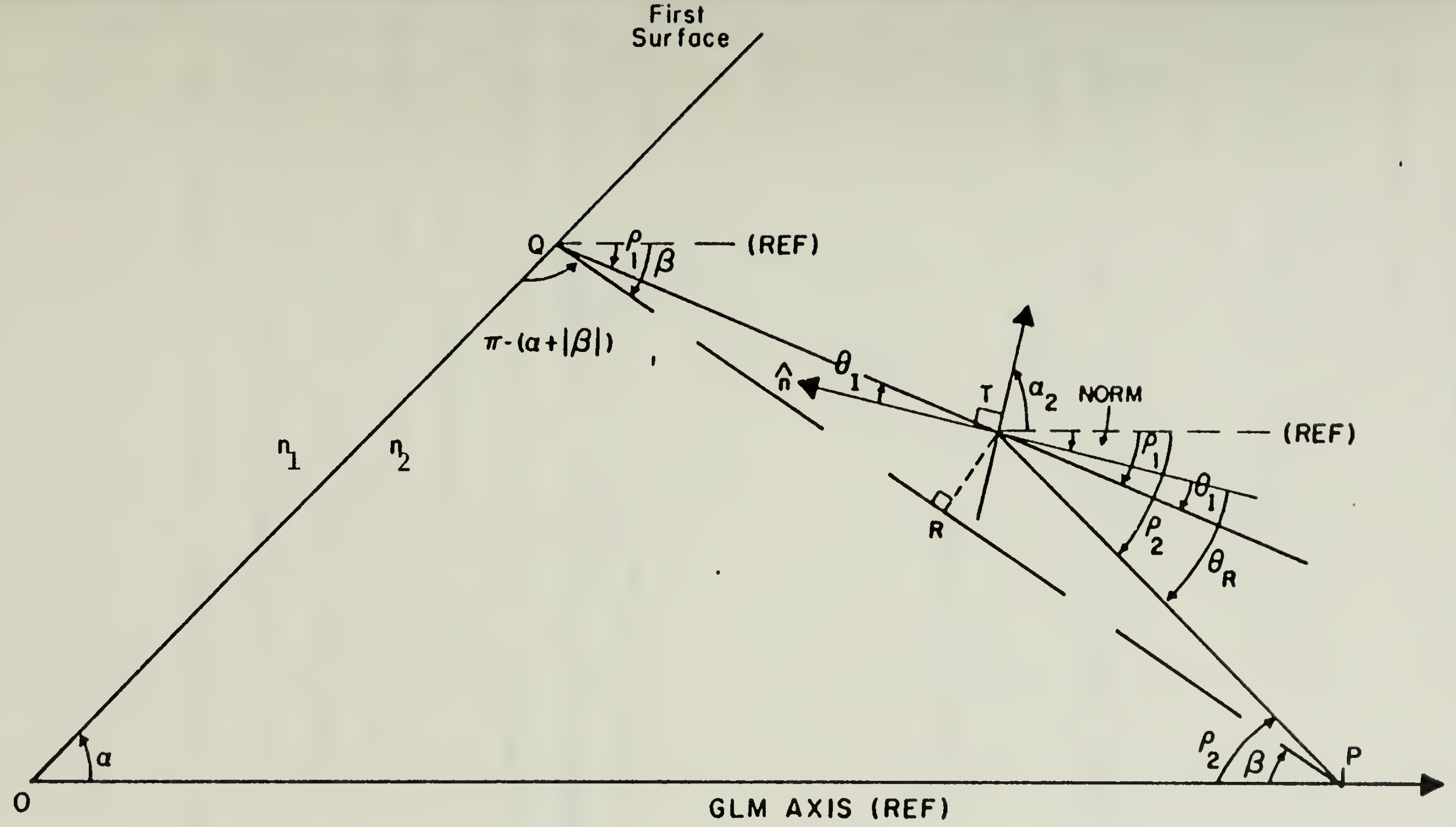


Figure F-4. Geometry and Symbols Used for Snell's Law in the High Region as Applied to the Iterative Solution Method for Second Surface Generation.

As shown in figure F-4 the angle of incidence in the high region is

$$\theta_I = |\rho_2| - |\rho_1| \tag{F-7}$$

and the ray angle in medium 3 is

$$\rho_2 = \frac{\pi}{2} - \alpha_2 + \theta_R \tag{F-8}$$

Adding equations (F-7) and (F-8) and substituting Snell's Law for $\theta_R$ yields

$$\theta_I = \frac{\pi}{2} - (|\rho_1| + \alpha_2) + \arcsin\left(\frac{n_3}{n_2}\sin\theta_I\right) \tag{F-9}$$

now $\alpha_2 = \frac{\pi}{2} - |NORM|$

hence

$$\alpha_2 = \frac{\pi}{2} - |\rho_1| + \theta_I \tag{F-10}$$

Equation (F-9) is a transcendental equation in $\theta_I$ and must be solved graphically or by numerical methods. Therefore equations (F-9) and (F-10) must be solved for each iteration of $\theta_I$ until a satisfactory value of $\alpha_2$ is determined. From the geometry in figure F-4 an initial guess for $\theta_I = 2\,|\rho_1|$ should cause fast convergence to the desired value of $\alpha_2$. Changing equations (F-9) and (F-10) into a form suitable for numerical applications yields

$$\theta_{I,n+1} = \frac{\pi}{2} - (|\rho_1| \quad \alpha_{2\,{}_{1}n}) + \arcsin\left(\frac{n_3}{n_2}\sin\theta_{I,n}\right) \tag{F-11}$$

$$\alpha_{2,n+1} = \frac{\pi}{2} - |\rho_1| + \theta_{I,n+1} \tag{F-12}$$

for the angle of incidence and angle of the tangent line at Point T in the high region.

### 2. Predicting Succeeding Points on the Second Surface

The iterative solution method must involve a regression routine to use a parabola to predict the next point on the second surface. The regression routine solves a set of simultaneous equations consisting of the equation of the parabola and first derivative at $T_1(x_1,y_1)$ and $T_2(x_2,y_2)$ in the form

$$\begin{aligned} a + bx_1 + cx_1^2 &= y_1 \\ b + 2cx_1 &= y'_1 \\ a + bx_2 + cx_2^2 &= y_2 \\ a + 2cx_2 &= y'_2 \end{aligned} \qquad \text{(F-13)}$$

The set of equations is nonlinear in x, but is linear in the coefficients. Therefore, the techniques of linear algebra can be used on equation (F-13). The method used here is the Gaussian-Jordan row-reduction technique discussed in Anton [10]. The first step is to form the augmented matrix whose determinant is set equal to zero

$$\begin{vmatrix} 1 & x_1 & x_1^2 & y_1 \\ 0 & 1 & 2x_1 & y'_1 \\ 1 & x_2 & x_2^2 & y_2 \\ 0 & 1 & 2x_2 & y'_2 \end{vmatrix} = 0 \qquad \text{(F-14)}$$

The row-reduction technique diagonalizes the first three columns of determinant (F-14) with, in general, non-zero quantities in the fourth column in the form

$$\begin{vmatrix} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & C \\ 0 & 0 & 0 & \Delta \end{vmatrix} = 0 \qquad \text{(F-15)}$$

Since four equations have been used to determine the three coefficients a, b and c the determinant is over-specified and nonhomogeneous. Utilizing the fact that the expression represented by D at location (4,4) in (F-15) is equal to zero

$$D = x_1y_2' - x_1y_1' + x_2y_2' - x_2y_1' - 2y_2 + 2y_1 = 0 \qquad \text{(F-16)}$$

if $x_2 \neq x_1$, which is reasonable, since the problem demands that $x_2 \neq x_1$. Simplifying equation (F-16) and solving for

$\frac{y_2 - y_1}{x_2 - x_1}$ yields

$$\frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{Y_2^1 + Y_1^1}{2} \qquad \text{(F-17)}$$

Equation (F-17) states that the slope of the straight line connecting T1 and T2 in figure F-5 is the average of the slope of the parabola used to predict T2 evaluated at T1 and T2. This result is very simple, yet provides a tool to use in predicting the value of T2. Experience gained in using CHART has shown that the second surface of the conical lens is a smooth convex surface with no discontinuities; compare

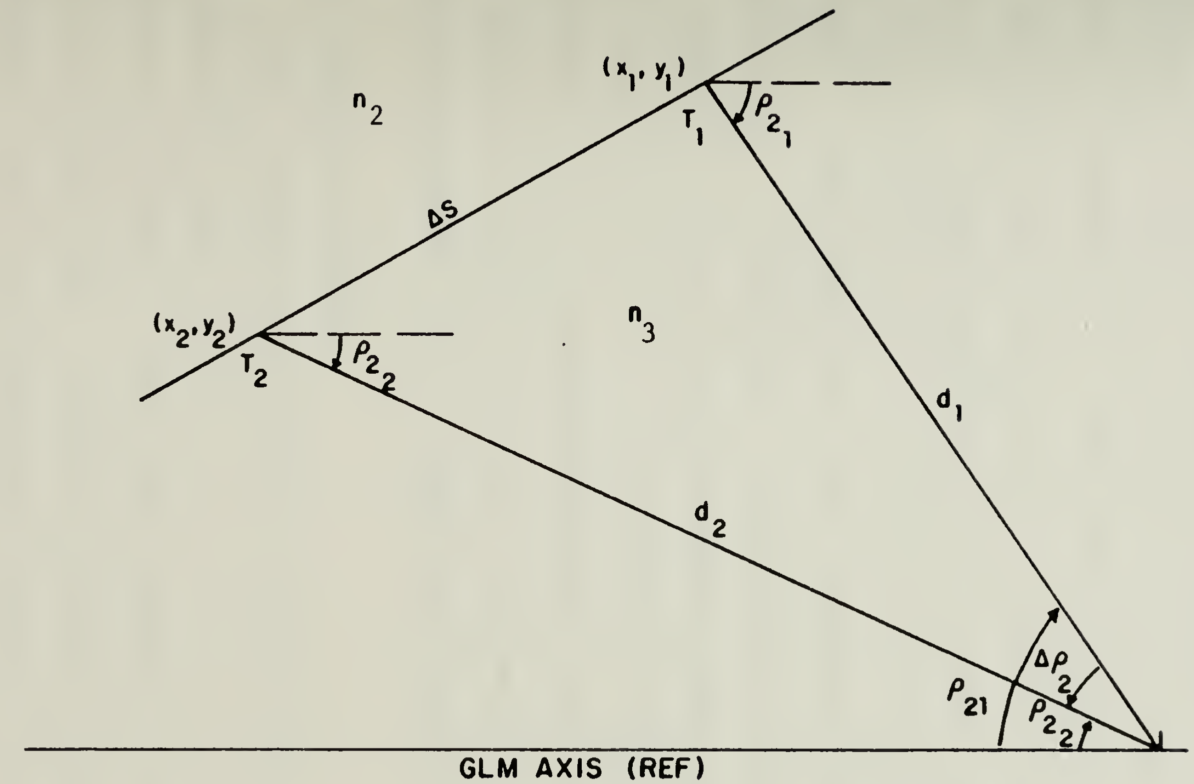


Figure F-5. Illustration of the Result of the Solution of the Simultaneous Equations Used to Predict Point T2 on the Second Surface Using the Iterative Method.

figures 14 and 20 in Chapter III. The exact shape of the second surface is a function of the cone half-angle $\alpha$, and the indices of refraction $n_1, n_2$ and $n_3$. The first guess of where T2 is generated by coordinate values of the intersection of the ray in the lens and the ray in medium 3 is shown in figure F-1. The set of equations (F-13) is solved using $(x_1, y_1)$ and $(x_2, y_2)$ to find $(\Delta Y/\Delta X)$ in equation (F-17). Since the slope at T1 is known the next guess of the actual location of T2 is aided by the experience of using CHART. The slope at T2 will increase as the first surface is approached. Therefore, calculating the slope of the line connecting T1 and T2 and comparing that value with the value from equation (F-17) will determine whether to increase or decrease $\Delta\rho_2$ to predict the next position of $T_2$. This procedure is continued until the coordinate values of T2 are determined within a specified tolerance.

## C. NEWTON'S METHOD

The recommended numerical method used to solve the transcendental equations for $\theta_I$ and $\alpha_2$ in section B-1 is Newton's method. A detailed explanation of the technique is not provided here. Scores of texts on applied numerical methods exist and should be consulted for further study.

Newton's method is simple and converges quickly if the function does not have any of the characteristics illustrated in figure F-6. If the function describing $\theta_I$ exhibits a local minimum in the range of interest, Newton's method will

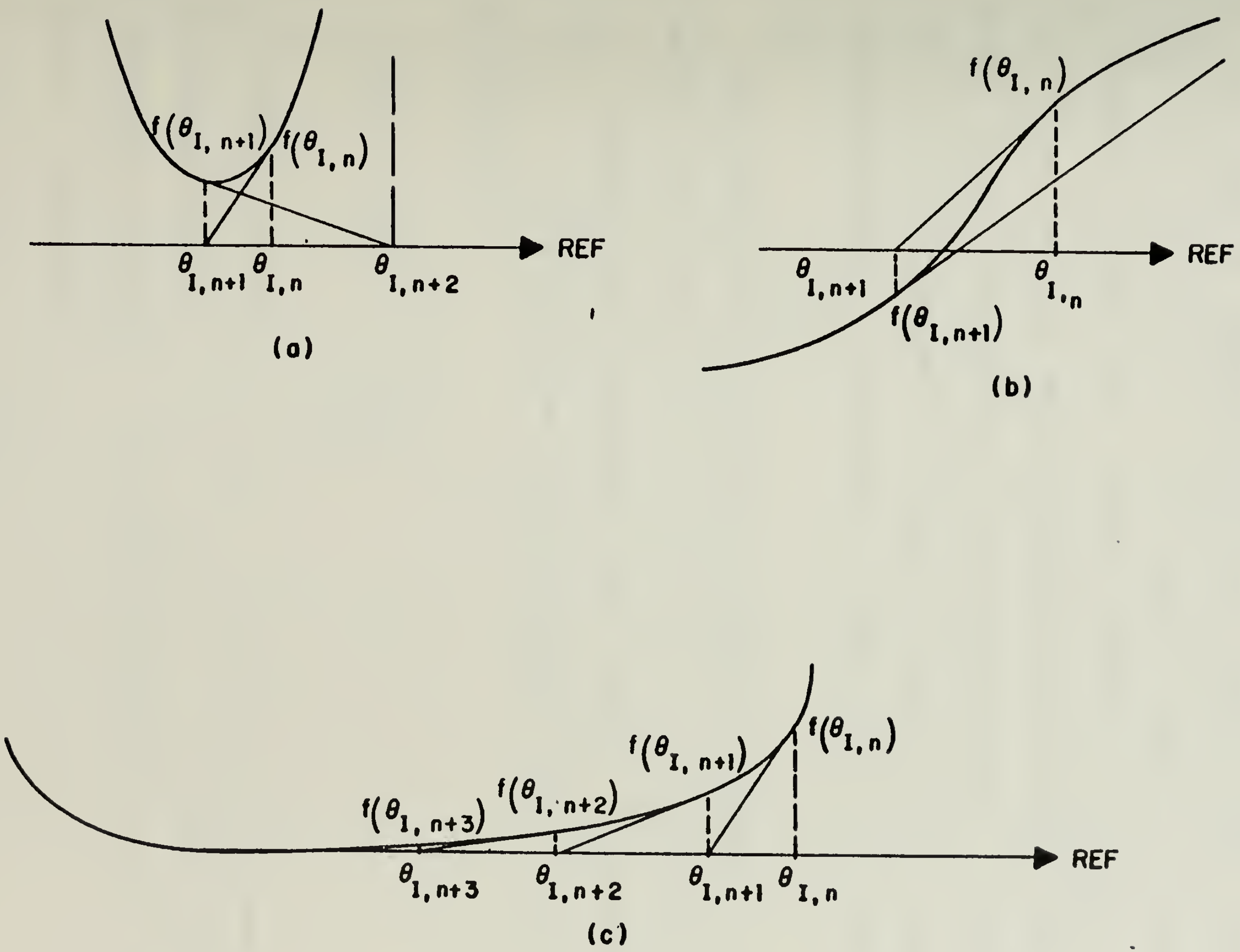


Figure F-6. Three undesirable Characteristics in Functions $f(\theta_I)$ Which Cause Newton's Method to Diverge.

diverge as illustrated in figure F-6a. If the function exhibits an inflection point, Newton's method will diverge as shown in figure F-6b. If the function has multiple roots in the range of interest, Newton's Method cannot determine which value is correct. The special case of a multiple root is shown in figure F-6c. Newton's method will approach the roots from one side then overshoot and oscillate back and forth, never converging on the desired value. A function which is "well behaved" is shown in figure F-7; compare the characteristics of the curve in figure F-7 and those of figure F-6.

The application of Newton's method as shown in figure F-7 involves the selection of a value of $\theta_I$ as the first guess $\theta_{I_1}$, finding the functional value $f(\theta_{I_1})$ and the slope $f'(\theta_{I_1})$. The prediction of the correct value of $\theta_{I_2}$ is via the relation

$$\theta_{I_2} = \theta_{I_1} - \frac{f(\theta_{I_1})}{f'(\theta_{I_1})} \tag{F-18}$$

The process is repeated until the difference between two succeeding values of $\theta_I$, $|\theta_{I_n} - \theta_{I_{n-1}}|$ is less than an acceptable tolerance. Transforming equation (F-18) into general form yields

$$\theta_{I_{n+1}} = \theta_{I_n} - \frac{f(\theta_{I_n})}{f'(\theta_{I_n})} \tag{F-19}$$

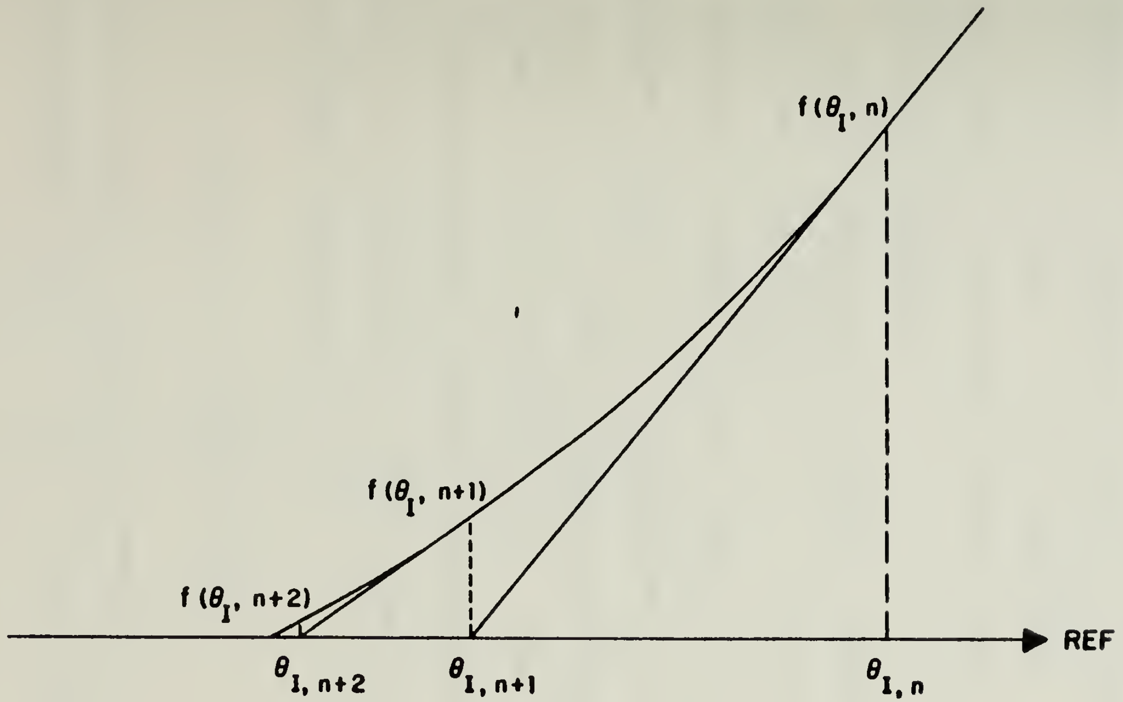


Figure F-7. A Desirable Function $f(\theta_I)$ to Use with Newton's Method.

# APPENDIX G

## LIMIT PROGRAM DESCRIPTION AND LISTING

The derivation of the relationship between the slopes encountered of the first and second surfaces of a GLM lens composed of homogenous material by the ray shown in figure G-1 is the subject of this appendix. The derivation is applicable to the case in which total reflection occurs at the second surface.

The incident ray in medium 1 forms an angle $\theta_{I_1}$ with the first surface normal $\hat{n}$, at point T1 and is refracted according to Snell's Law $\sin\theta_{R_1} = (n_1/n_2\sin\theta_{I_1}$. The first surface normal $\hat{n}$, is defined as the normal to the tangent line at T1. The angle which the tangent line makes with respect to the GLM axis is $\alpha_2$. The ray traverses the lens and intercepts the second surface at T2. The ray is refracted at T2 according to Snell's Law $\sin\theta_{R_2} = (n_2/n_3)\sin\theta_{I_2}$. For total reflection at T2

$$\sin\theta_{R_2} = 1 = \left(\frac{n_3}{n_2}\right) \sin\theta_{I_2} \tag{G-1}$$

Now the angular difference between $\hat{n}_1$ and $\hat{n}_2$ is $(\alpha - \alpha_2)$ and therefore $\theta_{I_2} = \theta_{R_1} + (\alpha-\alpha_2)$ Hence

$$\theta_{R_1} + (\alpha - \alpha_2) = \arcsin\left(\frac{n_3}{n_2}\right) \tag{G-2}$$

The angle with respect to the GLM axis of the tangent line at T2, $\alpha_2$, is then defined by

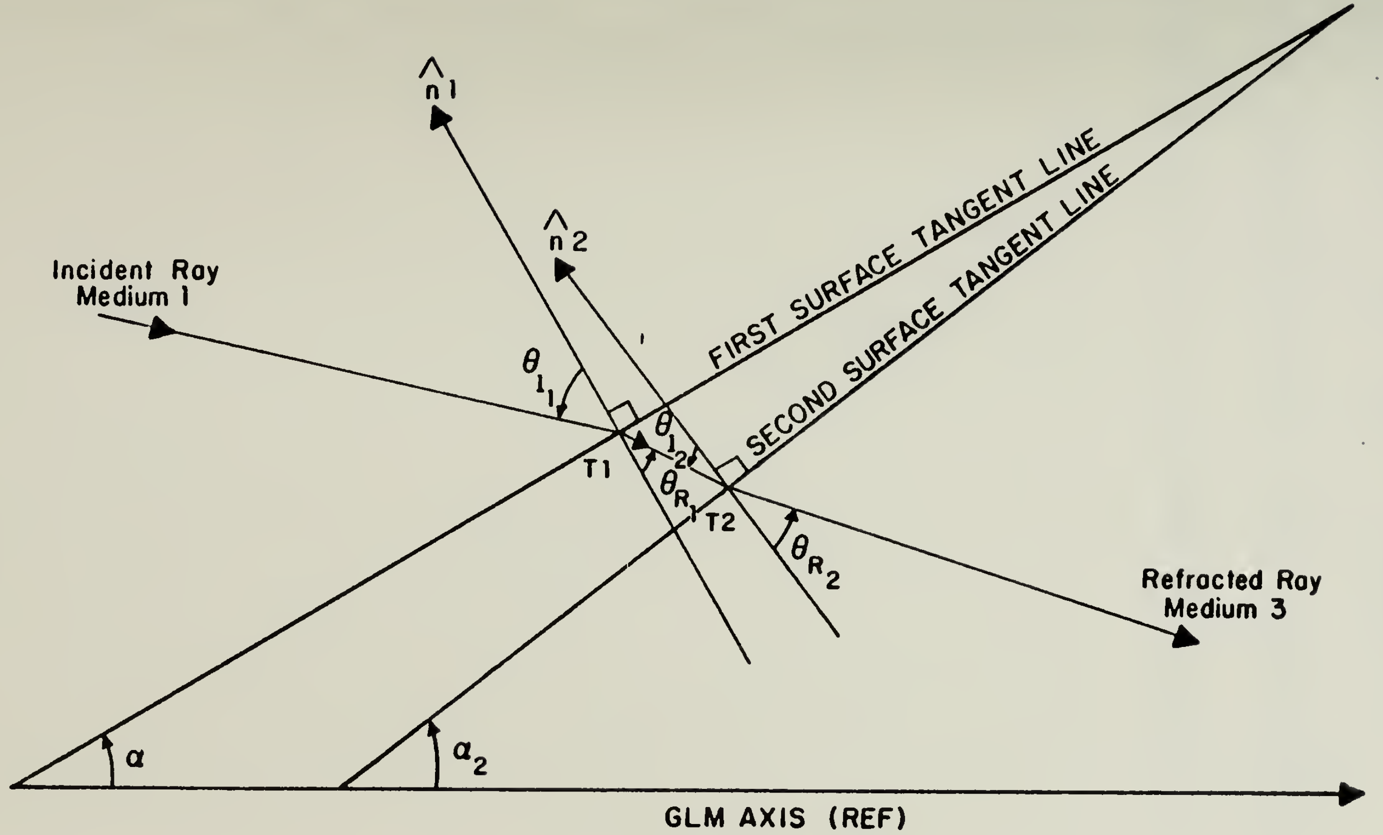


Figure G-1. Geometry and Symbol Definition for the Relationship of the Slope of the Second Surface as a Function of the Slope of the First Surface.

$$\alpha_2 = \arcsin\left(\frac{n_1}{n_2} \sin\theta_{I_1}\right) + \alpha - \arcsin\left(\frac{n_3}{n_2}\right) \qquad \text{(G-3)}$$

Noting that $\theta_{I_1} = \frac{\pi}{2} - \alpha$ then

$$\alpha_2 = \arcsin\left[\frac{n_1}{n_2} \sin\left(\frac{\pi}{2} - \alpha\right)\right] - \arcsin\left(\frac{n_3}{n_2}\right) + \alpha \qquad \text{G-4)}$$

Equation (G-4) is illustrated in figure G-2.

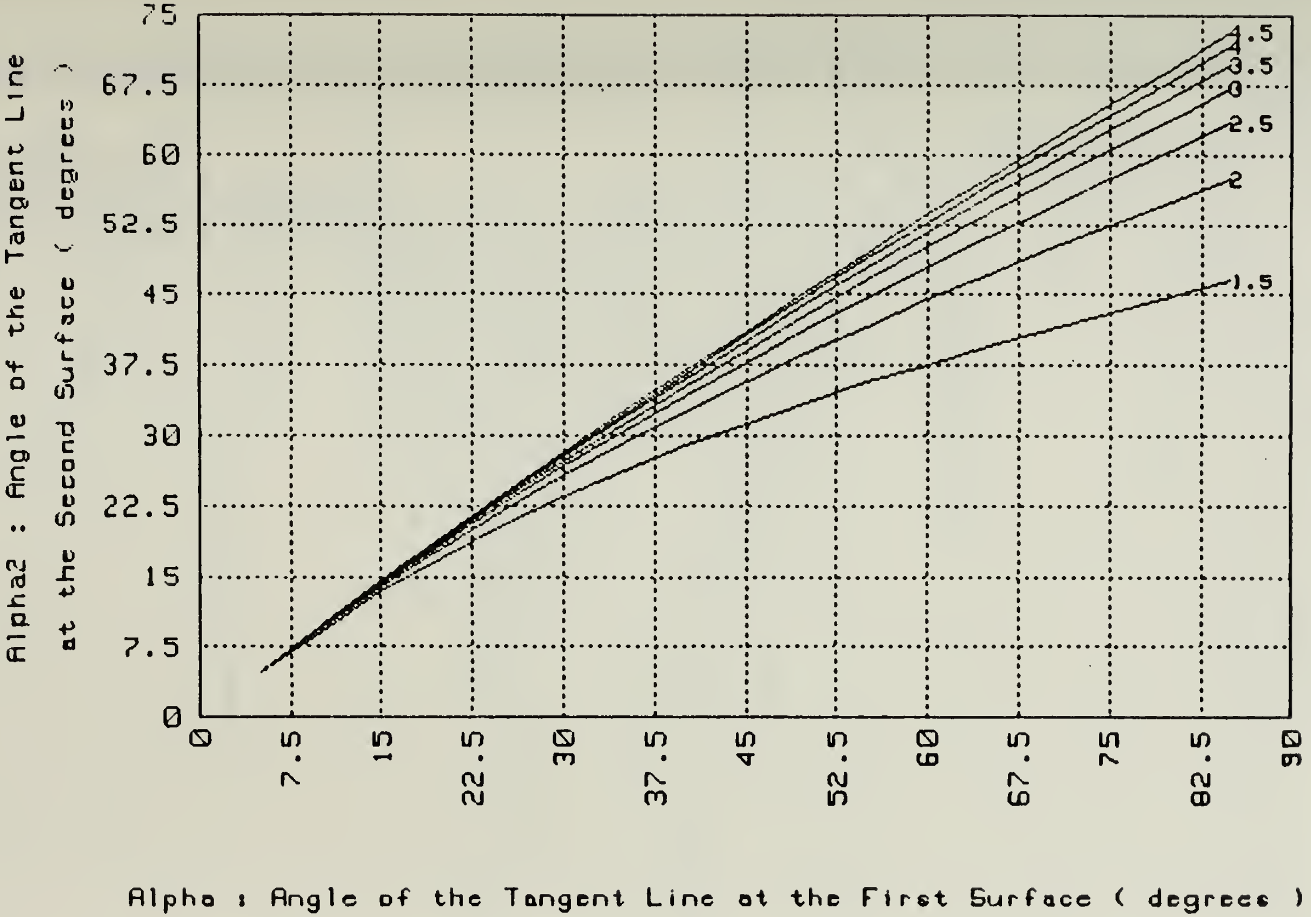


Figure G-2. The Slope of the Second Surface as a Function of the Slope of the First Surface for Various Values of the Lens Index of Refraction and the Case of Total Reflection at the Second Surface.

# LIMIT PROGRAM LISTING

```
10 PRINT "*********************************************************************"
20 PRINT "*                                                                   *"
30 PRINT "*                              LIMIT                                *"
40 PRINT "*                                                                   *"
50 PRINT "*********************************************************************"
60    DISP "THIS IS LIMIT"
70    WAIT 2500
80    OPTION BASE 0
90    INTEGER I,Linecount,Linemax
100   DEG
110   FIXED 2
120   PRINTER IS 16
130   Linecount=0
140   Linemax=48
150   N1=1
160   N3=1
170   INPUT "DO YOU WANT A HARD COPY? Y/N",Y$
180     IF Y$="Y" THEN PRINTER IS 0
190     GOSUB Header
200     CALL Graph
210   FOR N2=1.5 TO 4.5 STEP .5
220     Alpha=5
230     Thetai_1=90-Alpha
240     Sin_thetar1=N1/N2*SIN(Thetai_1)
250     Thetar_1=ASN(Sin_thetar1)
260     Alpha2=Alpha+ASN(N1/N2*SIN(90-Alpha))-ASN(N3/N2)
270     Thetai_2=Thetar_1+Alpha-Alpha2
280       IF Linecount>Linemax THEN GOSUB Header
290     PRINT USING 410;N2,Thetai_1,Thetar_1,Thetai_2,Alpha,Alpha2
300     Linecount=Linecount+1
310       MOVE Alpha,Alpha2
320      FOR Alpha=5 TO 85 STEP 5
330        Thetai_1=90-Alpha
340        Sin_thetar1=N1/N2*SIN(Thetai_1)
350        Thetar_1=ASN(Sin_thetar1)
360        Alpha2=Alpha+ASN(N1/N2*SIN(Thetai_1))-ASN(N3/N2)
370        Thetai_2=Thetar_1+Alpha-Alpha2
380       IF Linecount>Linemax THEN GOSUB Header
390        PRINT USING 410;N2,Thetai_1,Thetar_1,Thetai_2,Alpha,Alpha2
400       Linecount=Linecount+1
410        IMAGE 5X,DD.DD,5(5X,DDDD.DDD)
420        DRAW Alpha,Alpha2
430      NEXT Alpha
440      LDIR 0
450      LORG 2
460      CSIZE 2.5
470      LABEL USING 480;N2
480      IMAGE K
490        IF Linecount>Linemax THEN GOSUB Header
500      PRINT
510      Linecount=Linecount+1
520    NEXT N2
530      GOSUB Header_end
540    PRINTER IS 16
550    PRINT LIN(2),"PRESS CONT"
560    BEEP
570    WAIT 250
580    BEEP
590    PAUSE
600    EXIT GRAPHICS
610 GOTO Finished
620 Header:   IF Linecount>Linemax THEN GOSUB Header_end
630           IF Linecount>Linemax THEN PRINT PAGE;TAB(28);"TABLE I (CONT)";LIN(
2)
640           IF Linecount<=Linemax THEN PRINT PAGE;TAB(35);"TABLE I ";LIN(2)
650         Linecount=0
660         PRINT LIN(2)
```

```
670           GOSUB Char
680         PRINT LIN(1);SPA(5);"  n2        ThetaI(1)     ThetaR(1)      ThetaI(2)
    Alpha        Alpha2";LIN(1)
690           GOSUB Char
700         PRINT LIN(2)
710       RETURN
720 Char: FOR I=0 TO 79
730           IF I=79 THEN PRINT CHR$(228)
740           IF I=79 THEN 760
750          PRINT CHR$(228);
760       NEXT I
770     RETURN
780 Header_end: PRINT LIN(2)
790               GOSUB Char
800            RETURN
810 Dump_it: PRINTER IS 0
820          PRINT CHR$(27)&"&100T"
830          DUMP GRAPHICS
840          PRINT CHR$(27)&"&136T"
850          PRINTER IS 16
860        RETURN
870 Finished: Dump_crt$="N"
880           INPUT "DO YOU WANT A HARD COPY OF THE PLOT ? Y/N",Dump_crt$
890             IF (Dump_crt$="Y") OR (Dump_crt$="y") THEN GOSUB Dump_it
900           DISP "FINISHED"
910         END
920 SUB Graph
930   OPTION BASE 0
940   DEG
950   !
960   !
970   !  THE EXPRESSIONS USED TO DERIVE THE TIC MARKS WERE OBTAINED FROM THE
980   !  HP-9845B UTILITIES LIBRARY, TAPE NR 09845-10205, PROGRAM "REGPLT"
990   !
1000  !
1010  GCLEAR
1020  PLOTTER IS 13,"GRAPHICS"
1030  LDIR 0
1040  LORG 5
1050  LIMIT 0,184,0,100
1060  !
1070  DATA -2,-1,1,2
1080  READ Um,Dm,Md,Mu
1090  DATA .39794,.69897,.87506
1100  READ Log2,Log5,Log7
1110  !
1120  Xorg=Xmin=Yorg=Ymin=0
1130  Xmax=90
1140  Ymax=80
1150 Alpha: INPUT "WHAT IS THE MAXIMUM VALUE OF ALPHA ( DEFAULT = 90 DEGREES) ?"
,Xmax
1160 Alpha2: INPUT "WHAT IS THE MAXIMUM VALUE OF ALPHA2 ( DEFAULT = 80 DEGREES)
?",Ymax
1170    IF Xmax<=Xmin THEN BEEP
1180    IF Xmax<=Xmin THEN DISP "ALPHA MUST BE > 0. PLEASE RE-ENTER ALPHA."
1190    IF Xmax<=Xmin THEN WAIT 2500
1200    IF Xmax<=Xmin THEN Alpha
1210    IF Ymax<=Ymin THEN BEEP
1220    IF Ymax<=Ymin THEN DISP "ALPHA2 MUST BE > 0. PLEASE RE-ENTER ALPHA2."
1230    IF Ymax<=Ymin THEN WAIT 2500
1240    IF Xmax<=Xmin THEN Alpha2
1250 GRAPHICS
1260  !
1270  LIMIT 0,184,0,140
1280  LOCATE 20,132,28,100
1290  Lx=LGT(Xmax-Xmin)
1300  Ly=LGT(Ymax-Ymin)
```

```
1310  Xfudge=.02*(Xmax-Xmin)
1320  Yfudge=.02*(Ymax-Ymin)
1330 Ticmarks: Testxtic=FRACT(Lx)+(Lx<0)
1340           Testytic=FRACT(Ly)+(Ly<0)
1350           Xtic=10^(INT(Lx)-1)*(1+1.5*((Testxtic>Log2) AND (Testxtic<Log5))+
4*((Testxtic>=Log5) AND (Testxtic<=Log7))+6.5*(Testxtic>Log7))
1360           Ytic=10^(INT(Ly)-1)*(1+1.5*((Testytic>Log2) AND (Testytic<Log5))+
4*((Testytic>=Log5) AND (Testytic<=Log7))+6.5*(Testytic>Log7))
1370 Scale: SCALE Xmin,Xmax+.25*ABS(Xtic),Ymin,Ymax+.25*ABS(Ytic)
1380         CLIP Xmin,Xmax,Ymin,Ymax
1390        LINE TYPE 3
1400        Xmaj=Ymaj=1
1410        Minticsize=4
1420        GRID Xtic,Ytic,Xorg,Yorg,Xmaj,Ymaj,Minticsize
1430        LINE TYPE 1
1440        FRAME
1450 Labelx: LDIR 90
1460         LORG 8
1470         FOR A=Xorg TO Xmax STEP ABS(Xtic)
1480             MOVE A,Yorg-Yfudge
1490           LABEL USING 1570;A
1500         NEXT A
1510 Labely: LDIR 0
1520         LORG 8
1530         FOR A=Yorg TO Ymax STEP ABS(Ytic)
1540             MOVE Xorg-Xfudge,A
1550           LABEL USING 1570;A
1560         NEXT A
1570 IMAGE #,K
1580  LABEL USING 1590;""
1590  IMAGE /,K
1600 Label_: SETGU
1610         LDIR 0
1620         LORG 5
1630         Centerx=72
1640         Centery=64
1650         CSIZE 3
1660         MOVE Centerx,10
1670         LABEL "Alpha : Angle of the Tangent Line at the First Surface ( deg
rees )"
1680         LDIR 90
1690           MOVE 1.45,Centery
1700         LABEL "Alpha2 : Angle of the Tangent Line"
1710           MOVE 6.45,Centery
1720         LABEL "at the Second Surface ( degrees )"
1730         LDIR 0
1740         CSIZE 15/4.54
1750         SETUU
1760       SUBEXIT
```

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